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AN APPLICATION OF CONTROL THEORY TO SEASONAL
FOOD SECURITY

BY

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1 Background

In spite of many efforts undertaken by national governments and international agencies food security is still a major problem in many developing countries of the world especially in Sub-Saharan Africa. The overall discussion of this important issue continues on different levels. For example, in the case of the SADCC countries, Hay and Rukuni have indicated that most recently the focus has been on consumption growth and stabilization objectives and policies to achieve the objectives (Hay and Rukuni, 1988, p.1020). However, a review of the development of this issue would bring one to the conclusion that until now most of the efforts to achieve food security have been done in the design of a general food policy in a particular country or region. In contrast, this article deals with the specific design of an operational policy to pursue food security introducing seasonal stabilization scheme. The objective of this approach is to provide an analytical tool of seasonal planning in which import parity prices and variations in seasonal consumption requirements simultaneously determine optimal monthly releases from post harvest stocks, imports and hence seasonal consumption patterns. The general idea is to provide guidance for monthly decisions on such importance variables due to several constraints.

Furthermore, assuming government intervention in agricultural markets, parastatals need instructions on a quantitative basis. And since government authorities dealing with these problems feel insecure about the quantitative design of market intervention, economically based recommendations are urgently needed. Presumably, the planning problem emerges, when import and export parity prices diverge widely as it is the case in partly isolated markets in many regions of the world due to high transport costs and transfer problems. Furthermore, it is assumed that only a limited budget of foreign currency is available to spend on imports regarding the whole season. This assumption coincides with the practice of many countries whose government runs a foreign currency allocation policy.

The article is structured as follows: First, there will be given an introduction in the

problem. Second, a formalization of the issue will be conducted using mathematical tools. Third, the model will be solved and at least a fictive empirical application will be presented. The analytical result will be applied to a representative example for a typical food deficit country.

2 Introduction to the problem of seasonal determination of consumption, stockpiling and trade

The justification for the more narrowly formulated operational policy discussed below emerges from the fact that many countries are not able or willing to change their agricultural policy globally towards a more liberalized agricultural sector as is often recommended. Approaches similarly characterized by operational policy objectives have been conducted by Pinckney (1988) and Pinckney/Valdes (1988, p 1025–1034) regarding some institutional settings as given. However, they are operating on an annual decision making time framework which may yield different results from a model in which seasonal demand variations call for different trade and/or stock behavior at different times within the year. The former studies' contribution to this article can be seen mainly in the introduction of storage activities into operational policies.

Moreover, in a case study of Madagascar by Shuttleworth, Bull and Hodgkinson (1988 p. 140–153) one can see how severe misplanning in the seasonal delivery of food to the market can impair food security in a seasonal context. In this study the authors demonstrate that poorly balanced stock release and imports of rice —the major staple food in Madagascar — have resulted in a very heavy shortage in some periods of the year. Since that case study additionally involves a structural adjustment component, and it can be expected that many countries are going or will go through a similar process, a well designed operational policy can crucially contribute to the success of broader policy objectives. Especially, optimal

seasonal management in deliveries to the market and its balancing between imports and stock release can be a prerequisite for the success of the introduction of an austerity policy in many countries. Alternatively, occasional seasonal food shortages can result in political unrest due to almost no deliberate policy or badly designed policy as proved in so many cases.

More specifically, one has to keep in mind that food security policies operate under several limitations which can not be neglected. For example, many African governments keep a marketing authority in charge of the supply of major staple food products to the consumers. Given that limitations, then, in many cases a parastatal is confronted with the problem of planning post harvest deliveries to the market on a monthly basis which could come only from two sources, the release from internal stocks and imports from external sources (Lele and Candler, 1984, p. 207–201). Hence, on one side the approach which is considered in this article takes the institutional framework in agricultural market policy as given in many African countries. That comprises a monopolistic parastatal which controls stocks and imports of a particular commodity in that particular country. On the other side the achievement of a good operational policy in the seasonal management of food deliveries can be a prerequisite for the implementation of a general policy change towards a greater market orientation. This will alleviate the stress to consumers emerging from economic adjustments.

3 Economic problems involved in seasonal planning and formalizing post harvest decisions

The starting assumption of this article is provided by a given harvest and stock level which is under control of a parastatal. This parastatal needs some planning recommendations which leads to an optimal solution of seasonal planning in releases from stock and imports due to a number of constraints. Furthermore, it will be assumed that the objective of such

a parastatal should be to maximize consumer surplus as deviations from an overall fixed static condition. The static condition is initially defined on a year to year basis and might be determined by other goals which are of no interest in this study. Hence, it is not the objective of this approach to make recommendations towards a design of an overall price policy which should prevail in the long run. Presumably, in a seasonal context the average internal seasonal price level is regarded as given. The question of price policy discussed in this approach reduces to seasonal movement of food prices.

However, to come to the seasonal problem of formally planning imports, mostly one has to model foreign currency limitations in order to meet import requirements within one season in way of an additional constraint. In other words a limited budget in foreign currency has already been allocated to the parastatal. The question where it comes from shall be beyond this paper. Planer's problems shall only be as followed: The budget has to be broken down in import contracts over a 12 month period. Of course, this type of approach assumes that the economic planner cannot cope with a social welfare function in a pure neoclassical sense. His planning possibilities are predetermined by overall budget planning as often found in developing countries' government decisions on trade¹.

Again, in the context in which seasonal planning is of relevance, for example, the Ministry of Finance normally allocates only a specified amount of foreign currency for purpose of imports to a particular crop in its global planning. This amount of money can not be exceeded at all.

3.1 Planning problems

Of course, considering all restrictions mentioned above would even present no problems in

¹Perhaps some economist dislike this. However, it is very often the reality with which a planner is confronted in a developing country, and he needs some assistance.

