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Part II

by

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ALTERNATIVE THEORIES OF WAGE DETERMINATION AND UNEMPLOYMENT IN L.D.C.'s:

II. THE EFFICIENCY WAGE MODEL

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1. Introduction

In an earlier paper ("Alternative Theories of Wage Determination and Unemployment in L.D.C.'s: I. The Labor Turnover Model") we argued that to determine the desirability of such commonly recommended policies for alleviating urban unemployment as subsidizing wages or using a shadow price of labor that is less than the market wage required an explicit model of the determination of wages and unemployment in L.D.C.'s. We presented a model there in which firms in the urban sector pay a wage higher than the rural wage in order to reduce labor turnover; we showed that in that model, a wage subsidy actually lowered national output and increased the unemployment rate; and the "correct" shadow price of labor was, at least in some circumstances, just the urban wage. Some observers have suggested that labor turnover is one of the more important determinants of firms' wage policy in East Africa,[1] and if this is the case, our results certainly call into question the advisability of providing a wage subsidy and using low shadow prices for labor in such economies. On the other hand, development economists in South Asia have favored an alternative explanation of the phenomena of urban unemployment: the efficiency wage model, which dates back at least to the work of Leibenstein.[2] In spite of its common

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[1] Cf. W. Elkan, Migrants and Proletarians, Oxford University Press, 1960, a: An African Labor Force, East African Studies, Vo. 7, Kampala, 1955.

[2] H. Leibenstein, Economic Backwardness and Economic Growth, July, 1957.

acceptance,[1] there appears to be no general equilibrium analysis of its implications, particularly its policy implications for wage subsidies and shadow prices. This paper provides such an analysis.

An analysis of the implications of the efficiency wage for urban wages and employment requires, however, a detailed examination of the rural sector. Part A of this paper describes the rural sector and Part B links these results with the urban sector.

A. The Rural Sector

The economics of the rural sector--the allocation of labor, the supply of effort, the determinants of migration from the rural to the urban sector, etc.--depends critically on how the sector is organized, for instance, whether farms are individually owned or whether there are extended families, whether there is a large landless peasantry, whether individuals who migrate to the urban sector lose their rights to the land, etc.[2] For most of the analysis of this part of the paper, we shall assume land is owned by an extended family. We shall consider a variety of rules according to which food is allocated to the members of the family: in Section 2, we shall consider one "polar case"--one which plays a prominant role in the literature--where everyone receives his average product and efficiency considerations are completely ignored, while in Section 3 we shall describe the other polar case, where distribution is completely ignored and the farm maximizes its output. The analysis of Sections 2 and 3 will make clear that there are conflicts between equity and efficiency: the output in the equalitarian situation where everyone receives the same wage may be significantly lower than in the output maximizing solution. Moreover, in the completely equalitarian situation the social marginal product of an individual is negative: as individuals migrate from the rural sector, output actually increases.

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[1] We do not wish to discuss here the direct empirical evidence in support of and against this model; we shall note, however, that the model does correctly describe many aspects of the labor market in L.D.C.'s.

[2] For a more extended treatment of these issues, see J.E. Stiglitz, "Rural-Urban Migration, Surplus Labor, and the Relationship between Urban and Rural Wages," E. African Econ. Rev., Dec. 1969.

The analyses of Sections 2 and 3 are both special cases of the more general analysis of Section 4 where the family maximizes family "welfare" according to an additive (Benthamite) welfare function, i.e. the family takes cognizance of the trade-off between efficiency and equity. It is still true in this case that the social marginal product of an individual is negative. Different members of the family will receive different wages, but we are able to establish certain bounds on the degree of inequality. In particular, we can think of each individual's receipts as his marginal product plus a pro rata share of rents. The low wage individuals are less efficient than the high wage individuals; they receive less than their share of the rents pro-rated on the basis of percentage of the population, but more than their share of the rents pro-rated on the basis of percentage contribution to total effective labor supply.

In the final section of Part A, we describe the rural sector of an economy with landless laborers. Working individuals have a positive marginal product, but there may be unemployment; this is an equilibrium, i.e. the unemployed are unable to bid down the wages of the employed. It will be shown that the wage is identical to that of output maximizing family farm described in Section 3.

The models of this section show that presence of a positive wage for working individuals in a competitive labor market (and a corresponding positive marginal productivity) cannot be taken as evidence that labor is not in surplus (as some authors seem to have done); indeed in Sections 3 and 4, the laborers actually have a negative social marginal product although they receive a positive wage.

2. The Equalitarian Family Farm

Output in the representative farm in the rural sector is a function of the input of labor services, land, capital, and other factors of production. In this paper, we focus only on labor; all other factors of production are assumed to be fixed in the short run.[1] The efficiency wage hypothesis says that the services a laborer renders are a function of the wage he receives. One well-paid worker may do what two poorly paid workers can do. We let $\lambda(w)$ be the index of efficiency of a worker receiving a wage of w .[2] We hypothesize that λ has the shape depicted in Figure 1. There is a region of increasing returns, where, as the individual is brought above the "starvation" level additional increments in wages result increasing increments in efficiency, although eventually diminishing returns sets in. Although many observers have claimed that the efficiency curve has the shape depicted, direct empirical evidence is hard to come by and it remains a moot question. It should be emphasized that our results do depend critically on the existence of the initial region of increasing returns.[3]

Since everyone receives the same wage (i.e. they receive the average product) total labor services, E , are given by

$$E = \lambda(w)L \tag{2.1}$$

where L is the number of workers. Output on the farm is given by

$$Q = G(E) = G(\lambda(w)L) \tag{2.2}$$

where $G' > 0$, $G'' < 0$, there is a positive marginal product and diminishing returns to labor services. Since workers receive their average product

$$w = Q/L \ . \tag{2.3}$$

The nature of the equilibrium is depicted in Figure 2. At a given level of L , output per man is initially an increasing function of w and then a decreasing function of w . Equilibrium requires output per man to equal w .

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[1]Most of our results will still be true in the more general case where these other factors are allowed to vary, as they certainly will in the long run; our primary interest here is in the short run analysis (rather than with capital accumulation, or intersectoral capital movements) and hence the assumption of other factors being fixed may not be unreasonable. In any case, the more general analysis would obfuscate the simple points we wish to establish here.

[2]Since Part A is concerned exclusively with the rural sector, we need not burden ourselves here with distinguishing between w_r , the wage in the rural and w_u , the urban wage, or λ_r and λ_u . In Part B, subscripts will be required.

[3]We return to the remark made in footnote 1, p. 2 above: although the assumptions of the model are hard to verify directly, many of its implications are roughly in accord with the observed patterns of wages and employment.

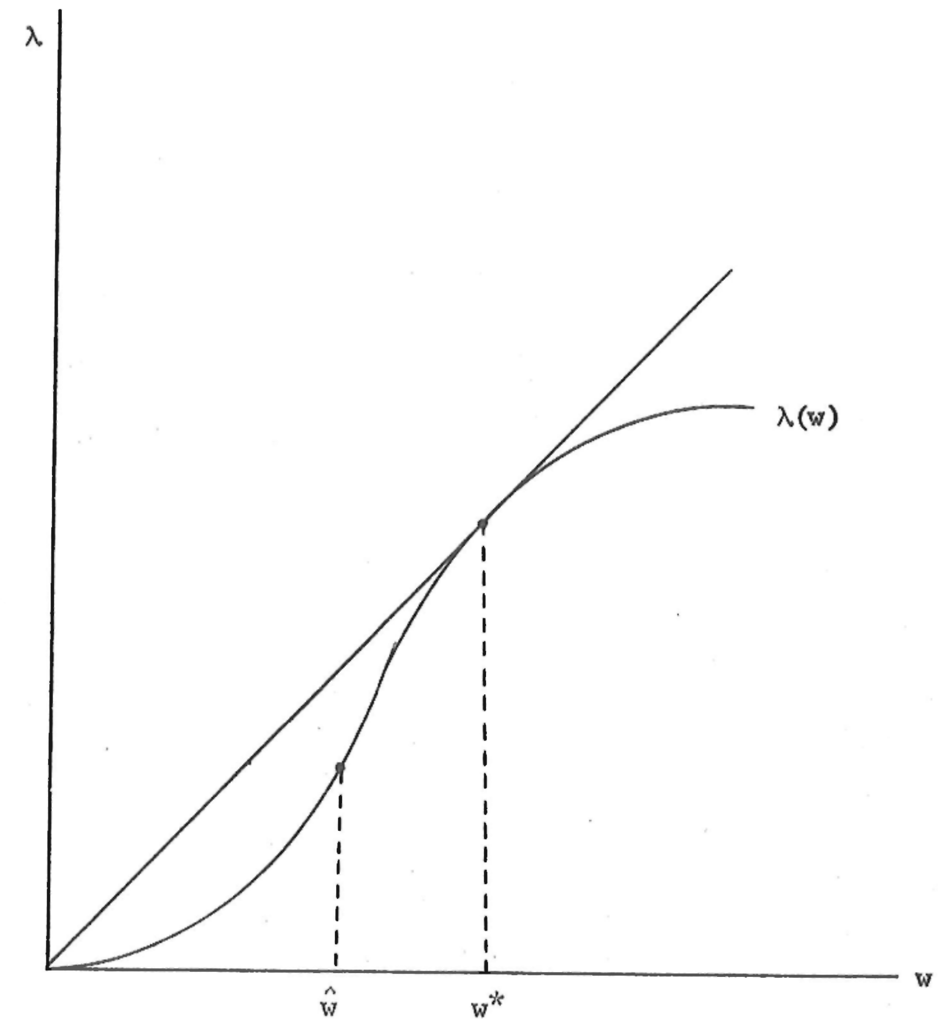

FIGURE 1

The Efficiency Curve

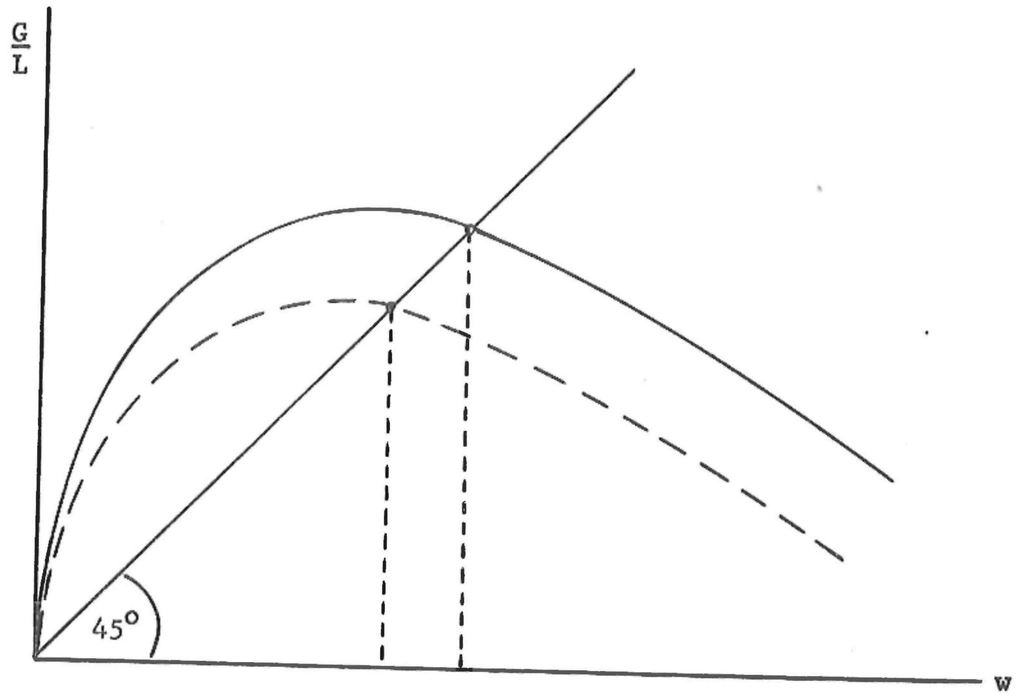

FIGURE 2a

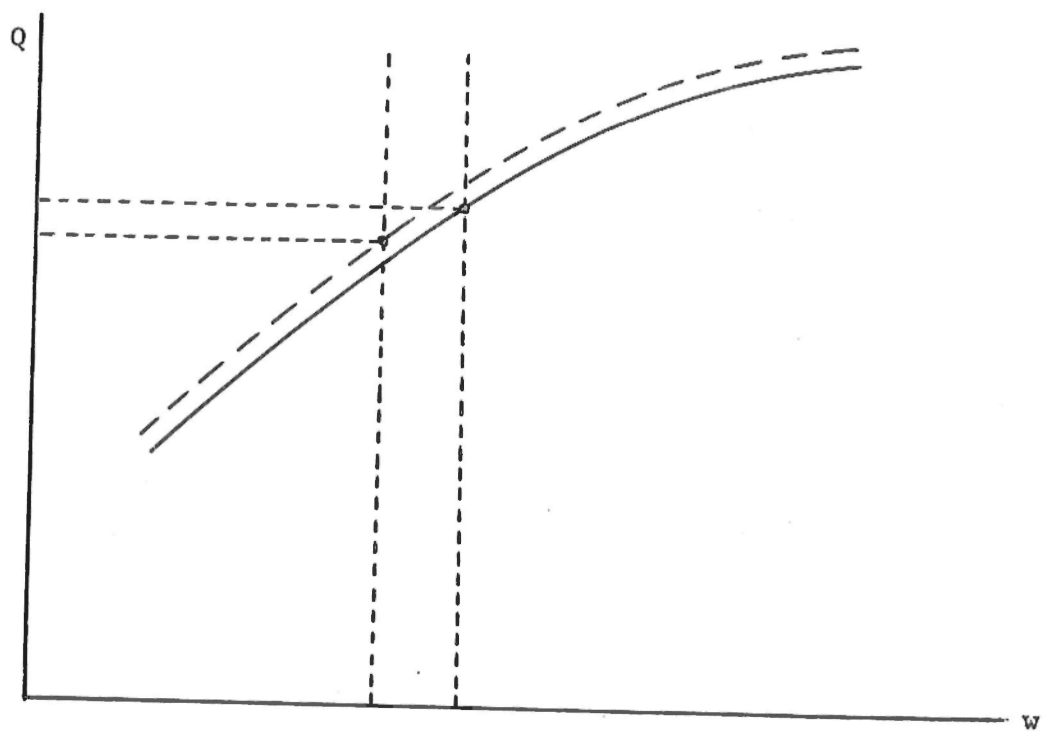

FIGURE 2b

Equilibrium in "Wage Equal Average Product" Farm:
Increase in Workers Reduces Output

Consider now the effect of an increase in the number of laborers, L ; clearly at each value of w, the output per man diminishes (because of diminishing returns); as the dotted line in Figure 2a makes clear, output per man (and hence the wage) is reduced. But not only does an increase in the number of laborers reduce output per man, but it actually may reduce total output, as Figure 2b illustrates. To see this, we take the logarithmic derivative of (2.3) to obtain

$$1 + \frac{d \ln w}{d \ln L} = \frac{G'L}{G} \lambda \left(1 + \frac{\lambda' w}{\lambda} \frac{d \ln w}{d \ln L} \right)$$

i.e.

$$\frac{d \ln G}{d \ln L} = \frac{d \ln G}{d \ln E} \frac{d \ln E}{d \ln L} = \frac{d \ln G}{d \ln E} \left(\frac{d \ln \lambda}{d \ln L} + 1 \right)$$

$$= \frac{\alpha \left(1 - \frac{\lambda' w}{\lambda} \right)}{1 - \alpha \frac{\lambda' w}{\lambda}} \gtreqless 0 \quad \text{as} \quad \frac{\lambda' w}{\lambda} \gtreqless 1 \qquad (2.4)$$

where $\alpha = G'E/G$, the imputed share of labor, i.e. the share of labor if marginal productivity pricing were used.