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planning, if there are no stocks from internal harvest which deteriorate with a specific rate, no cost of money holding, progressively increasing storage costs, no variation in consumption needs and an unchanging world market price. The result of planning would be simply result in an equal distribution of consumption in each month. Moreover, as Koester (1982, p.65) has shown in the EC-case of wheat exports which is similar to our import problem in its general structure, one would require no seasonal planning import policy at all. In that special case it merely would be necessary to find the month with cheapest import parity prices; when the whole seasonal requirement should be imported.

In order to apply that findings to our problem, the marketing authority would just be recommended to import the whole need in the last months when they are running out of stocks or as already mentioned the cheapest months. However, if the assumptions of that particular case are not guaranteed the problem becomes more complicated. Then the economic planner of a staple food crop has to balance stock extraction and imports due to the relevant economic costs on a monthly basis. For example, he has to be aware of the fact that his stockpiling is increasingly costly and stock deterioration takes place to the end of the season at a certain rate. Mainly, he wants to know how long should the market be supplied by food from own stocks and when should stock extraction be supplemented by additional imports. Furthermore, each month's individual decision on stock release depends on the forgoing decision because previous stock release behavior restricts new decisions.

This is similar to imports. Since, as already mentioned, most developing countries run a foreign currency allocation policy by their reserve bank instead of liberal imports depending on world market prices multiplied by the exchange rate, imports shall be restricted to a certain amount of foreign currency. To stress the planning implications of that budget restriction which means in a time dependent context, that foreign currency already spent in a previous month cannot be used any longer in a month when more severe shortage might arise, we have to look at planner's problem again. If he takes not in consideration the fact of a limited budget in his early decisions of a particular season at least there will be no

more money available in months prior to the new harvest. This fault has mainly contributed to severe shortages in some countries in the end of a particular season. Moreover, many importing countries in the developing world which face expensive food imports must cope with an import parity price far above internal prices because of high transport costs to their major consumption areas. This is well reported by Kingsbury (1989, p. 259–276), for example, in the SADCC region. That, of course, creates the necessity of planning imports in a partly isolated market and is one of the basic justifications of this approach. Hence, this approach tries to combine internal stockpiling policies and trade policies due to constraints like high transport cost, foreign currency shortages and allocations. In other words the approach demonstrates how seasonal trade fits into the planning of stock release.

In the context of the financial constraint let's again refer to the necessity of a well designed operational policy in order to guarantee broader policy. In some cases the financial constraint of imports actually is part of required adjustment policy from IMF since food imports contribute largely to balance of payments deficits. Hence, the approach applied here is not only of relevance from the point of view of an importing country but could be included in some external policy recommendations to pursue the sustainability of such policies. Then the import bill is fixed not only by the ministry of finance but additionally limited by external adjustment requirements in conjunction with agreed monetary policies.

3.2 Formalizing the issue of simultaneous stock extraction and imports due to the constraints

Formally, the problem constitutes of the maximization of welfare gains from a welfare function. This function contains the consumption level $c(t)$ which is equal to stock reduction in a particular period, and imports available, $m_1(t)$. Both variables directly and

positively affect the welfare function. Furthermore, the welfare function comprises costs which negatively contribute to the final objective of welfare maximization. These are costs of stockpiling due to the amount of staple food in stocks and costs due to inappropriate timing of money expenditure resulting in interest costs and opportunity cost of money due to the money hoarding.

The total amount in foreign currency is treated as fixed and given (3). Hence, it is not directly a part of the welfare function but regarded as a constraint. This emerges from the planner's problem of seasonal planning. Within that planning framework the decision of foreign currency allocation due to the import of a particular product has already been made without contacting our planner². His role as an economist is constrained to this particular problem.

However, since the value function stands for a whole season one has to sum up over 12 months which means in continuous modeling to work with the integral:

$$(1) \quad W(t) = \int_0^{12} \exp(-\rho t) f\{ (c(t)-\bar{c}), (m_1(t)-\bar{m}_1), (S(t)-\bar{S}), (R(t)-\bar{R}) \} dt$$

where:

$W(t)$: Welfare function of the whole period

$\exp(-\rho t)$: Discounting future consumer surplus in continuous case

$c(t)$: monthly delivery of staple food by way of stock release to be planned

$m_1(t)$: monthly imports of close substitute of local staple food to be planned

²It must be stressed, that this approach is not operating with a social welfare function in a pure neoclassical economic sense regardless of any institutional constraints. In contrast, it is the planners task only to deal with the seasonal problem which restricts his degrees of freedom towards imports by way of a given allocation in foreign currency.

$S(t)$: stocks to be planned

$R(t)$: reserves of foreign currency to be spent on future imports inducing opportunity costs

Moreover, the monthly welfare generation must be discounted by $\exp(-\rho t)$.

This objective function can be optimized taking in consideration the movement of stocks

$$(2) \quad S(t) = S(0) - \int_0^t c(\tau) d\tau + \int_0^t \nu m_2(\tau) d\tau - \int_0^t \pi S(\tau) d\tau$$

where:

$S(t)$: stocks at time t (t counted in months)

$S(0)$: initial stocks

$\nu m_2(\tau)$: imports of close substitute to be taken into stocks in order to increase future availability of stocks; ν : blend coefficient

$\pi S(\tau)$: stock deterioration in any particular subperiod

and the movement of money extraction

$$(3) \quad R(t) = R(0) - \int_0^t p^w(\tau) [m_1(\tau) + m_2(\tau)] d\tau$$

where additionally:

$R(t)$: foreign currency reserve at time t (t counted in months)

$R(0)$: initial foreign currency reserve to be spent in purchases of imports

$p^w(\tau)$: import parity price

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the solution requires a fairly complex approach where $p^w(\tau)$ is the import parity price. The appropriate method is control theory which provides a continuous form solution for every specific time period in the season considered. In some sense this approach is similar to Sethi and Thompson's inventory model (1981, p.144). However, some comments must be made on the model in general. First, the model assumes perfect knowledge of price movement on the world market, stock deterioration coefficient and seasonal demand structure in its initial version presented here. But, it could be extended to models incorporating stochastic components, for example using a certainty equivalent approach (Buccola and Sukume, 1987). This especially means the incorporation of a fluctuating world market prices. Second, the annual harvest is given as initial stock. Third, a continuous time model is applied in contrast to a programming approach (Pinckney, 1988 and Pinckney & Valdes, 1988). This provides the advantage of an analytical solution being more flexible with stockpiling in a seasonal approach than those undertaken by Pinckney and Pinckney & Valdes dealing with an annual model. These authors have neglected the seasonal problem discussed in this paper. However, one can regard both decision problems as separate. Nevertheless, annual import amounts in money terms calculated by Valdes and Pinkney can provide information on $R(0)$ required in the model of this paper.

For interpretation of (2) let us use its first derivative in time.

$$(4) \dot{S}(t) = rS(t) - c(t) + \nu m_2(t)$$

where additionally:

$\dot{S}(t)$: change in stocks at a particular subperiod which stands for $S_t - S_{t-1}$ in continuous modeling

This implies that every particular extraction from stocks $\dot{S}(t)$ is equal to the deterioration

of stocks $\pi S(t)$, and the monthly delivery of staple food which is identical with consumption $c(t)$. Furthermore, intakes to stocks from imports $m_2(t)$ making a positive contribution to available stocks must be added. Since this is an intertemporal equation the intake $m_2(t)$ is transferred into stocks and can be used first in next period. In principle this is the same with $S(0)$ in (2) — the initial stock. However it can be used for release in the whole post harvest period but can not be exceeded as a physical constraint for the marketing authority in that particular season investigated. As we will see the solution will be due to this stock. Hence each years pattern of stock release can be derived from information of available stocks.

A similar interpretation holds for (3) and its first derivative in time:

$$(5) \dot{R}(t) = [m_1(t) + m_2(t)] p^W(t)$$

where additionally:

$\dot{R}(t)$: change in money reserves at a particular subperiod which stands for $R_t - R_{t-1}$
in continuous modeling

The first derivative of equation (3) in time (equation 5) expresses the monthly expenditure of money $\dot{R}(t)$ for the two purposes direct consumption from import $m_1(t)$ and intake into stocks $m_2(t)$. It is evaluated at the border parity price $p^W(t)$ which is assumed to be time dependent in our model and might be above internal prices due to transfer costs. However, if one summarizes overall periods by integration the money available $R(0)$ cannot be exceeded.

4 Model building and solution procedure

The objective function and the constraints must be solved in one approach which can be

done by the application of control theory comparable to what Sethi and Thompson (1981) used in their production and inventory model. To specify the objective function a quadratic approach will be used. This coincides with linear demand and supply functions which provides a first order approximation to consumer surplus very often applied in welfare economics (Bale and Lutz, 1983). Hence, in terms of limited deviations from the fixed situations this approach provides realistic results and is analytically tractable.

In general a Hamilton-equation must be optimized:

(6)

$$H(t) = \exp[-\rho t] \left\{ \alpha_1 [c(t) - \bar{c}]^2 + \alpha_2 [m_1(t) - \bar{m}_1]^2 + \alpha_3 [m_1(t) - \bar{m}_1] [c(t) - \bar{c}] - \alpha_4 [S(t) - \bar{S}]^2 - \alpha_5 [R(t) - \bar{R}]^2 + \exp[\rho t] \lambda^*(t) [-\pi S(t) - c(t) + \nu m_2(t)] + \exp[\rho t] \mu^*(t) [p_t^w(t) [m_1(t) + m_2(t)]] \right\}$$

where additionally:

$\lambda^*(t)$: stands for a time dependent Lagrange multiplier in order to integrate stock movements, and

$\mu^*(t)$: stands for a time dependent Lagrange multiplier in order to integrate money reserve movements

Again, in this presentation of the model the implicit objective function is transferred into explicitly quadratic forms which coincide with linear demand and supply function mainly used in graphical approaches³. It provides second order approximation to the real function. The coefficient ρ expresses the discounting rate to be regarded by the planner. To find the objective function the graphical approaches for determining consumer surplus and supply rents must be translated into algebraic presentation (See Appendix I).

³However, another advantage of a non-linear cost and welfare function can be seen in a sense that it does not provide one-zero-decisions of a bang bang type.

Furthermore, the specification contains the dynamic constraints

$$(7) \dot{S}(t) = \pi S(t) - c(t) + \nu m_2(t) \quad \text{for stockpiling and}$$

$$(8) \dot{R}(t) = [m_1(t) + m_2(t)] p_t^w \quad \text{for the financial capacity which can be exhausted.}$$

This model specification comprises the distinction between $m_1(t)$ and $m_2(t)$, the opportunities to provide imports for direct consumption $m_1(t)$ or imports which can be used to increase stock availabilities per month by $m_2(t)$. This means that imports can be used for blending original stocks from harvest. However, direct consumption of imports $m_1(t)$ is characterized by different tastes which can be seen from the goal function separating consumption $c(t)$ from $m_1(t)$. Due to consumer preferences, this is reasonable, if a smaller price elasticity of consumption exists for locally produced and stored good. For example, white maize is a special staple food in Eastern and Southern Africa.

From optimization in control theory given by

$$(9a) H_x = -\dot{\lambda}$$

$$(9b) H_u = 0$$

$$(9c) H_\lambda = \dot{x}$$

one obtains the following equations as particular conditions in our case:

$$(10a) \alpha_4 [S(t) - \bar{S}] + [\rho + \pi] \lambda(t) = \dot{\lambda}(t)$$

$$(10b) \alpha_5 [R(t) - \bar{R}] + \rho \mu(t) = \dot{\mu}(t)$$

$$(10c) \nu m_2(t) - c(t) - \pi S(t) = \dot{S}(t)$$

$$(10d) p_t^w m_1(t) + p_t^w m_2(t) = \dot{R}(t)$$

$$(10e) \alpha_1 [c(t) - \bar{c}] + \alpha_3 [m_1(t) - \bar{m}] = -\lambda(t)$$

$$(10f) \alpha_3 [c(t) - \bar{c}] + \alpha_2 [m_1(t) - \bar{m}] = \bar{p}_t^w \mu(t)$$

$$(10g) \nu \lambda(t) + \bar{p}_t^w \mu(t) = 0$$

This is a system with 7 unknown independent variables of which some are in first order. It has been arranged and solved in Appendix II. After some manipulations we obtain equation (11) in which $\lambda(t)$ can be calculated:

However, so far it has not been taken enough care of some major problem concerning the general model construction as mentioned in the introduction of planner's problems. Firstly, a seasonal change in the price formation on the world market can heavily influence the solution. This environment in which policy recommendation operates can be included using a time dependent variable $p^w(t)$ as follows.