The point where $\lambda' = \lambda/w$ plays a central role in the subsequent discussion. In Figure 1, it is the point where a line through the origin is tangent to the efficiency curve. It has the property that "wage payments per effective unit of labor" are minimized there, i.e. the solution to

$$\min \frac{w}{\lambda} \qquad (2.5a)$$

is just

$$\frac{1}{\lambda} - \frac{w\lambda'}{\lambda^2} = 0$$

or

$$\lambda'(w^*) = \frac{\lambda(w^*)}{w^*} . \qquad (2.5b)$$

We shall call the solution to (2.5b) w^*, and refer to it as the "efficiency wage."

Thus (2.4) can be rewritten[1]

$$\frac{d \ln G}{d \ln L} \gtrless 0 \quad \text{as} \quad w \gtrless w^* . \qquad (2.4')$$

Our concern here, of course, is with economies which are sufficiently poor that $w < w^*$, so the social marginal productivity of a laborer is negative.[2]

It should be clear, however, that although the social marginal productivity of a laborer in the rural sector is negative, the apparent "private" marginal productivity, $G'\lambda$, is positive. Each person is contributing something on the margin to production. It is only the fact that his presence in the rural sector decreases the income per capita, and hence the productivity of the other workers in the rural sector, that makes his social marginal productivity negative.

3. The Income Maximizing Family

We shall now consider the extended family which allocates its income among its members so as to maximize family income, paying no attention to equity. If $P(w)$ is the percentage of the family workers receiving at least a wage of w, then the effective labor supply E, of the family is just[3,4]

$$E = L\int \lambda(w)dP(w) . \qquad (3.1)$$

To maximize family income, $G(E)$, we simply maximize E subject to the constraint that

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[1]Since $d \ln w/d \ln L = -(1-\alpha)/(1 - (\alpha\lambda'w/\lambda)) < 0$, and $\alpha < 1$, it is clear that $\alpha\lambda'w/\lambda < 1$.

[2]This argument is very different for that presented by Stiglitz, "Rural-Urban Migration..." op.cit. and A.K. Sen, "Peasants and Dualism with or without Surplus Labor," JPE, 74 (Oct. 1966), 425-50, where the social physical marginal productivity may be negative because with reduced population, workers work harder, and output actually may increase.

[3]The integral is best interpreted as a Stieltjes integral.

[4]In most of the subsequent discussion, we shall let $P(w)$ take on any values between zero and one; obviously, if there are L individuals in the family, P can only take on values $1/L$, $2/L$, etc., the slight loss in realism is more then compensated for by the gain in analytic tractability.

$$\int w dP(w) = G(E) \tag{3.2}$$

i.e. that the distribution to members of the family add up to the family income. The problem of finding a function, $P(w)$ which maximizes an integral (3.1) subject to an integral constraint is a classical problem in the calculus of variations. It turns out in this case that the solution is degenerate. We shall first show that there are only two wage groups, and indeed, only one group receiving a positive wage; after establishing this, we shall show that the group receiving positive wages is paid w^*, the efficiency wage (eq. (2.5) above).

To see that there are only two wage groups, consider any two wages which are actually received by some individuals, w_1 and w_2. Consider the experiment of giving one person at a wage of w_1 one more unit. The effective labor supply goes up by $\lambda'(w_1)$. This does not take away one full unit from the resources available to other groups, since output will go up now by $G'\lambda'$. Hence, for national product exhaustion, we need to take away net, say from a group with w_2 wage, an income of $1 - G'\lambda'(w_1)$. Each unit we take away gross leads to a reduction in effective labor supply by $\lambda'(w_2)$ and of output of $G'\lambda'(w_2)$. Hence the net reduction is $1 - G'\lambda'(w_2)$. Accordingly, if the effective labor supply is to have been maximized, as was hypothesized, it must be true that this rearrangement of wage payments does not affect the effective labor supply, i.e.

$$\lambda'(w_1) = \frac{\lambda'(w_2)(1 - G'\lambda'(w_1))}{1 - G'\lambda'(w_2)} \tag{3.3}$$

that is

$$\left(\frac{1}{\lambda'} - G'\right) = \text{constant}. \tag{3.4}$$

Given our assumptions about $\lambda(w)$ it is clear that there are two solutions to (3.4), as depicted in Figure 3. But the first is a local minimum--further increases in the wage for this group increase productivity by more than the cost. Hence, there is only one interior wage.

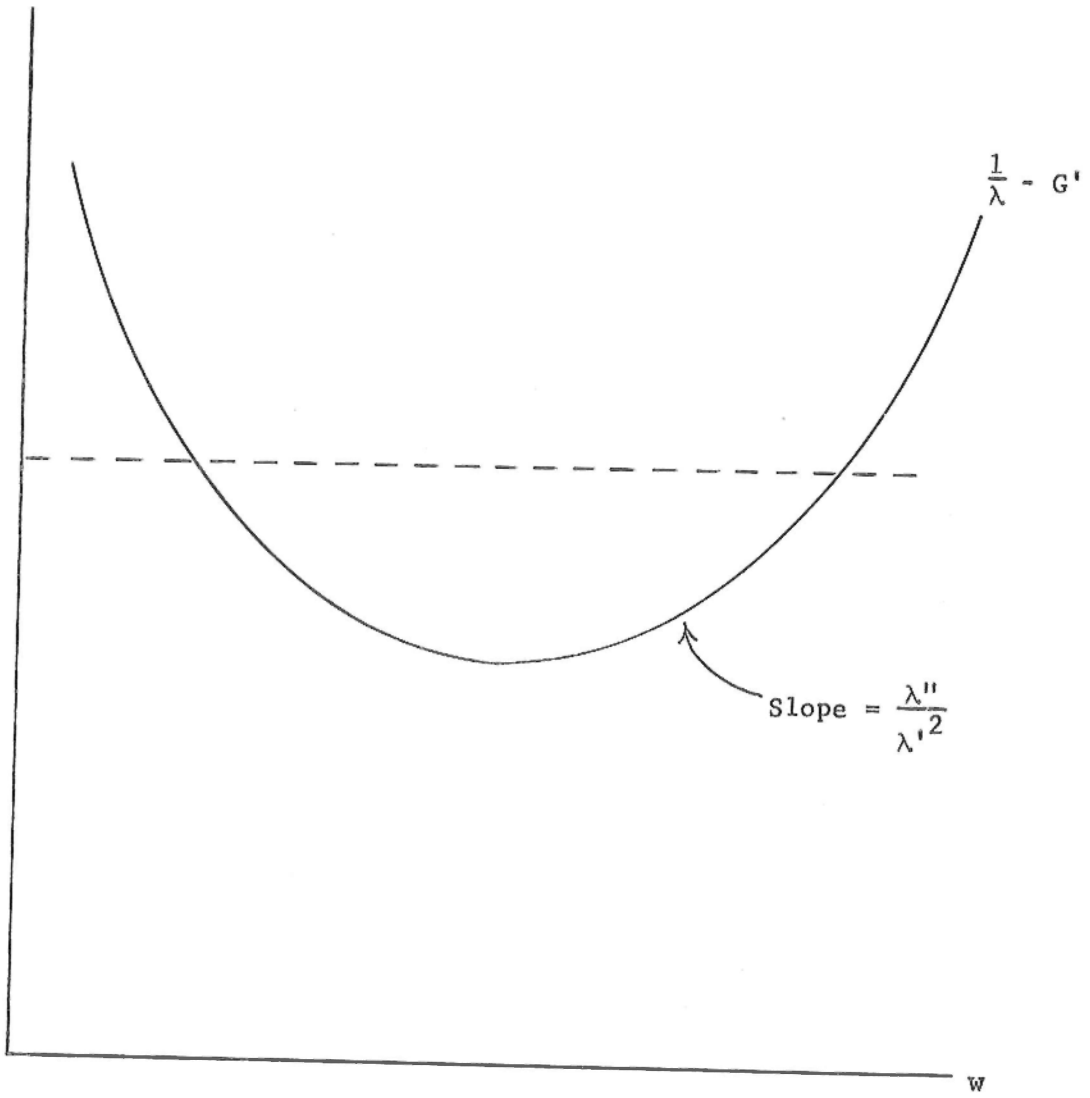

FIGURE 3

Wage Determination in Output Maximizing Farm

We shall now show that the wage received by the workers is in fact the efficiency wage. We rewrite our problem as

$$\max_{\{p,w\}} p\lambda(w) \tag{3.5}$$

where p is the proportion of the population receiving the wage w. The constraint is now simply

$$G(p\lambda L) = wpL . \tag{3.6}$$

The nature of the solution can be seen most simply if we define

$$\frac{w}{\lambda(w)} = y$$

so we wish to

$$\max \quad E$$

$$\text{s.t.} \quad G(E) = Ey$$

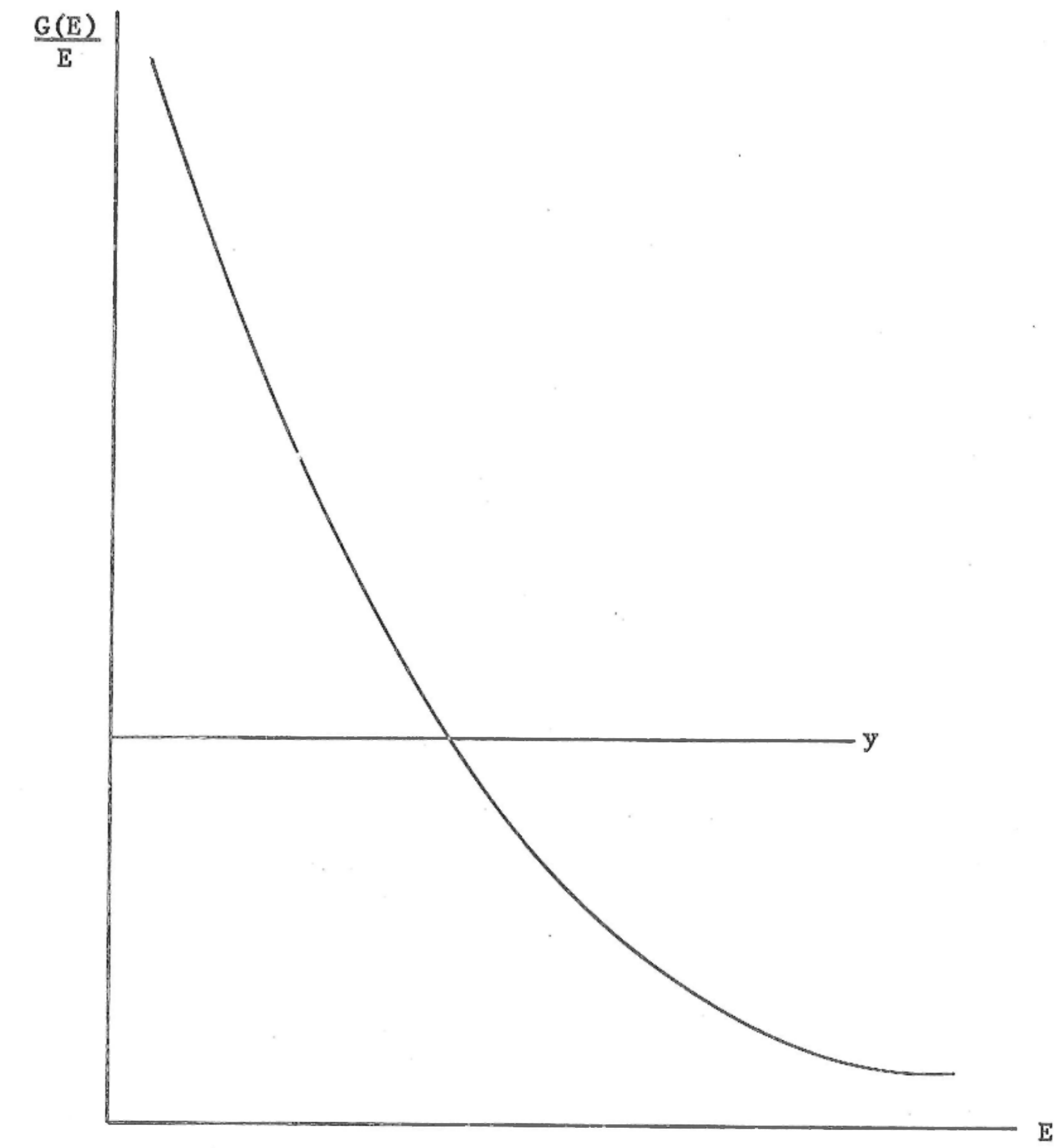

FIGURE 4

Hence E is maximized when y is minimized, i.e.

$$\min \frac{w}{\lambda(w)}$$

which simply yields the efficiency wage. The employment rate is

$$p = \frac{E^*}{\lambda(w^*)L} .$$

In this case the social marginal productivity of labor is exactly zero; an extra worker is simply unemployed and output is unchanged. An increase in efficiency which does not change the efficiency wage (as in the case of a technical improvement which increases the efficiency of a worker at each wage proportionately (see Figure 5) increases employment.[1] Then

$$\frac{\hat{\lambda}'w}{\hat{\lambda}} = \frac{m\lambda'w}{m\lambda} = \frac{\lambda'w}{\lambda} = 1$$

$$\frac{d \ln y^*}{d \ln m} = -1$$

$$\frac{d \ln E^*}{d \ln m} = \frac{1}{1-\alpha}$$

$$\frac{d \ln p}{d \ln m} = \frac{1}{1-\alpha} - 1 > 0 .$$

On the other hand, there can be other kinds of technical improvements which increase the efficiency wage a great deal but have a relatively small impact on the cost per unit effective labor; such a technical change will actually reduce the level of employment.