Hence, there is a need for manipulation of equation (10d); (following this we are dealing with an approximation of (10d)).

$$(10d') \left[\bar{m}_1 + \bar{m}_2 \right] p^w(t) + \bar{p}^w \left[m_1(t) - \bar{m}_1 \right] + \bar{p}^w \left[m_2(t) - \bar{m}_2 \right] = \dot{R}(t)$$

Graphically this means that the joint product $p^w(t) * m_i(t)$ will not be taken into account.

Of course, in situations of a constant world market price equation (10d') is exactly the same like (10), just easing the solution of (10) introducing deviations from fixed average values comparable with the other equations.

Applying (10d') instead of (10d) a solution of the the equation system of (10) can be conducted by inserting the equations in each other. This leads to the following second order differential equation in $\lambda(t)$

$$(11) \left[\alpha_1 \alpha_2 - \alpha_3^2 \right] \left[\left[\frac{\nu}{p^w} \right]^2 \frac{\alpha_4}{\alpha_5} + 1 \right] \ddot{\lambda}(t) - \left[\alpha_1 \alpha_2 - \alpha_3^2 \right] \left[\left[\frac{\nu}{p^w} \right]^2 \frac{\rho \alpha_4}{\alpha_5} + \rho \right] \dot{\lambda}(t) - \left[\alpha_1 \alpha_2 \pi(\rho + \pi) + \alpha_3 \alpha_4 \nu - \nu \alpha_4 \alpha_3 - \nu^2 \alpha_1 \alpha_4 - \alpha_3^2 \pi(\rho + \pi) + \alpha_2 \alpha_4 \right] \lambda(t) = - \left[\alpha_1 \alpha_2 - \alpha_3^2 \right] \frac{\nu}{p^w} \alpha_4 \left[\bar{m}_1 + \bar{m}_2 \right] p^w(t) + \left[\alpha_1 \alpha_2 - \alpha_3^2 \right] \alpha_4 \left[(\bar{m}_1 + \bar{m}_2) \nu - \pi \bar{S} \right] - \left[\alpha_2 - \alpha_3 \nu \right] \alpha_4 \left[\alpha_1 \bar{c} + \alpha_3 \bar{m}_1 \right] + \left[\alpha_3 - \alpha_1 \nu \right] \alpha_4 \left[\alpha_3 \bar{c} + \alpha_2 \bar{m}_1 \right]$$

containing $p^w(t)$ as some exogenous time dependent variable. The procedure is

demonstrated in Appendix II. Hence, the final solution of system (10) is of the form

$$(12') \beta_1 \dot{\lambda}(t) + \beta_2 \ddot{\lambda}(t) + \beta_3 \lambda(t) = \beta_0 + \beta_4 \bar{c} + \beta_5 \bar{m}_1 + \beta_6 \bar{m}_2 + \beta_7 \bar{S} + \beta_8 \bar{P}^w(t)$$

regarding $\lambda(t)$ as basic variable. If we have the basis variable we can cope with all other variables. The above equation provides a formula which is according to standard procedures of differential equation solutions documented elsewhere (Tu, 1982). A numerical solution will be given as an example below. However, it might be that some people believe that this approach is too narrow with regard to specific problems of particular countries. To demonstrate that it is fairly open to often announced problems a discussion of some extensions of the approach will follow. This shall demonstrate that many additional issues can be addressed with the same framework.

5 Discussion of possibilities to extend the theoretical approach

The model contains some possibilities to extend the approach. For example it might be that a seasonal moving of $\bar{c}(t)$ occur. Formally this must be integrated in the constant term $\beta_4 \bar{c}$. For the purpose of receiving an analytical solution this requires a functional representation which can be solved by applying a trigonometrical representation:

$$(13) \bar{c}(t) = \gamma_0 + \gamma_1 \cos(\varphi t + \xi)$$

where: γ_1 , φ , and ξ are coefficients in order to describe a seasonal variation

An seasonal variation in average food requirements may be necessary if one looks at some empirical findings about food requirements in some regions in Sub-Saharan Africa. For

example, Kumar (1988 p. 1054) reports for Zambia that the planting and preharvest agricultural labour requirement demand for an increase of 120 p.c. in calorie intake. Since this additional food demand mostly cannot be provided by the farmers themselves they must go for food on the market. The reason for seasonal shortages in private peasant farmer's stockpiling is an issue which is still under investigation (Stanning, 1988). It might be that it is caused by a unique price within a season in many countries. Hence it could be changed by a different seasonal pattern in official deliveries to the market. But, there are some reasons which will be unchanged like post harvest sales for paying debts and school fees. Hence there is still a strong argument in favour of a varying food requirement in seasonal context. Even, if one takes other food sources into account or assumes that peasant farmers run out of own stocks in the end of season. However, it needs an empirical investigation. Furthermore, a change in seasonal consumption pattern after the application of the operational policy has to be discussed in order to make qualitative improvements of policies.

Another problem in planning which might occur is characterized by devaluation of foreign currency hoarded for purchasing inputs. It can be solved within our given framework. Theoretically this issue just requires that the money stock depreciates like physical stocks. Furthermore, it is possible to extend the model to two or more products which enables one to investigate seasonal export and import strategies of a specific country or region. In the case of zero sum expenditure/revenue assumptions a country could optimize its gains from trade by exporting in a situations of a seasonal harvest pattern of a particular crop whilst importing a different product of which it faces a seasonal shortage. This research might be especially worthwhile in neighboring countries which follow a specific movement in weather conditions, for example a north-south pattern of rainfall. However, that involves some more advanced approaches with different constraints.

The same argument of advanced calculations can be applied to the very often stated case of divergence of import and export parity prices if one deals with simultaneous imports and

exports in particular crops. To guarantee the correct price for a particular activity, for example in imports, one only requires a method involving non equality conditions. In principle the mathematics are manageable and available.

Might be that these extensions are dependent on the power of the already developed approach. To demonstrate the power an empirical example will be provided next.

6 Calculating an numerical example

The empirical application is due to a simply constructed fictive example of a country roughly importing 10 percent p.c. of its overall seasonal need. To enable calculations of the coefficients we are working with elasticity assumptions which are transferred by their definitions into α_1 -values.

On the consumption side it is assumed that the demand elasticity due to internal price is

$\epsilon_{c(t),p(t)}^D = -0.2$ and the average internal price is 250,- currency units per tonne. This might be the case for example for white maize in several Southern and East African

countries. At a presumed consumption in the starting condition of $\bar{c} = 100000t$ this gives

$\alpha_1 = -0.00705$. Similar calculations are being done with the price elasticity of the directly consumed import good which shall be $\epsilon_{m_1(t),p(t)}^D = -0.3$ indicating that consumers have

less preference for imports (say yellow maize). The cross price elasticity between direct

consumption and consumption from imports which is given by $\epsilon_{m_1(t),p(t)}^D = 0.1$. Presumed

consumption directly from imports will start at an average of 5000 t and intake into stocks

shall be equal to $m_2(t) = 5000 t$. The average internal price will be 200,- currency units per

tonne and the import parity price shall be fixed at 300,- currency units. This empirical

perception coincides with findings from Kingsbury (1988, p.268) who reported diverse

deviations between internal prices and import parity prices, for example, on a regional

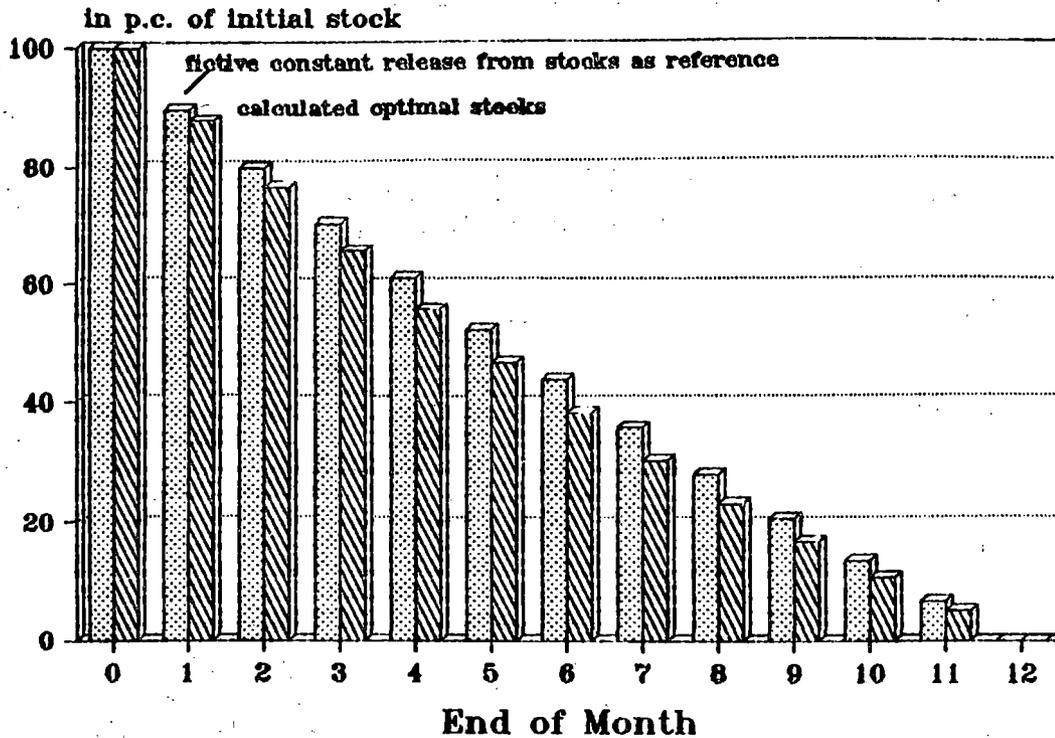
basis for maize in Zambia.

Furthermore, post harvest stocks are starting at 1200000 t, and in the initial situation we are beginning our calculations at $\bar{R}=36$ million currencies units. Average storage costs are 5,- currency units per tonne and month and the prevailing interest rate on a monthly base is 0.01 p.c. per month. At least, our stock depreciation will be 0.04 p.c. per month and the necessary conversion rate into stocks will be $\nu=0.8$ units to substitute one stock unit.

These basic input data provide us the necessary coefficients of the model whose solutions can be seen from the following figures. Again, the calculation in this fictive case study is fairly simple without any extension to seasonal movements in import parity or consumer needs, just providing a preliminary basis for discussion:

Figure 1:

Optimal Stocks due to Model Calculation



Source: Own calculations due to assumpt.

Figure 1 shows the optimal stocks due to the model calculations. It can be seen that in comparison to a fictive stock reduction which would guarantee constant consumption stocks due to model calculations should be reduced faster. However, that reference does not take into account that government might fail in its policy at all.