[1] The new efficiency curve, $\hat{\lambda}$, is related to the old curve by $\hat{\lambda} = m\lambda$ where m is a constant.

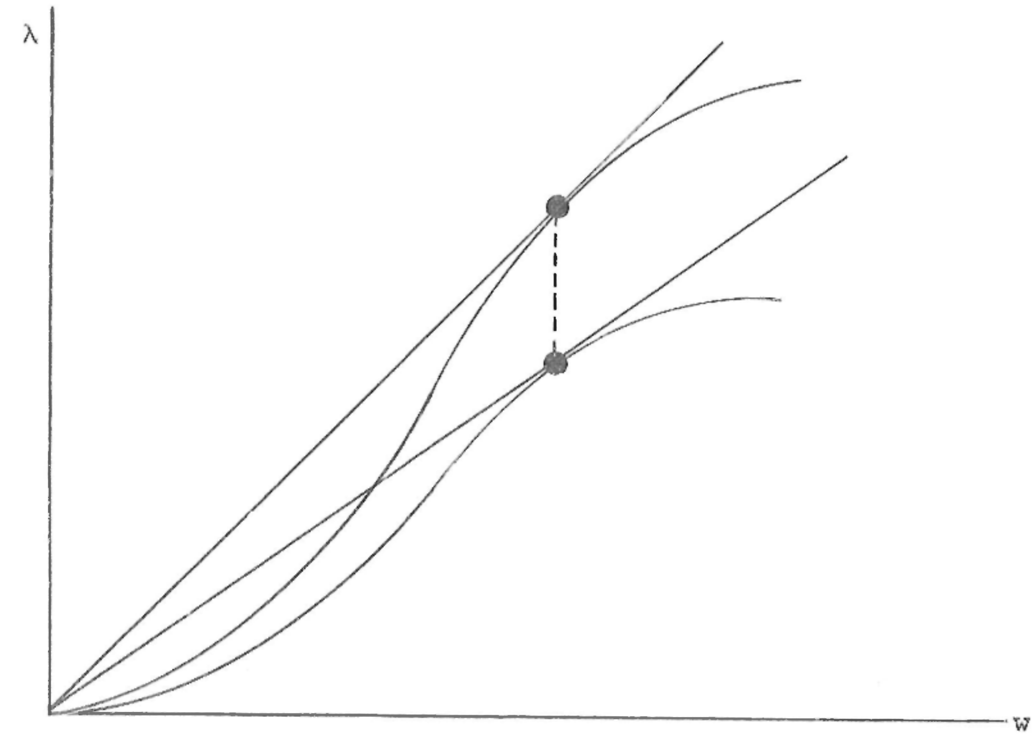

FIGURE 5a

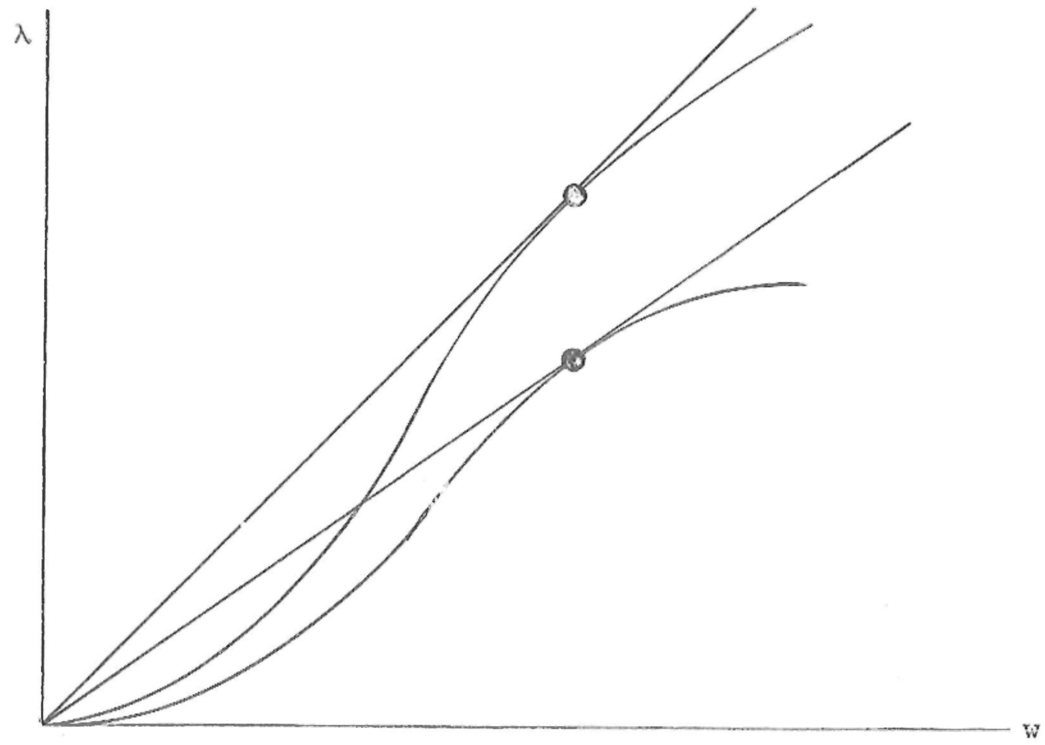

FIGURE 5b

Effect of Technical Change on Efficiency Wage

4. Maximization on Family Welfare

Obviously, the allocation of income among the members of the family described in the previous section, which leaves a proportion of the family with no wages, is probably unacceptable: most families would insist on supporting even their unproductive members at some, perhaps sub-subsistance level. This will, of course, reduce the total family income. One way of "capturing" the family trade-off between equity and efficiency is for the family to maximize a family welfare function. Let $V(w)$ be the utility associated with an income of w. We wish to[1]

$$\max \int V(w)dP(w) \tag{4.1}$$

subject to the income constraint (3.2). Our approach to solving this problem will be similar to that employed in Section 3. We first ascertain if wage levels w_1 and w_2 are paid to different individuals, what relations must exist between w_1 and w_2. We shall show that although there is only one level of wages that can be paid that exceeds $\hat{w}$, the point of inflection in the efficiency curve (see Figure 1), there may be many different wage levels paid to the "unproductive" (i.e. those for whom $w < \hat{w}$). We then simplify the problem to assume that there is only one low productivity wage. We let w_2 be the low wage, w_1 be the high wage. We then establish the following propositions:

[1]It is obvious that if V is linear, i.e. $V(w) = a + bw$, then maximization of (4.1) is equivalent to maximizing

$$\int w dP(w)$$

which, by (3.2), is equivalent to maximizing $G(E)$, family output. If $V(w)$ is of the form $-w^{-\rho}$, then maximizing

$$-\int w^{-\rho} dP(w)$$

is equivalent to maximizing

$$[\int w^{-\rho} dP(w)]^{-1/\rho}$$

and in the limit, as $\rho \to \infty$, this approaches

$$\min w$$

i.e. we obtain the completely equalitarian solution of section 2, where everyone receives his average product.

$$\hat{w} < w_1 < w^* . \tag{4.2}$$

w_1, the higher of the two wage levels, is less than the efficiency wage. This is as expected, since although an extra dollar to this group contributes more to productivity, it contributes less to "utility" since there is diminishing marginal utility.

$$\lambda(w_1)G' + \frac{R}{L} < w_1 < \lambda(w_1)G' + \frac{\lambda(w_1)}{p\lambda(w_1) + (1-p)\lambda(w_2)} \frac{R}{L} \tag{4.3a}$$

$$\lambda(w_2)G' + \frac{R}{L} > w_2 > \lambda(w_2)G' + \frac{\lambda(w_2)}{p\lambda(w_2) + (1-p)\lambda(w_2)} \frac{R}{L} \tag{4.3b}$$

where

$R = G - G'E$, the implicit rent on the land

and

p = proportion of population receiving a high wage.

The income received by any individual can be thought of as consisting of a wage payment, equal to his marginal productivity, $\lambda G'$, plus a share of the rent, R. Individuals in the more productive group receive more than a proportionate share of the rent. On the other hand, they contribute more to output. They receive less than their proportionate contribution to output.

$$\frac{dG}{dL} < 0 . \tag{4.4}$$

The social marginal productivity of a laborer in the rural sector is negative, i.e., increasing the number of workers in the rural sector reduces output.

$$\frac{dw_1}{dL} > 0 , \quad \frac{dw_2}{dL} < 0 . \tag{4.5}$$

As L increases, and output falls inequality in the rural sector increases: w_1 increases while w_2 decreases. Equality is a "luxury" of the well-off.

To see the relationship between the different wages paid to different individuals, consider two groups, with wages w_1 and w_2. The gain from giving one person at w_1 an increase in wage of one is V'. The resource cost is only $1 - G'\lambda'(w_1)$. To raise this revenue, we must take away $\frac{1 - G'\lambda'(w_1)}{1 - G'\lambda'(w_2)}$ from one person at w_2 with the consequent loss of utility. Hence we require for welfare maximization

$$V'(w_1) = \frac{1 - G'\lambda'(w_1)}{1 - G'\lambda'(w_2)} V'(w_2)$$

for all w_1, w_2 actually paid, i.e.

$$\frac{V'(w)}{1 - G'\lambda'(w)} = \text{constant} \tag{4.6}$$

for all w actually paid.

(4.6) is plotted in Figure 6. The logarithmic derivative of (4.6) is

$$\frac{V''}{V'} + \frac{G'\lambda''}{1 - G'\lambda'} . \tag{4.7}$$

The first term is always negative, the second is positive for $w < \hat{w}$ negative for $w > \hat{w}$. Accordingly, for $w > \hat{w}$, (4.6) is declining, establishing that there can be only one wage level in excess of $\hat{w}$. As w approaches zero, (4.6) approaches a positive infinite level, provided

$$\lim_{w \to 0} V'(w) \to \infty$$

marginal utility becomes infinite as w goes to zero, since under our assumptions concerning the efficiency curve,

$$\lim_{w \to 0} \lambda'(w) \to \epsilon \geq 0 .$$

Moreover as $w \to \infty$, (4.6) approaches zero, if

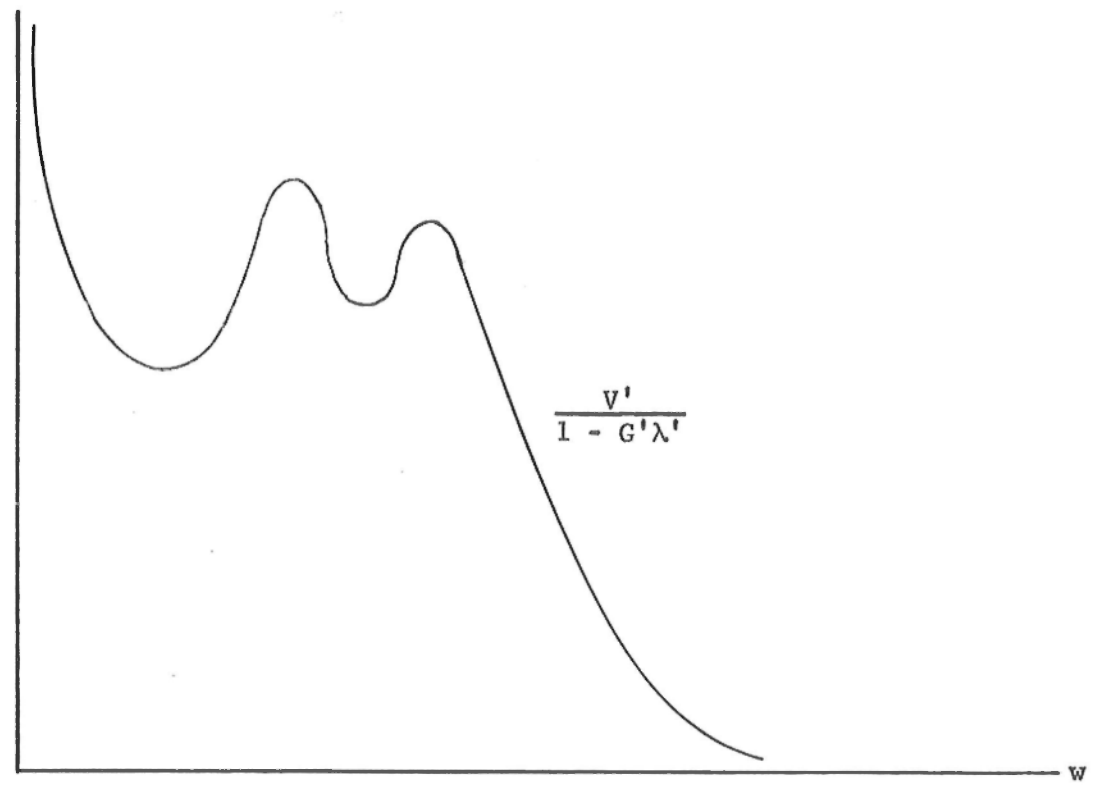

FIGURE 6a

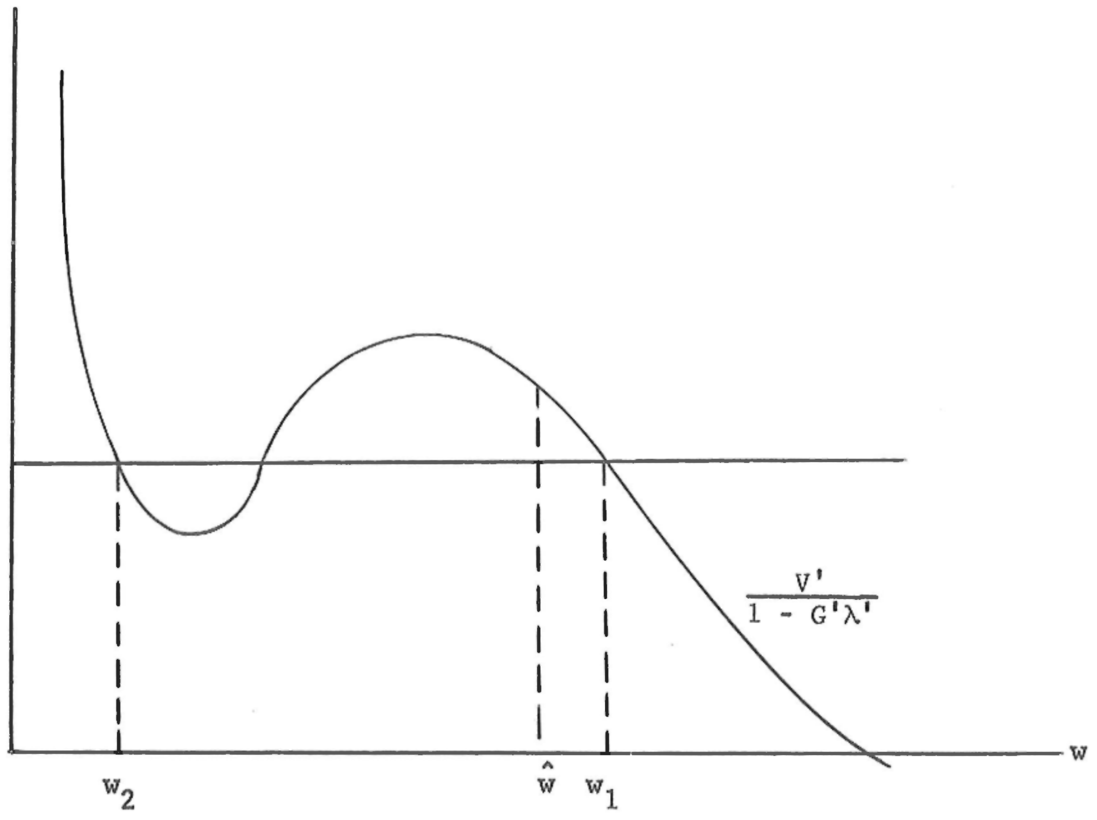

FIGURE 6b

Wage Determination in Family Welfare Maximizing Farm

$$\lim_{w \to \infty} V'(w) \to 0$$

and if

$$\lim_{w \to \infty} \lambda'(w) \to 0$$

i.e. there is a maximum attainable level of efficiency.

For $0 < w < \hat{w}$, there may be any number of local minimum or maximum; For instance, in Figure 6b, there are three wages at which $V'/1 - G'\lambda'$ equals the particular constant represented by the dotted line. Only two of these, denoted by w_1 and w_2, are utility maximizing wages; the intermediate point represents a local minimum.[1]

If $G' = 0$, i.e. the marginal productivity of labor services is zero, then of course there is no trade off between equity and efficiency; as (4.7) makes clear, $V'/1 - G'\lambda'$ is a monotonically declining function of w so there is only one wage: everyone receives his average product. On the other hand, if the economy is very well off, in the sense that if everyone received the efficiency wage, the average product would exceed w^*:

$$\frac{G(\lambda(w^*)L)}{L} \geq w^* , \tag{4.8}$$

again there is no trade off between efficiency and equity.

If workers get paid their average product, we have established in Section 2 that

$$\frac{dE}{dL} \gtrless 0 \quad \text{as} \quad L \lessgtr L^*$$

where

$$G(\lambda(w^*)L^*) = w^*L^* .$$

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[1]As we take wages from it and give it to say the group receiving a wage w_2, $V'/1 - G'\lambda'$ for the former group decreases, and for the latter increases, so further shifts are desirable.

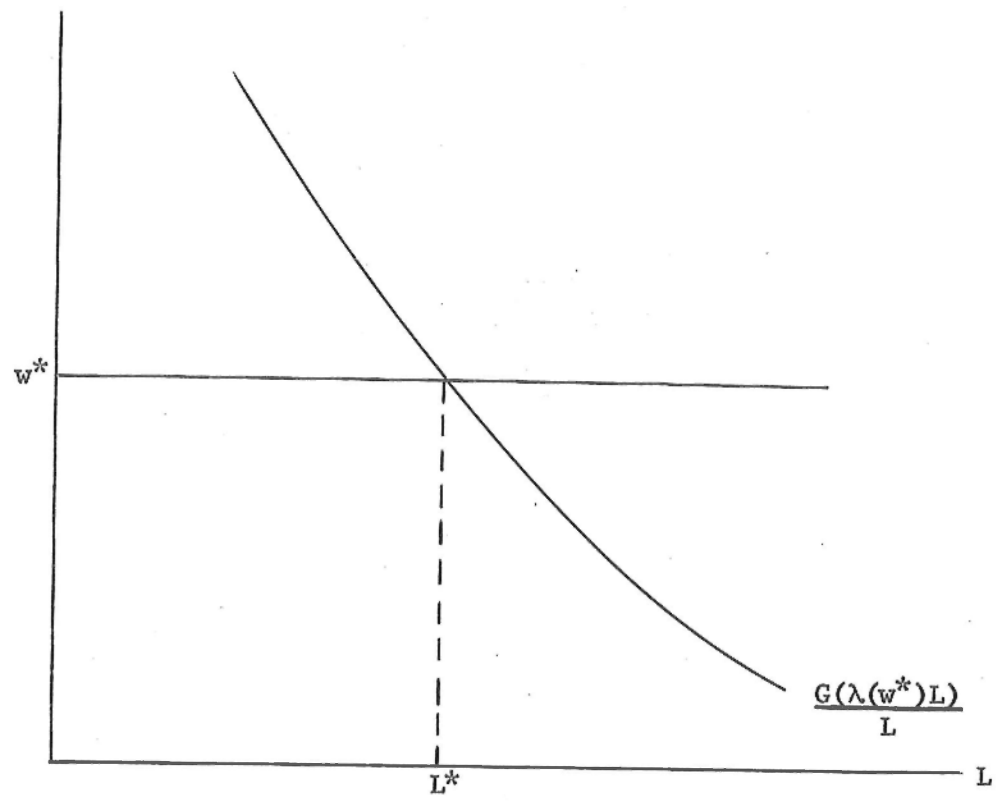

FIGURE 7a

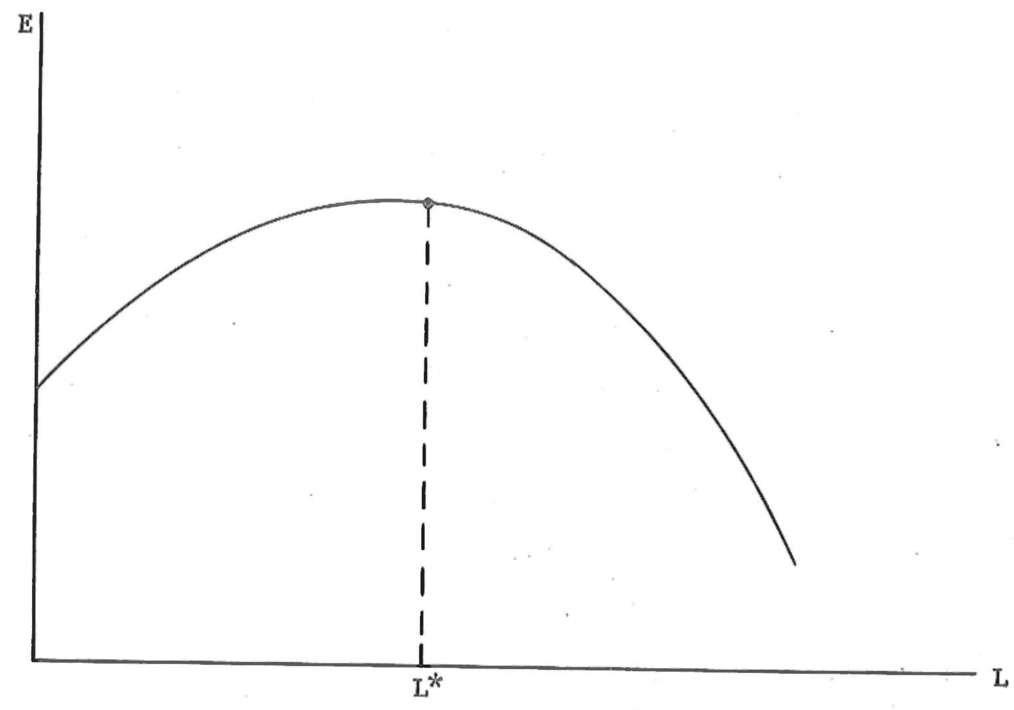

FIGURE 7b

Effective Supply of Labor Reaches Maximum at $L = L^*$

Hence $E \leq E^* \equiv (w^*)L^*$. Hence our analysis applies to situations when

$$G'(E^*) > 0$$

and

$$L > L^* \ . \tag{4.9}$$

Even in these situations, if the degree of equality preference (as measured by $-V''/V'$ or $-V''w/V'$) is sufficiently great, then again workers will receive their average product.[1,2] Our concern in this section is with the behavior of families when the degree of equality preference is sufficiently weak that the gains in total family income induce them to give different members of the family[3] different incomes.

We consider the case where there are exactly two wage groups. The extension to other cases is straightforward. To see more precisely the characteristics of the solution, we reformulate the problem to read

$$\max(pV(w_1) + (1-p)V(w_2))L \tag{4.10}$$

subject to

$$L(pw_1 + (1-p)w_2) = G[(p\lambda(w_1) + (1-p)\lambda(w_2))L] \tag{4.11}$$

where it will be recalled, p is the proportion of the population in the upper group. The first order conditions are simply

$$V'(w_i) + \nu(G'\lambda' - 1) = 0 \ , \quad i = 1, \ 2 \tag{4.12}$$

$$V(w_1) + \nu(G'\lambda(w_1) - w_1) = V(w_2) + \nu(G'\lambda(w_2) - w_2) \tag{4.13}$$

---

[1] For a discussion of the use of these as measures of degrees of equality preference see, A.B. Atkinson, "On the Measurement of Inequality," J. Econ. Theory, 2 (1970), 244-63.

[2] That is, $V'/1 - G'\lambda'$ will then be a monotonically declining function (see eq. (4.7)).

[3] It is clear that what is crucial is the sign of V''/V' relative to λ'', the degree of "increasing returns" to wages.

where ν is the shadow price associated with the constraint (4.10).

To see that $w_1 < w^*$, we multiply (4.12) by w_i and subtract the result from (4.13) to obtain

$$(V_1 - V_1'w_1) - (V_2 - V_2'w_2) = \nu G'[(\lambda_2 - \lambda_2'w_2) - (\lambda_1 - \lambda_1'w_1)] \qquad (4.14)$$

where

$$V_1 \equiv V(w_1)\ , \quad \lambda_1 = \lambda(w_1),\ \text{etc....}$$

Since $d(V - V'w)/dw = -V''w > 0$, the left hand side of (4.14) is positive. For w less than the efficiency wage w^*, $\lambda' > \lambda/w$, so $\lambda_2' > \lambda(w_2)/w_2$. Hence $\lambda_1 - \lambda_1'w_1 < 0$ if the R.H.S. of (4.14) is to be positive, i.e.,

$$\lambda'(w_1) > \frac{\lambda(w_1)}{w_1}\ ,$$

w_1 must be less than the efficiency wage.

To establish (4.3), we let S_i equal the amount by which the wage exceeds the marginal product:

$$S_i = w_i - G'\lambda_i\ . \qquad (4.15)$$

We can rewrite (4.13) then as

$$S_1 - S_2 = \frac{V_1 - V_2}{\nu}\ . \qquad (4.16)$$

But

$$pS_1 + (1-p)S_2 = R/L \qquad (4.17)$$

where R/L is the "rent" $(G - G'L)$ per capita. Substituting (4.17) into (4.16) we obtain

$$S_1 = (1-p)\frac{(V_1 - V_2)}{\nu} + \frac{R}{L} > \frac{R}{L}$$

$$S_2 = p\frac{(V_2 - V_1)}{\nu} + \frac{R}{L} < \frac{R}{L} .$$

On the other hand, the more productive contribute, on a per capita basis, more to output. If they were to receive income proportionate to their contribution to output, they would receive an amount

$$\frac{\lambda_1}{p\lambda_1 + (1-p)\lambda_2} \cdot \frac{G}{L} .$$

The difference between what the first group receives and its proportionate contribution to output is

$$w_1 - \frac{\lambda_1}{p\lambda_1 + (1-p)\lambda_2}(pw_1 + (1-p)w_2)$$

$$= \frac{(1-p)w_1w_2}{p\lambda_1 + (1-p)\lambda_2}\left(\frac{\lambda_2}{w_2} - \frac{\lambda_1}{w_1}\right) < 0 .$$

Similarly

$$w_2 - \frac{\lambda_2}{p\lambda_1 + (1-p)\lambda_1}(pw_1 + (1-p)w_2) > 0 .$$

The third and fourth propositions come from straightforward (but tedious) differentiation of the first order conditions for welfare maximization.[1]

---

[1]The family's optimal decisions are described by the four equations in the four unknowns, w_1, w_2, p, and ν, and the parameter L.