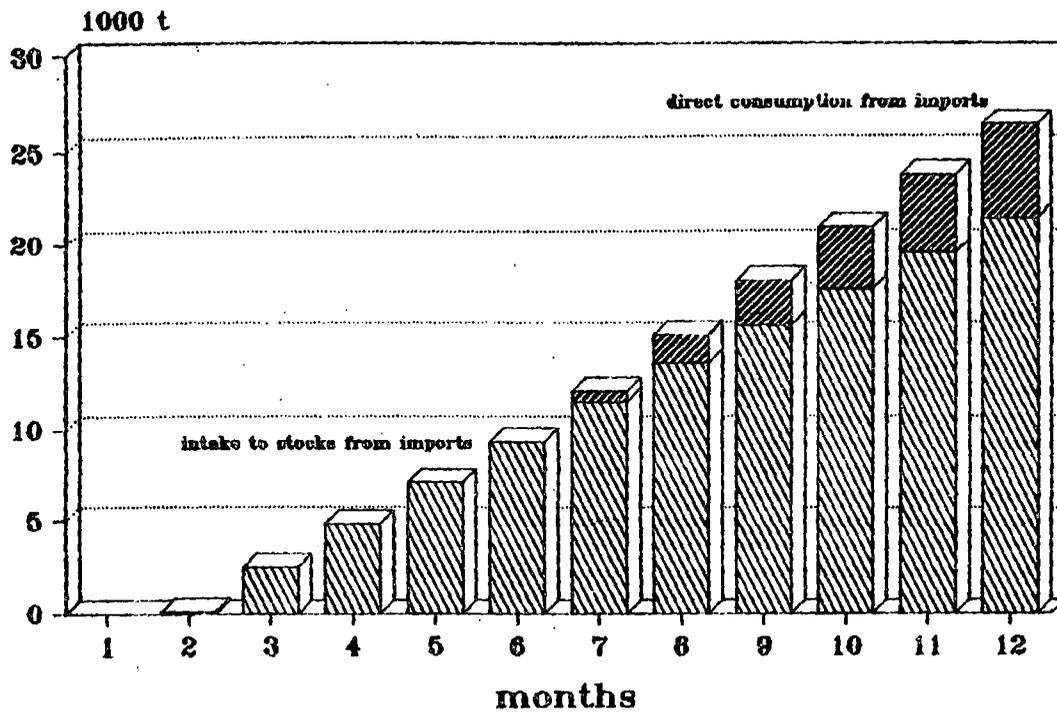
In figure 2 intake to stocks and direct consumption from imports are displayed.

To explain the late beginning of direct consumption of imports one must know from calculations that before the middle of season imports have been negative. This actually would have implied an export due to higher import parity prices. However, the increase of

imports towards the end of the season can be explained by high storage costs in the early months which recommends less imports and higher stock extraction, whilst in the end extraction must be supplemented by more imports to meet consumption.

Figure 2:

Imports due to Model Calculations Intake to Stocks and Direct Consumption

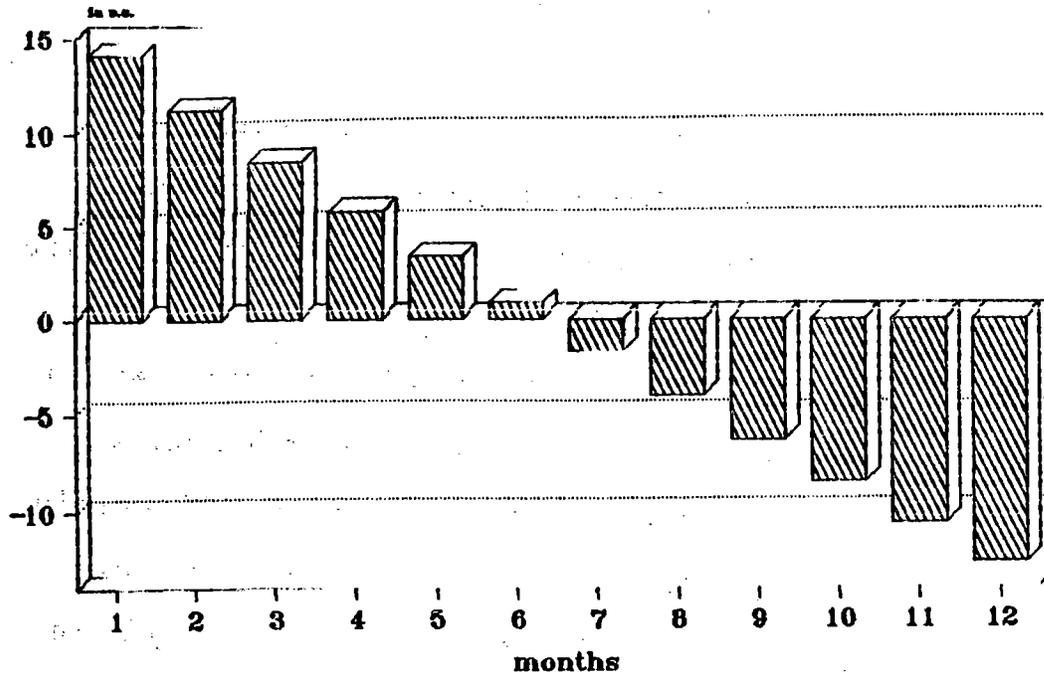


Source: Own Calculations due to assumpt.

Figure 3 displays the seasonal consumption pattern due to the model results. It can be seen that total supply regarding average seasonal consumption to the market should be roughly 15 p.c. higher in early season, diminishing to midst of season and being less than -12.p.c in the last month.

Figure 3:

Seasonal Consumption due to Model Result in p.c of average consumption



Source: Own calculations due to assumpt.

All in all the conclusion drawn from this exercise should be that significant seasonal movements in stocks, imports and consumption can be calculated with the planning approach presented over here. Depending on the assumptions and numerical environment, of course, the results will differ widely and even sensitivity analyses would be highly recommendable in application of the model to policy conduct. Moreover, nothing has been said about the implementation of the operational policy yet. As far as stock releases are concerned this could be done by deliveries to private traders. Then, they will realize different situations in the provision of food to the market on a monthly basis. Which, of course, under perfect competition conditions would lead to a seasonal price movement. To deal with the institutional issues involved in the design an auction system could be

suggested.