$$V'(w_1) + \nu(G'\lambda'(w_1) - 1) = 0$$

$$V'(w_2) + \nu(G'\lambda'(w_2) - 1) = 0$$

$$V_1 - V_2 + \nu(G'(\lambda_1 - \lambda_2) - (w_1 - w_2)) = 0$$

$$G((p\lambda_1 + (1-p)\lambda_2)L) - (pw_1 + (1-p)w_2)L = 0 .$$

Totally differentiating, we obtain (after some simplification)

$$\begin{bmatrix} -(1+a_1) & -1 & -1 & -b_1 \\ -1 & -(1+a_2) & -1 & -b_2 \\ -1 & -1 & -1 & -c \\ -b_1 & -b_2 & -c & 0 \end{bmatrix} \begin{bmatrix} p\lambda_1' dw_1 \\ (1-p)\lambda_2' dw_2 \\ (\lambda_1 - \lambda_2)dp \\ d\nu \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ e \end{bmatrix} [p\lambda + (1-p)\lambda] d \ln L$$

where

$$a_i = \frac{V_i'' + \nu G'\lambda_i''}{\nu G''L} > 0 , \quad i = 1, 2$$

$$b_i = \frac{-V_i'}{\lambda_i' \nu^2 G''L} > 0 , \quad i = 1, 2$$

$$c = \frac{-(V_1 - V_2)}{(\lambda_1 - \lambda_2)\nu^2 G''L} > 0$$

$$e = \frac{G - G'E}{(p\lambda_1 + (1-p)\lambda_2)\nu G''L} > 0$$

$$\frac{d \ln E}{d \ln L} = 1 + \frac{1}{p\lambda_1 + (1-p)\lambda_2} \quad p\lambda_1' \frac{dw_1}{d \ln L} + (1-p)\lambda_2' \frac{dw_2}{d \ln L} + (\lambda_1 - \lambda_2) \frac{dp}{d \ln L}$$

$$\sim a_1 a_2 c(c-e) \sim \frac{(w_1 - w_2) - G'(\lambda_1 - \lambda_2)}{\lambda_1 - \lambda_2} - \frac{\{pw_1 + (1-p)w_2 - G'(\lambda_1 p + \lambda_2(1-p))\}}{p\lambda_1 + (1-p)\lambda_2}$$

$$= \frac{w_1 w_2}{(\lambda_1 - \lambda_2)(p\lambda_1 + (1-p)\lambda_2)} \left(\frac{\lambda_2}{w_2} - \frac{\lambda_1}{w_1} \right) < 0$$

$$\frac{dw_1}{dL} \sim a_2(e-c)(c - b_1) < 0 ,$$

since, as we have just established, $e > c$ and

$$c - b_1 \sim \frac{1}{\nu} \left(\frac{V_1 - V_2}{\lambda_1 - \lambda_2} - \frac{V_1'}{\lambda_1'} \right) = \frac{G'\lambda_1' - 1}{\lambda_1'} - \frac{G'(\lambda_1 - \lambda_2) - (w_1 - w_2)}{\lambda_1 - \lambda_2}$$

$$= \frac{w_1 - w_2}{\lambda_1 - \lambda_2} - \frac{1}{\lambda_1'} > 0 .$$

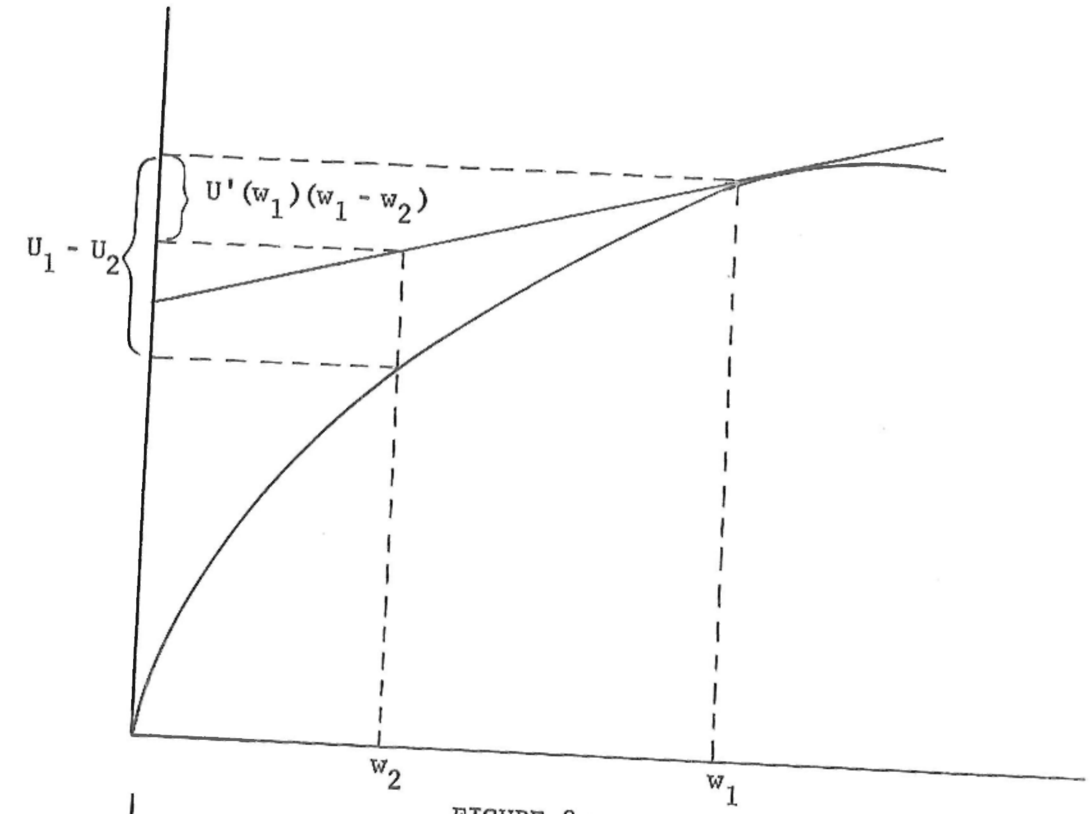

FIGURE 8a

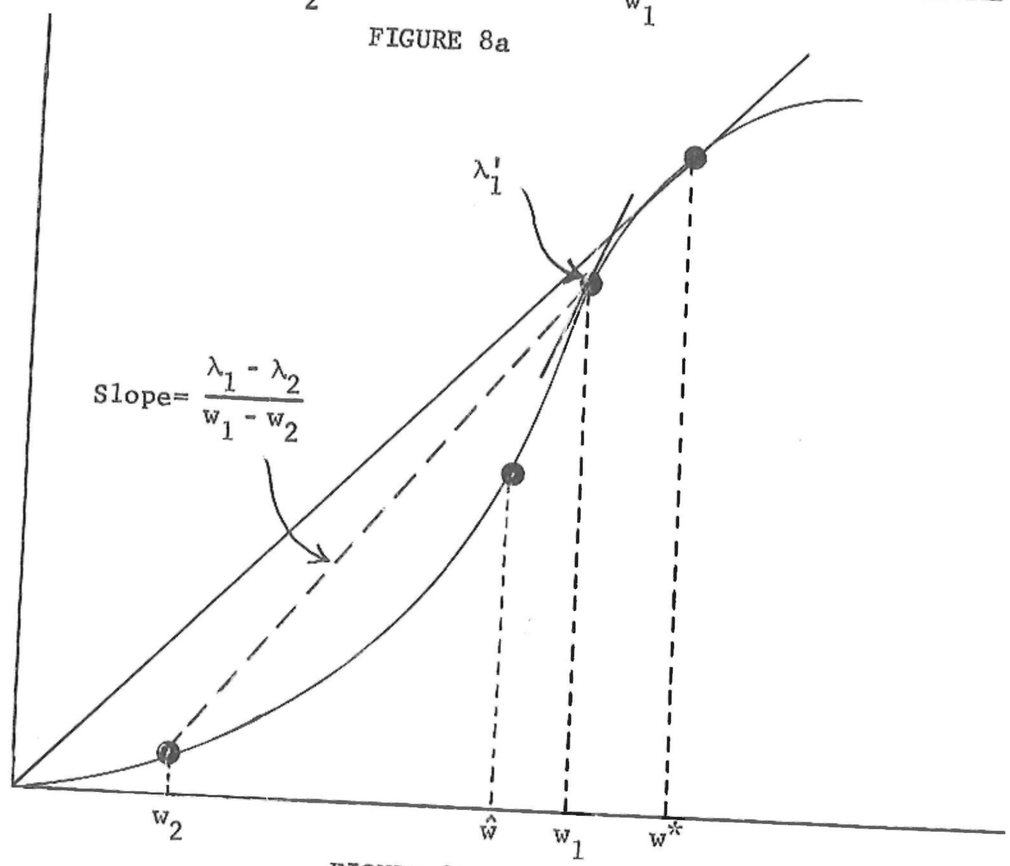

FIGURE 8b

5. Landless Peasantry: The Plantation Economy

If land in the rural sector is owned by a landlord class, and these landlords act to maximize profits, the equilibrium wage is identical to that of the output maximizing family farm. To see this, observe that the landlords will pay a wage to minimize costs per unit of effective labor, i.e.

$$\min \frac{w}{\lambda} \,. \tag{5.1}$$

The solution to (5.1) is what we have called the efficiency wage, and what we earlier established was the amount received by workers on the output maximizing farm. On the other hand, the level of employment is different. The landlord hires laborers up to the point where

$$G'\lambda(w^*) = w^* \tag{5.2}$$

The latter assertion follows from the fact that (using the concavity of V and (4.13))

$$\frac{V_1'(w_1 - w_2)}{\nu} < \frac{V_1 - V_2}{\nu} = (w_1 - w_2) - G'(\lambda_1 - \lambda_2)$$

which may be rewritten (using (4.12))

$$-G'\lambda_1' = \frac{V_1'}{\nu} - 1 < \frac{-G'(\lambda_1 - \lambda_2)}{w_1 - w_2}$$

or

$$\lambda_1' > \frac{\lambda_1 - \lambda_2}{w_1 - w_2} \,.$$

Similarly,

$$\frac{dw_2}{dL} \sim a_1(e-c)(c - b_2) > 0$$

since

$$c - b_2 \sim \frac{w_1 - w_2}{\lambda_1 - \lambda_2} - \frac{1}{\lambda_2'} < 0$$

(see Figure 8).

Since E has decreased and L increased, it is clear that average wages must decrease; w_1 has increased and w_2 decreased. It is easy to see that whether p increases or decreases depends in the relative magnitude of a_1 and a_2, about which we have made no assumptions. Accordingly, dp/dL may be positive or negative.

the wage is equal to the marginal productivity, while in the family farm, the wage is equal to the average product of the working members of the family:

$$\frac{G}{pL} = w^*$$

(where p is the proportion of the family working). Clearly, there is less employment and output in the plantation economy than in the output maximizing family farm. (See Figure 9.)

There is one objection which may be raised to this analysis. If the reasons for the efficiency curve is, at least partially, nutritional rather than psychological, and the workers on the plantation share their income with non working or poorer relatives, the landlord will reap, through the increased efficiency of his workers, only a part of the benefits of paying high wages. The implications of these sharing arrangements are discussed in Section 10.

PART B

We now return to the original objective of our study, the analysis of urban employment and the impact of various government wage and employment policies.

As we noted earlier, the effects of these policies depend critically on the nature of the rural sector, in particular, on whether there is a surplus of laborers (as we have discussed it in the preceding part) in agriculture. Sections 6-8 are concerned with an economy in which there is not a surplus of labor; Section 6 presents the basic model and describes the competitive equilibrium; Section 7 describes the second best situation where the government cannot control migration but sets the urban wage and employment levels optimally; while Section 8 describes the "third best" situation where the government uses wage subsidies to induce firms to hire more workers. Section 9 extends these results to economies in which there is a surplus of laborers. Section 10 considers the complications introduced by "sharing."

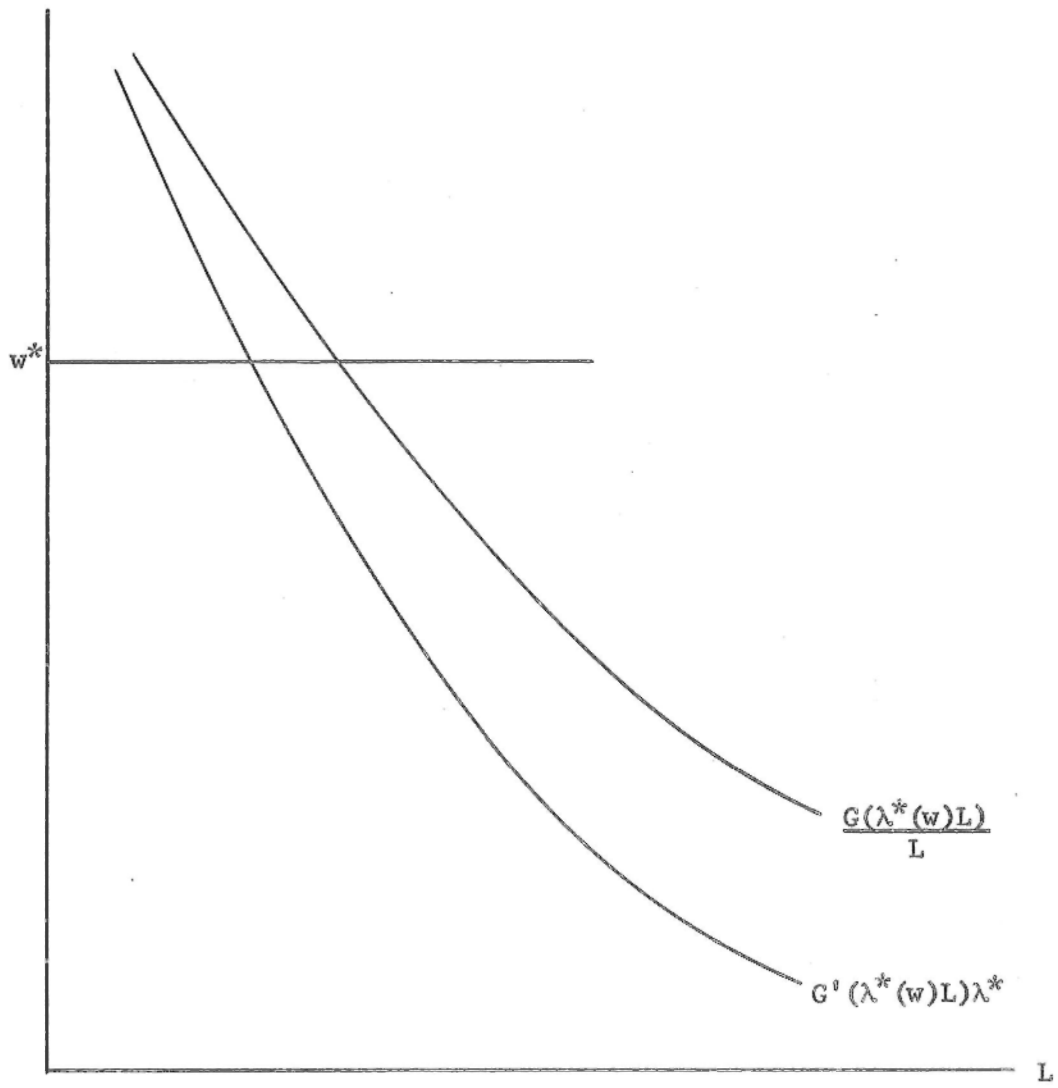

FIGURE 9

Comparison of Plantation and Output Maximizing Farms

6. The Competitive Determination of the Urban Wages and Employment

The basic model is identical to that presented in our earlier paper, except for the equation determining the urban wage. In our earlier study, by increasing wage rates, firms were able to reduce labor turnover and hence labor costs. Thus, for each level of unemployment and rural wage, there was an optimal urban wage, generally in excess of the rural wage. This high wage induced migration from the rural to the urban sector, which resulted in urban unemployment.