7 Summary

This paper contributes to the design of an operational policy to pursue seasonal food security in a developing country. It is assumed that a parastatal is confronted with the planning of releases from stocks, imports for direct consumption and imports for intakes to stocks on a month to month basis knowing initial stocks. Due to an additional budget constraint in foreign currency, import parity prices and stock deteriorations a function is constructed which provides monthly values of the variables of concern. The method to solve the problem can be seen in an application of control theory which enables analytical planning in a dynamic context. Formally, the model is simultaneously solved for all variables. Furthermore, a fictive numerical example which is close to the reality in many Sub-Saharan countries is calculated. One of the findings shows that a 15 p.c. higher consumption than average should prevail in the first month, whilst it should come down to 12 p.c in the last month of the season.

Appendix I:

Some points must be clarified concerning the objective function. First the positive part in the objective function works with the consumer surplus which in a linear demand equation model results in a quadratic presentation. This is a first order approximation to a utility function mainly used in economic approaches. It combines the amount of $\bar{c}(p-\bar{p})$ and $1/2 \alpha_1 (c-\bar{c})^2$. Since lower prices are related to a loss of the selling agency this increase in consumer surplus is balanced against the payment. That means overall social welfare is

constituted by the quadratic component. Second the cost functions of storage costs and opportunity cost of money holding are created by the supply functions of these activities. Assuming a linear supply function which again shall represent a first order approximation the use of the supply elasticity $\epsilon_{S,1}$ provides the α_4 coefficient by

$$\alpha_4 = -\frac{1}{\epsilon_{S,1}} \frac{T}{\bar{S}^*} \text{ with } T \text{ average cost per tonne and } \bar{S}^* \text{ average stock}$$

Then a linear supply function $S^S = \gamma_0 + \gamma_1 I$ can be translated in the required expression of $\alpha_4(S - \bar{S})^2$ using $\bar{S} = \bar{S}^*(1 - \epsilon_{S,1} T)$.

Appendix II:

Calculation procedure:

There are some opportunities to simplify the system in (10). One procedure will be presented here which results in a first and second order equation of $\lambda(t)$:

To do this we first regard (10g). Either it is

$$(A1) \lambda(t) = -\frac{\bar{p}^w}{\nu} \mu(t) \quad \text{or} \quad \mu(t) = -\frac{\nu}{\bar{p}^w} \lambda(t)$$

With regard to this information one can eliminate $\dot{\lambda}(t)$ and $\dot{\mu}(t)$ out of (10a) and (10b)

$$(A2) \dot{S}(t) = -\frac{\alpha_5}{\alpha_4} \frac{\bar{p}^w}{\nu} \dot{R}(t) - \frac{\pi}{\alpha_4} \lambda(t) + \bar{S} + \frac{\alpha_5}{\alpha_4} \frac{\bar{p}^w}{\nu} \bar{R}$$

which can be expressed in first differential as

$$(A3) \dot{S}(t) = -\frac{\alpha_5}{\alpha_4} \frac{\bar{p}^w}{\nu} \dot{R}(t) - \frac{\pi}{\alpha_4} \lambda(t)$$

A next step is to eliminate $m_2(t)$ out of (10c) and (10d):

$$(A4) \nu \left[\bar{m}_1 + \bar{m}_2 \right] p^w(t) + \nu \bar{p}^w m_1(t) - \nu \bar{p}^w \left[\bar{m}_1 + \bar{m}_2 \right] - \nu \dot{R}(t) = -\bar{p}^w c(t) - \bar{p}^w \pi S(t) - \bar{p}^w \dot{S}(t)$$

This solved for $\dot{R}(t)$ gives

$$(A5) \dot{R}(t) = \frac{\bar{p}^w}{\nu} \dot{S}(t) + \frac{\bar{p}^w}{\nu} \pi S(t) + \bar{p}^w m_1(t) + \frac{\bar{p}^w}{\nu} c(t) + \left[\bar{m}_1 + \bar{m}_2 \right] p^w(t) - \bar{p}^w \left[\bar{m}_1 + \bar{m}_2 \right]$$

Hence by inserting in (A3) it follows:

$$(A6) \left[1 + \frac{\alpha_5}{\alpha_4} \left[\frac{p^w}{\nu} \right]^2 \right] \dot{S}(t) = -\frac{\alpha_5}{\alpha_4} \left[\frac{p^w}{\nu} \right]^2 \left[\nu m_1(t) + c(t) + \pi S(t) + \frac{\nu}{p^w} \left[\bar{m}_1 + \bar{m}_2 \right] p^w(t) - \nu \left[\bar{m}_1 + \bar{m}_2 \right] \right] - \frac{\pi}{\alpha_4} \dot{\lambda}(t)$$

Since from we get from (10a)

$$S(t) = -\frac{\rho + \pi}{\alpha_4} \lambda(t) + \frac{1}{\alpha_4} \dot{\lambda}(t) + \bar{S} \quad \text{which gives}$$

$$\dot{S}(t) = -\frac{\rho + \pi}{\alpha_4} \dot{\lambda}(t) + \frac{1}{\alpha_4} \ddot{\lambda}(t),$$

the solution for $\lambda(t)$ is:

$$(A7) - \left[\frac{\alpha_4}{\alpha_5} \left[\frac{\nu}{p^w} \right]^2 + 1 \right] \frac{\rho + \pi}{\alpha_4} \dot{\lambda}(t) + \left[\frac{\alpha_4}{\alpha_5} \left[\frac{\nu}{p^w} \right]^2 + 1 \right] \frac{1}{\alpha_4} \ddot{\lambda}(t)$$

$$= -\frac{\alpha_4}{\alpha_5} - \frac{\pi}{\alpha_4} \left[\frac{\nu}{p^w} \right]^2 \dot{\lambda}(t) + \pi \frac{\rho + \pi}{\alpha_4} \lambda(t) - \frac{\pi}{\alpha_4} \dot{\lambda}(t) - c(t) - \nu m_1(t) \\ - \frac{\nu}{p^w} [\bar{m}_1 + \bar{m}_2] p^w(t) + \nu [\bar{m}_1 + \bar{m}_2] - \pi \bar{S}$$