Here, by increasing the urban wage, firms increase the efficiency with which workers work. The efficiency curve for urban workers has the same basic shape as that described earlier for rural workers; there is an initial region of increasing returns followed by a region of diminishing returns. Thus, the efficiency of an urban worker is given by

$$\lambda_u = \lambda_u(w_u) , \quad \lambda_u' \geq 0 , \quad \lambda_u'' > 0 \quad \text{for} \quad w_u < \hat{w}_u , \quad \lambda_u'' < 0 \text{ for } w_u > \hat{w}_u \tag{6.1}$$

where w_u is the urban wage (of the representative firm in the urban sector). Firms wish to minimize the cost per effective unit of labor, i.e.

$$\min \frac{w_u}{\lambda_u} \tag{6.2}$$

the solution to which requires

$$\frac{\lambda_u' w_u}{\lambda_u} = 1 . \tag{6.3}$$

We denote the wage satisfying (6.3) as the urban efficiency wage, w_u^*. (Note that the efficiency curve for agricultural sector may be quite different from that in the urban sector, and accordingly the efficiency wage may be quite different; the efficient organization of production in the urban sector may, for instance, require 8 to 10 hour working days, in the rural sector shorter working days may be efficient except at peak times.)

Employment is then determined in a straightforward manner: the short run production function in the urban sector is[1]

$$Q_u = F(\lambda_u L_u) , \quad F' \geq 0 , \quad F'' \leq 0 \tag{6.4}$$

where $\lambda_u L_u$ is the effective labor supply, and L_u is the number of laborers. Labor is hired to the point where the wage is equal to the marginal product:[2]

$$w_u^* = F'\lambda_u . \tag{6.5}$$

For simplicity, in this and the next two sections we shall assume that there is no efficiency wage curve applicable to the rural sector, i.e. wages in the rural sector are sufficiently high that they have reached their asymptotic efficiency levels (for performing agricultural tasks).[3] The rural wage is equal to the marginal product of labor in the rural sector

$$w_r = G'(L_r) \tag{6.6}$$

where L_r is the number of workers in the rural sector. If w_u is greater than w_r, workers will be induced to migrate from the rural to the urban sector; but as they migrate the unemployment rate in the urban sector increases. This serves to discourage further migration. We postulate that corresponding to any urban-rural wage differential, there is an equilibrium level of unemployment. We write the relationship in the form

$$\frac{w_u}{w_r} = \varphi\left(\frac{1}{1-U}\right) , \quad \varphi' > 0 , \tag{6.7}$$

where U is the unemployment rate.

---

[1]We again assume that capital stock is fixed and unaffected by the contemplated policy changes we are about to describe.

[2]As in the earlier part of this paper, we assume an open economy so that we can normalize the price of all goods at unity.

[3]Without loss of generality, we can let this value of $\lambda_r = 1$, so $E_r = \lambda_r L_r = L_r$.

Under certain circumstances, when workers are hired either completely randomly or in the order in which they arrive in the city, and there is no growth in urban employment, if workers migrate to the point where the expected urban wage is equal to the rural wage, (6.7) takes on a simple form

$$w_u(1-U) = w_r \ . \tag{6.7'}$$

Finally, laborers must live either in the urban or rural sector. If L^s is the total number of laborers, N_u workers and job seekers in the urban sector, then

$$L^s = N_u + L_r \tag{6.8}$$

where

$$N_u = \frac{L_u}{1-U} \ . \tag{6.9}$$

The description of the competitive equilibrium is straightforward. Knowing w_u^*, from (6.5), we can immediately determine L_u. Then substituting (6.9) into (6.8) and the result into (6.6) gives us with (6.7) two equations in two unknowns which we have plotted in Figure 10. Under our assumptions, it is clear that there is a unique equilibrium. Note that increments to the total labor force do not affect the urban wage or urban employment; rural wages fall and urban unemployment rises.

7. Optimal Wage and Employment Policy

We shall show that if the government chooses the wage and employment level in the urban sector to maximize national output, and if the "expected" urban wage, $w_u/1-U$, equals the rural wage, it will set the wage at w_u^*: the competitive wage is identical to the optimal wage. The competitive level of employment is, however, less than the optimum.

Formally, the problem of the government is to choose w_u, L_u and (implicitly) U to maximize the value of national income

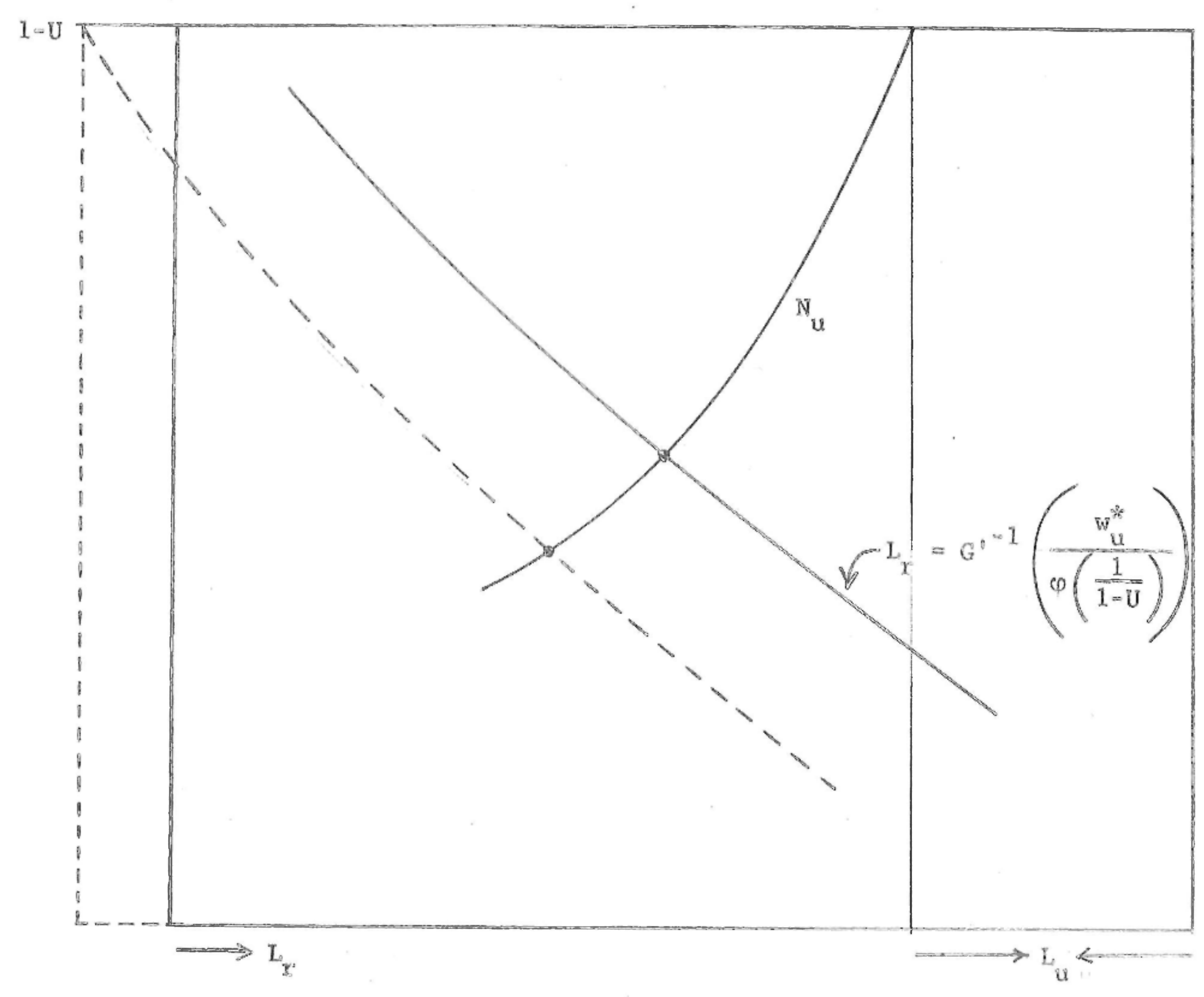

FIGURE 10

Competitive Equilibrium
(dotted lines show effect of increased population)

$$(7.1) \qquad F(\lambda_u(w_u)L_u) + G\left(L^s - \frac{L_u}{1-U}\right)$$

subject to the free migration constraint (6.7) which we can write as

$$(7.2) \qquad G'\left(L^s - \frac{L_u}{1-U}\right) = w_u/\varphi\left(\frac{1}{1-U}\right) .$$

Letting μ be the Lagrange multiplier associated with the constraint, we obtain as our first order conditions

$$(7.3a) \qquad F'\lambda_u' L_u - \frac{\mu}{\varphi} = 0$$

$$(7.3b) \qquad F'\lambda_u - \frac{G'}{1-U} - \frac{\mu G''}{1-U} = 0$$

$$(7.3c) \qquad -\frac{G'}{(1-U)^2} L_u + \mu\left(-\frac{G''L_u}{(1-U)^2} + \frac{w_u}{\varphi^2}\frac{\varphi'}{(1-U)^2}\right) = 0 .$$

Substituting (7.3b) into (7.3c) and the result into (7.3a), we obtain

$$(7.4) \qquad \frac{d \ln \lambda_u}{d \ln w_u} = \frac{\varphi(1-U)}{\varphi'} .$$

Thus, if

$$(7.5) \qquad \varphi = \frac{k}{1-U}$$

then the optimum wage is just equal to the efficiency wage

$$(7.4') \qquad \frac{\lambda_u' w_u}{\lambda_u} = 1 .$$

A special case of (7.5) is (6.7'), the case described earlier when the expected urban wage equals the rural wage. More generally, the optimal urban wage, w_u^0, is greater or less than the competitive (efficiency) wage, w_u^c, as $\beta \gtrless 1$, where we define

$$(7.6) \qquad 1/\beta \equiv \varphi(1-U)/\varphi' .$$

If $G'' = 0$ (constant marginal productivity of labor in the rural sector) from (7.3b)

$$F'\lambda_u = \frac{G'}{1-U} = \frac{w_r}{1-U} .$$

Thus, in our special case where migration continues until the rural wage equals the expected urban wage, not only is the competitive wage optimal, but the equilibrium level of unemployment is also optimal.

More generally, we have

$$F'\lambda_u = \frac{w_u}{\varphi(1-U)\left\{1 + \frac{1}{\eta_r}\frac{N_u}{L_r\beta}\right\}} \tag{7.7}$$

where

$$\eta_r \equiv - \frac{G'}{G''L_r} \tag{7.8}$$

is the elasticity of demand for labor in the rural sector. Empirically, $\varphi(1-U)$ appears to be greater than or equal to unity, so that the shadow price of labor is less than the urban wage. How much less depends on the elasticity of demand for labor and the magnitude of $\varphi(1-U) - 1$. If hiring one more worker in the urban area results in some migration from the rural area, which raises the rural wage, the unemployment <u>rate</u> will be <u>reduced</u>. The smaller η_r, the more the unemployment rate is reduced, hence the lower the loss in output due to induced unemployment; this effect is more important the larger the urban sector is relative to the rural sector. If hiring one more urban worker results in $1/1-U$ workers migrating from the rural sector and if the marginal product of a rural worker is $w_r = w_u/\varphi$ clearly the loss in rural output (ignoring the effect on the unemployment rate) is $w_u/\varphi(1-U) \leq 1$ as $\varphi(1-U) \geq 1$. (Thus, if $w_u = w_r/1-U$, the loss in output is just the rural wage.) This effect is probably quantitatively more important than the effect on the unemployment rate, as we shall note in more detail below.

The shadow price of labor in the urban sector may be greater or less than the rural wage

$$F'\lambda_u = \frac{G'}{(1-U)\left(1 + \frac{1}{\eta_r}\frac{N_u}{L_r\beta}\right)} \gtrless G'$$

as

$$1 + \frac{1}{\eta_r}\frac{N_u}{L_r\beta} \lessgtr \frac{1}{1-U} \, .$$

These results imply that the competitive allocation will entail hiring too few workers in the urban sector and a higher unemployment rate than is optimal.

The magnitude of the deviation between optimal and competitive levels depends essentially on the magnitude of $\varphi(1-U) - 1$.

Consider the following numerical values:

$$\frac{N_u}{L_r} = 5\% \, .$$

the urban work force is 5% of the rural work force;

$$\eta_r = 2$$

e.g. we have Cobb-Douglas production function with the coefficient of labor equal to 1/2; $\beta = 1$ and

$$\varphi(1-U) = 1.2$$

e.g. $w_u/w_r = 1.5$ when the unemployment rate is 20%. Then the shadow price on labor is

$$\frac{w_u}{1.2\left(1 + \frac{.05}{2}\right)} \approx .83$$

i.e. it is 83% of the urban wage, but it is

$$\frac{1}{.8\left(1+\frac{.05}{2}\right)} \approx 1.2$$

times the rural wage.

For this example, the shadow price of labor in the urban sector is half way between the urban and rural wages.

If, on the other hand, the expected urban wage is equal to the rural wage, then the shadow price on urban labor is $N_u/\eta_r L_r$ less than the market wage, i.e. in our numerical example, only 2-1/2% less.

8. Wage Subsidies

A wage subsidy does not affect the wage the competitive firm pays, i.e. it is not shifted at all. If the ad valorem wage subsidy is at the rate τ, the firm minimizes

$$\frac{w(1-\tau)}{\lambda(w)} \tag{8.1}$$

i.e.

$$\frac{(1-\tau)}{\lambda} - \frac{w(1-\tau)}{\lambda^2}\lambda' = 0$$

or

$$\lambda' = \lambda w$$

which does not depend at all on τ. On the other hand, firms will now hire laborers up to the point where

$$F'\lambda(w^*) = w^*(1-\tau) . \tag{8.2}$$

Hence, if $\varphi(1-U)/\varphi' = 1$, the optimal wage subsidy is just

$$1-\tau = \frac{1}{\varphi(1-U)}\Bigg/\left(1+\frac{1}{\eta_r}\frac{N_u}{L_r}\right) \tag{8.3}$$

i.e. in our numerical example, a 17% wage subsidy is called for. If the expected urban wage equals the rural wage, the subsidy is negligible--in our example, only 2.5%.

With these subsidies, the economy can attain exactly the same equilibrium as in the situation where it controls wages and employment directly.

(This is not the case if β is not equal to unity; then the optimal wage differs from the competitive wage, and a wage subsidy has no effect on the competitive wage. The optimal wage subsidy is still given by

$$(8.4) \qquad (1-\tau) = \frac{1}{\varphi(1-U)\left(1+\frac{1}{\eta_r}\frac{N_u}{N_r}\frac{1}{\beta}\right)} .$$

Note that even if β is greater than unity, a wage subsidy is called for, provided $\varphi(1-U) \geq 1$.)[1]

9. Surplus Laborers in the Rural Sector

We now consider the situations we focused on in Part A, where there is an efficiency curve for labor in the rural sector as well. In the situation described in Section 5, where there is unemployment among the landless peasantry, the relative wages in the urban and rural sector determine the allocation of the unemployed; for instance, if workers were picked on a daily basis from the job seekers, equilibrium would require

$$(9.1) \qquad w_u(1 - U_u) = w_r(1 - U_r)$$

where U_u and U_r are the urban and rural unemployment rates. In this case, there is no loss of output from increasing urban unemployment, simply a shift in the location of the unemployed. Accordingly, optimal government policy requires hiring labor (subsidizing wages)at least to the point where there

---

[1]To derive (8.4), we take the derivative of

$$F(\lambda_u L_u) + G\left(L^s - \frac{L_u}{1-U}\right)$$

with respect to L_u , recalling that U is a function of L_u (eq. 7.2) and that $w_u = w_u^*$.

is no rural unemployment, and then somewhat beyond that.[1]

In the cases described in Sections 2-4, we must know how the decision to migrate from the rural to the urban sector is made.[2] The simplest hypothesis is that the marginal individual migrates if the expected urban income exceeds his wage in the rural sector. In the case of the equalitarian family, this means that

$$w_u(1-U) = G/L_r \; . \tag{9.2}$$

The competitive solution is depicted in Figure 11. Note that as L_r increases, average rural income falls; since urban employment is given by (6.5), as L_r increases, the unemployment rate decreases, and expected urban wages rise. Hence there is a unique equilibrium.

Since now the social marginal productivity of a rural laborer is negative, optimal government policy clearly requires hiring workers in the urban sector --and thus inducing migration and urban unemployment--at least to the point where the social marginal productivity is zero. More precisely, optimal government policy requires[3]

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[1] This ignores, of course, any long run (e.g. consumption-investment) implications of such a policy.

[2] See J.E. Stiglitz, "Rural-Urban Migration..." op.cit.

[3] Throughout this discussion, we employ the simplifying assumption that $w_u(1-U) = w_r$. The extension to the more general case is straightforward: we wish to

$$\max F(\lambda_u(w_u)(L - L_r)(1-U)) + G(L_r\lambda_r(w_r))$$

where

$$w_r = \frac{G(L_r\lambda_r(w_r))}{L_r} = \frac{w_u}{\varphi(1/(1-U))}$$

We thus obtain

$$F'\lambda_u' L_u + \frac{\mu}{\varphi} = 0$$

$$-F'\lambda_u(1-U) + \lambda_r G'\left(1 + \frac{\lambda_r'}{\lambda_r} w_r \frac{d \ln w_r}{d \ln L_r}\right) - \mu \frac{d w_r}{d L_r} = 0$$

$$-F' \frac{\lambda_u L_u}{1-U} - \frac{\mu w_u}{\varphi^2} \frac{\varphi'}{(1-U)^2} = 0 \; .$$

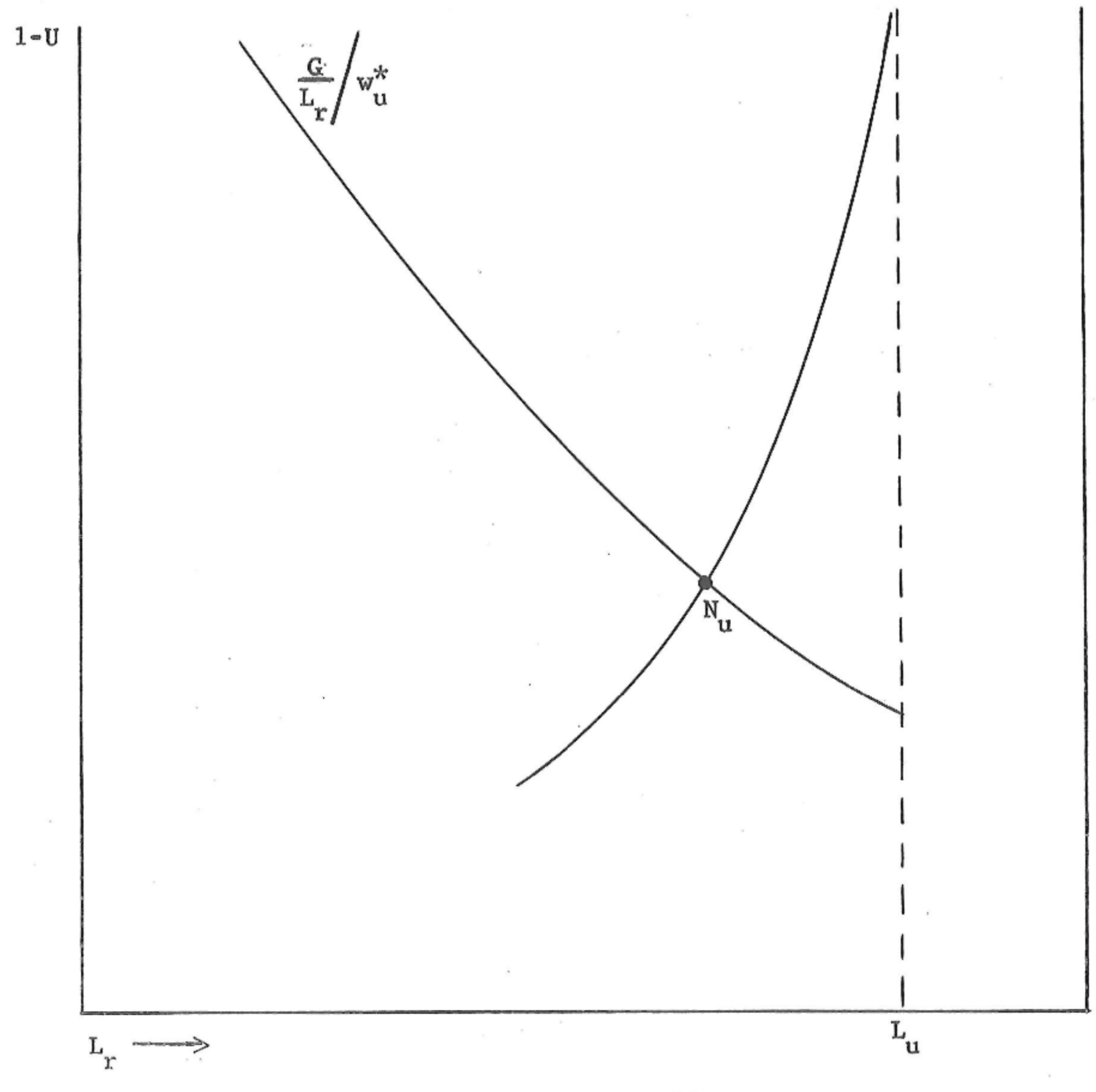


FIGURE 11

Competitive Equilibrium with Equalitarian Family Farms

(9.3)
$$\frac{\lambda_u' w_u}{\lambda_u} = 1$$

(9.4a)
$$F'\lambda_u = \frac{G'\lambda_r\left(1 + \frac{d \ln \lambda_r}{d \ln w_r}\frac{d \ln w_r}{d \ln L_r}\right)}{1 - U - \frac{L_u}{L_r}\frac{d \ln w_r}{d \ln L_r}}$$

(9.4b)
$$= w_u \frac{\alpha\left(1 - \frac{\lambda_r' w_r}{\lambda_r}\right)}{\left(1 - \alpha\frac{\lambda_r' w_r}{\lambda_r}\right)\left(1 + \frac{N_u}{L_r}\frac{(1-\alpha)}{1 - \alpha\frac{\lambda_r' w_r}{\lambda_r}}\right)}$$

where $\alpha = G'\lambda_u L_u/G$ the "share" of labor in the rural sector. The first condition is the familiar result that the urban wage should be the efficiency wage.

Optimality requires migration to continue beyond the point where $\lambda_r' w_r/\lambda_r = 1$, as (9.4b) makes clear. Since dw_r/dL_r is negative, it is clear that (provided U is small) the marginal productivity of labor in the urban sector (the shadow price of labor) is <u>less</u> than $G'\lambda_r$, the private marginal productivity of a laborer in the rural sector (i.e. ignoring his effect on average output and hence on productivity); indeed it is less than $\lambda_r G'\left(1 + \frac{\lambda_r' L_r}{\lambda_r}\frac{dw_r}{dL_r}\right)$, the social marginal productivity of a laborer in the rural sector, and w_u , the wage in the urban sector. The reason for this

---

Since
$$\frac{d \ln w_r}{d \ln L_r} = \left(\frac{w_r - G'\lambda_r}{(G'\lambda_r' - 1)L_r}\right)\frac{L_r}{w_r}$$
$$= \frac{1 - \alpha}{\alpha\frac{\lambda_r' w_r}{\lambda_r} - 1}$$
we obtain (9.4) upon substitution, when $\beta = 1$.

is that hiring additional urban workers, by raising the rural wage, actually reduces the unemployment rate. This effect is stronger here than in the case discussed in Section 7 because rural workers receive their average product (rather than marginal product) and, as workers leave the rural sector, the efficiency of the representative worker increases.

As in the earlier analysis, since the competitive wage is optimal, by the appropriate wage subsidy, the optimal allocation can be obtained.

10. Sharing

In discussing the consequences of alternative wage levels on the individual worker's productivity, so far we have not taken into account the consequences of "sharing" on productivity behavior. If the reasons for greater productivity are essentially nutritional, then a given wage increment will have less effect on productivity the greater the number of individuals among whom the income increment is to be shared. If, for instance, the urban employed shared their income equally with the urban unemployed, the competitive wage would be

$$\lambda_u'(1-U) = \frac{\lambda_u}{w_u} \tag{10.1}$$

which implies a higher competitive wage than in the earlier analysis. Indeed, observing that[1] $w_u/w_r = \varphi(1/1-U)$ we obtain

$$\frac{\lambda_u'(w_r\varphi(1/1-U)(1-U))}{\lambda(w_r\varphi(1/1-U)(1-U))}\, w_r\varphi(1-U)(1-U) = 1 \tag{10.2}$$

so that the optimal urban wage is such that $w_r\varphi(1/1-U)(1-U) = w_u(1-U)$ is equal to w^*, hence

$$w_r\varphi(1/1-U)(1-U) = w^* \tag{10.3}$$

$$w_u = \frac{w^*}{1-U} \tag{10.4}$$

where w^* is the solution to

---

[1] One might argue that one ought to replace (6.7) by $\frac{w_u(1-U)}{w_r} = \varphi\left(\frac{1}{1-U}\right)$. Then instead of (10.3) we obtain, $w_r\varphi = w_u^*$. (10.4) is unaltered.

(10.5) $$\frac{\lambda_u'(w^*)w^*}{\lambda_u(w^*)} = 1 .$$

For instance, in the special case where

(10.6) $$\varphi(1/1\text{-}U) = 1/1\text{-}U$$

then

(10.3') $$w_r = w^* .$$

Competitive equilibrium will require that the rural wage be equal to w^*. More generally, since $\varphi(1/1\text{-}U)(1\text{-}U) > 1$, $w_r < w^*_r$. In our numerical example (p. 25) with $\varphi = 1.5$ when $U = .2$, $w_r = \frac{5}{6} w^*_u$, and $w_u = 1.25w^*_u$.