This is a second order differential equation in $\lambda(t)$. Such second order equations can be solved using an easy formula. However, the equation above comprises terms of $m_1(t)$ and $c(t)$ which have to be balanced with equation (10e) and (10f). To do this we will construct a 3 by 3 matrix.

$$(A8) \begin{bmatrix} \frac{\pi (\rho + \pi)}{\alpha_4} & 1 & \nu \\ -1 & \alpha_1 & \alpha_3 \\ -\nu & \alpha_3 & \alpha_2 \end{bmatrix} \begin{bmatrix} \lambda(t) \\ c(t) \\ m_1(t) \end{bmatrix} =$$

$$\begin{bmatrix} \left[\left[\left(\frac{\nu}{p^w} \right)^2 \frac{\rho}{\alpha_5} + \frac{\rho}{\alpha_4} \right] \dot{\lambda}(t) - \left[\left(\frac{\nu}{p^w} \right)^2 \frac{\alpha_4}{\alpha_5} + 1 \right] \frac{1}{\alpha_4} \dot{\lambda}(t) \right] \\ 0 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} -\frac{\nu}{p^w} [\bar{m}_1 + \bar{m}_2] p^w(t) + \nu [\bar{m}_1 + \bar{m}_2] - \pi \bar{S} \\ \alpha_1 \bar{c} \\ \alpha_3 \bar{c} \end{bmatrix} \begin{bmatrix} \alpha_3 \bar{m}_1 \\ \alpha_2 \bar{m}_1 \end{bmatrix}$$

Again, this is a system of 3 equations in matrix representation $Ay=a$ for which a solution is given by $y=A^{-1}a$. This means to solve the system one has to build the inverse of the matrix a which is A^{-1} and multiply with a . The result is provided in the next representation: (A9)

$$\begin{bmatrix} \lambda(t) \\ c(t) \\ m_1(t) \end{bmatrix} = \frac{\begin{bmatrix} \alpha_4(\alpha_1\alpha_2 - \alpha_3^2) & + (\alpha_3\nu - \alpha_2^2)\alpha_4 \\ \alpha_4(\alpha_2 - \alpha_3\nu) & \nu^2\alpha_4 - (\rho + \pi)\pi\alpha_2 \\ -\alpha_4(\alpha_3 - \alpha_1\nu) & \alpha_3\pi(\rho + \pi) - \alpha_4\nu \end{bmatrix}}{\begin{bmatrix} -\alpha_1\alpha_2\pi(\pi + \rho) & -\alpha_3\alpha_4\nu + \alpha_3\alpha_4\nu + \nu^2\alpha_1\alpha_4 \end{bmatrix}}$$

$$\frac{\begin{bmatrix} (\alpha_3 - \alpha_1\nu)\alpha_4 \\ (\alpha_3\pi(\rho + \pi) - \alpha_4\nu) \\ -(\alpha_1\pi(\rho + \pi) - \alpha_4) \end{bmatrix} \left[\begin{bmatrix} \left[\left(\frac{\nu}{p-w} \right)^2 \frac{\rho}{\alpha_5} + \frac{\rho}{\alpha_4} \right] \lambda(t) \\ \alpha_1 \bar{c}_1 \\ \alpha_3 \bar{c}_1 \end{bmatrix} \right]}{+\alpha_3^2\pi(\pi + \rho) - \alpha_2\alpha_4}$$

$$\left[\begin{aligned} & - \left[\left(\frac{\nu}{p-w} \right)^2 \frac{\alpha_4}{\alpha_5} + 1 \right] \frac{1}{\alpha_4} \lambda(t) - \frac{\nu}{p-w} \left[\bar{m}_1 + \bar{m}_2 \right] p^w(t) - \nu \left[\bar{m}_1 + \bar{m}_2 \right] - \pi \bar{S} \\ & \alpha_3 \bar{m}_1 \\ & \alpha_2 \bar{m}_1 \end{aligned} \right]$$

which gives as final result a differential equation in $\lambda(t)$:

$$\begin{aligned} (A10) \quad & \left[\alpha_1\alpha_2 - \alpha_3^2 \right] \left[\left[\left(\frac{\nu}{p-w} \right)^2 \frac{\alpha_4}{\alpha_5} + 1 \right] \lambda(t) - \left[\alpha_1\alpha_2 - \alpha_3^2 \right] \left[\left(\frac{\nu}{p-w} \right)^2 \frac{\rho\alpha_4}{\alpha_5} + \rho \right] \right. \\ & \lambda(t) - \left[\alpha_1\alpha_2\pi(\rho + \pi) + \alpha_3\alpha_4\nu - \nu\alpha_4\alpha_3 - \nu^2\alpha_1\alpha_4 - \alpha_3^2\pi(\rho + \pi) + \alpha_2\alpha_4 \right] \lambda(t) = - \left[\alpha_1\alpha_2 - \alpha_3^2 \right] \\ & \frac{\nu}{p-w} \alpha_4 \left[\bar{m}_1 + \bar{m}_2 \right] p^w(t) + \left[\alpha_1\alpha_2 - \alpha_3^2 \right] \alpha_4 \left[\left(\bar{m}_1 + \bar{m}_2 \right) \nu - \pi \bar{S} \right] - \left[\alpha_2 - \alpha_3\nu \right] \alpha_4 \\ & \left[\alpha_1 \bar{c} + \alpha_3 \bar{m}_1 \right] + \left[\alpha_3 - \alpha_1\nu \right] \alpha_4 \left[\alpha_3 \bar{c} + \alpha_2 \bar{m}_1 \right] \end{aligned}$$

which is the used as equation (11) in the main text.

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