The rest of the market equilibrium is easy to describe. For simplicity, we assume there is no efficiency curve in the rural sector (i.e. we revert to the model of Sections 6-8). In the special case when (10.6) is true, so $w_r = w^*$, at that wage there is a given "employment" in the rural sector, $\hat{L}_r$. Hence, the number of individuals in the urban sector is just

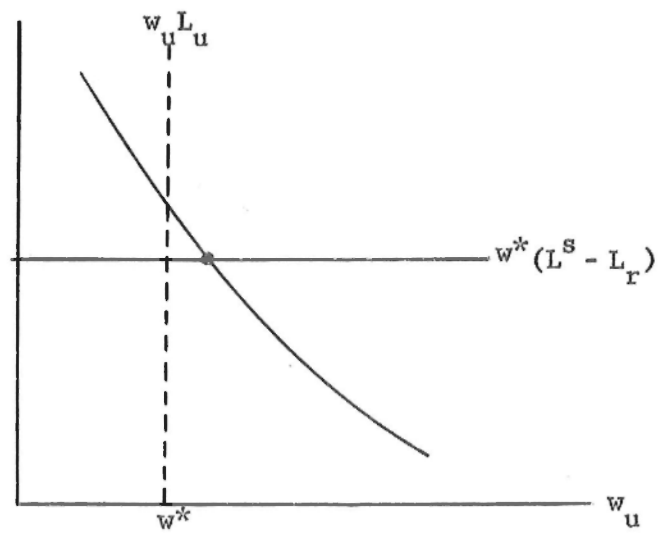
$$L^s - \hat{L}_r = N_u = \frac{L_u}{1\text{-}U} .$$

Substituting in from (10.3), we obtain

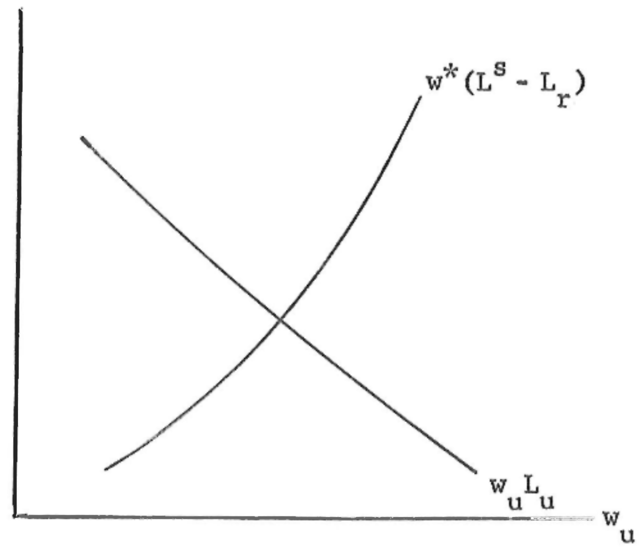
(10.7) $$w^*(L^s - \hat{L}_r) = L_u w_u .$$

The R.H.S. of (10.7) is an increasing, constant, or decreasing function of w_u as the elasticity of demand for labor is less than, equal to, or greater than unity in the urban sector. In the "normal case" say with a Cobb-Douglas production function, the equilibrium will be depicted as in Figure 12a.

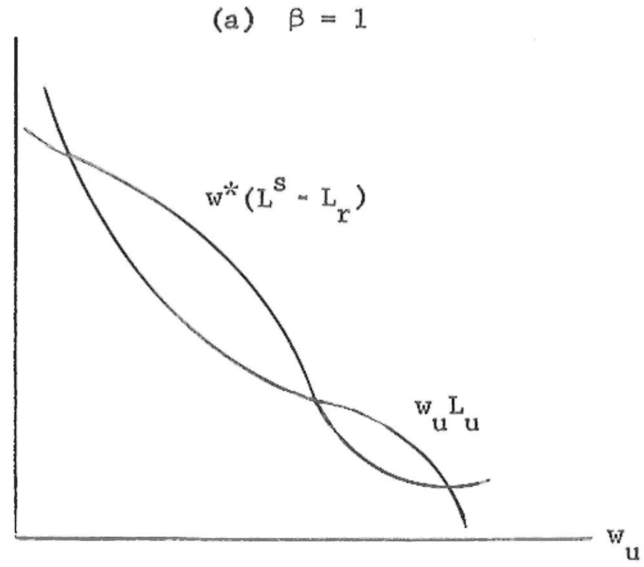
As Figure 12d makes clear, it is possible that there be no equilibrium in this economy. At $w_u = w^*$, there is an excess supply of workers. As the workers migrate into the urban sector, firms increase the wage they pay workers to get the optimal level of efficiency and because of the increased



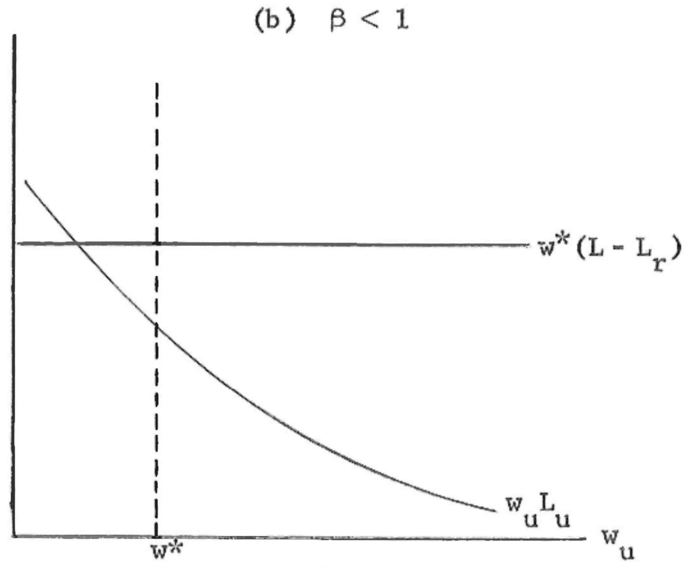
(a) $\beta = 1$



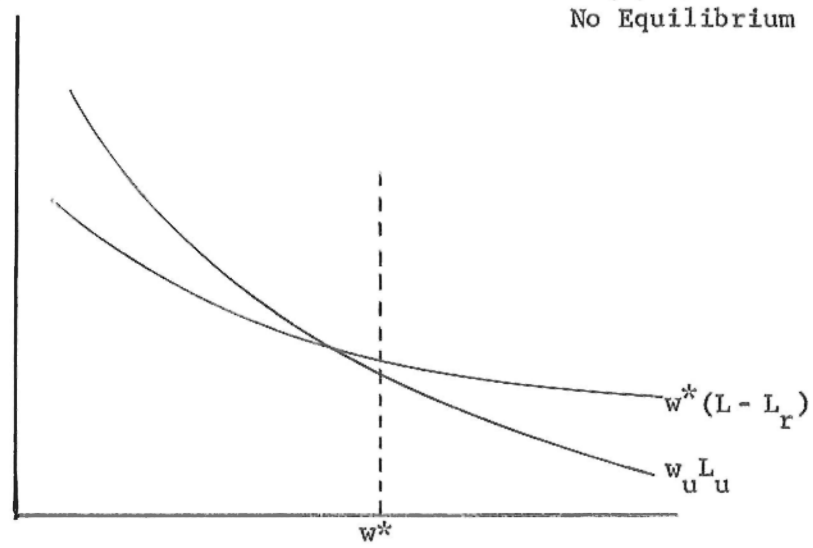
(b) $\beta < 1$



(c) $\beta > 1$



(d) $\beta = 1$
No Equilibrium



(e) $\beta > 1$
No Equilibrium

FIGURE 12

Competitive Equilibrium with Sharing

labor costs reduce the number of workers. This simply increases the number of unemployed, necessitating a further increase in wages to maintain "optimal efficiency."

This is a result of our extreme assumption that income is shared equally among all individuals in the urban sector. If only a fraction of income is shared equally, then the L.H.S. of (10.7) diminishes as well, as the urban wage increase (the unemployment rate increases). To see this, assume that a fraction f of income is shared equally. The worker consumes then

$$w_u((1-f) + (1-U)f) \tag{10.8}$$

and the optimal wage is

$$w_u(1 - fU) = w^* \tag{10.9}$$

so

$$w_r = \frac{w^*}{\varphi\left(\frac{1}{1-U}\right)(1-U)} \frac{(1-U)}{(1 - fU)}$$

$$\frac{d \ln w_r}{d \ln (1-U)} = \left(\beta - 1\right) + 1 - \frac{(1-U)f}{1 - fU}$$

$$= \left(\beta - 1\right) + \frac{1 - f}{1 - fU} .$$

If $\beta = 1$, this is unambiguously positive. Instead of (10.7) we now have

$$w^* \frac{(1-U)}{1 - fU} (L^s - L_r) = w_u L_u . \tag{10.7'}$$

Thus the L.H.S. diminishes as U increases, and equals zero at a finite value of w_u , provided the elasticity of demand for rural workers is bounded away from zero. It is also clear by the same token there may be multiple equilibrium (particularly if $\beta > 1$) and the economy may be caught in a high unemployment, high sharing "trap." (See Figure 12c)

The analysis of the optimal wage and employment policy is somewhat more complicated than in the earlier case. An increase in w_u results in an increase in efficiency only if the induced migration is "small." This will be the case if the elasticity of demand for labor in the rural sector is small.

We wish to

$$(10.8)\qquad \underset{\{L_u,\ 1/1-U\}}{\text{maximize}}\ F\left(\lambda_u\left(G'\left(L - \frac{L_u}{1-U}\right)(1-U)\varphi\right)L_u\right) + G\left(L - \frac{L_u}{1-U}\right)$$

yielding

$$(10.9a)\qquad F'(\lambda_u - L_u\lambda_u' G''\varphi) = \frac{G'}{1-U}$$

$$(10.9b)\qquad [(1-U)\varphi F'\{-\lambda_u' L_u G'' + \lambda_u'(1-U)(\beta - 1)G'\} - G']U(1-U) = 0\ .$$

Thus, either $U = 0$, in which case $w_u = w_r$, or

$$(10.10)\qquad -\frac{\lambda_u' w_r}{\lambda_u} = \frac{1}{(1-\beta)\varphi(1-U)}$$

and

$$(10.11)\qquad F'\lambda_u = \frac{w_r}{(1-U)\left(1 + \frac{N_u}{L_r\eta_r(\beta-1)}\right)}$$

From (10.10) if $\beta \leq 1$, $w_u = w_r$. For $\beta > 1$, the nature of the solution may be seen most easily for the case where β and η_r are constant. We solve (10.10) and (10.11) for U and w_r . Rewriting (10.11) we have

$$(10.12)\qquad \lambda_u(1-U)F'((L - L_r)(1-U)\lambda_u(w_r))\left(1 + \frac{L^s - L_r}{L_r}\frac{\lambda_u'(w_r)w_r\varphi(1-U)}{\lambda_u\eta_r}\right) = w_r\ .$$

(10.12) must hold whether $U = 0$ or $U > 0$. If one of the values of w_r for which $U = 0$ in (10.12) is such that $\lambda_u'(w_r)w_r/\lambda_u(w_r) < 1/(\beta-1)\varphi(1-U)$ then optimality requires full employment. (13b) illustrates such a case, while (13a) illustrates a case where there is no full employment equilibrium. The optimal wage is then given at E with the corresponding optimal level of unemployment. The competitive solution may entail either too high or too low a wage; but in any case the shadow price of labor in the urban sector is less than the urban wage (provided $(1-U)\varphi < 1$, as we have assumed earlier) and indeed, normally even less than the rural wage.

The reason for the ambiguity is that the competitor ignores two effects of increasing his wage and employment. Both of these result from the fact that as he increases the urban wage, the unemployment rate changes. Because there is more urban employment in the optimal allocation than in the competitive allocation, at any given urban wage, there is less unemployment (since the rural wage is higher); hence not only are workers more productive, but the return to increasing wages is higher, since there are fewer among whom to share the wage increase. This leads to a higher wage in the optimal situation as compared to the competitive situation. On the other hand, the competitor ignores the fact that as he (and all other like-minded firms) increases his wages, he increases the unemployment rate, so there is a significant loss in output from the induced migration from the rural sector.

In analyzing these effects of a wage subsidy, we must consider separately two different cases.

(a) An equilibrium exists with a finite level of unemployment; there, a wage subsidy increases the demand for labor at each value of w_u, hence increases the equilibrium unemployment rate, as depicted in Figure 14.

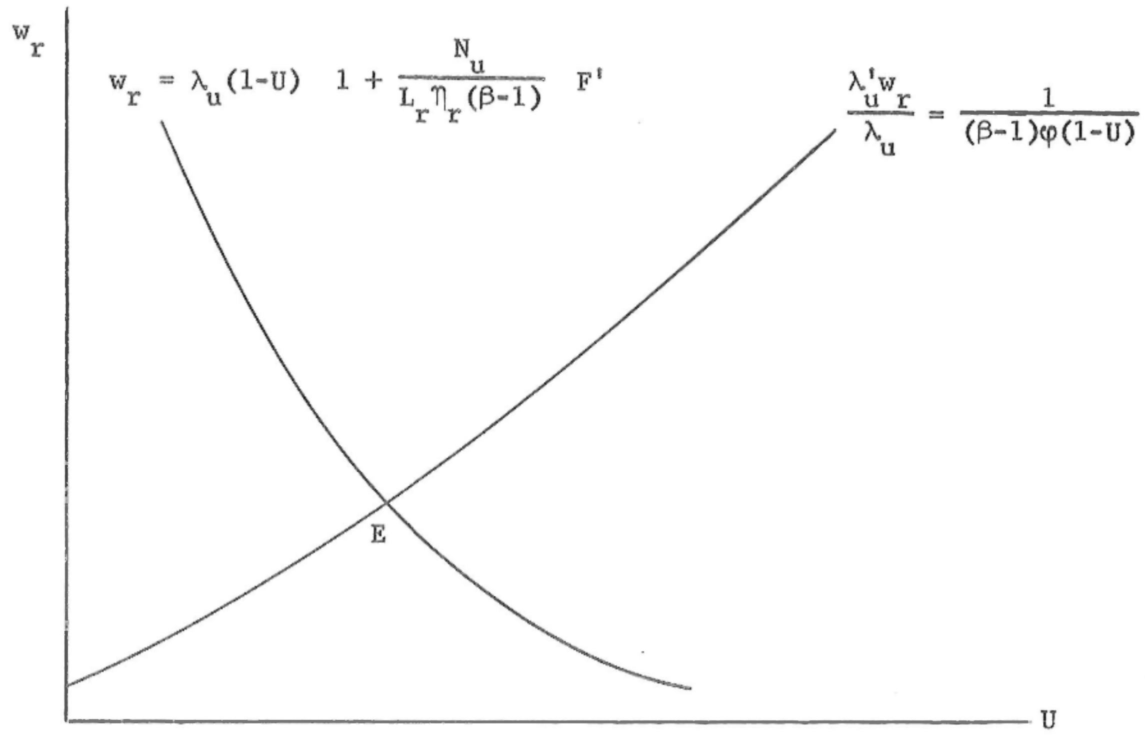


FIGURE 13a

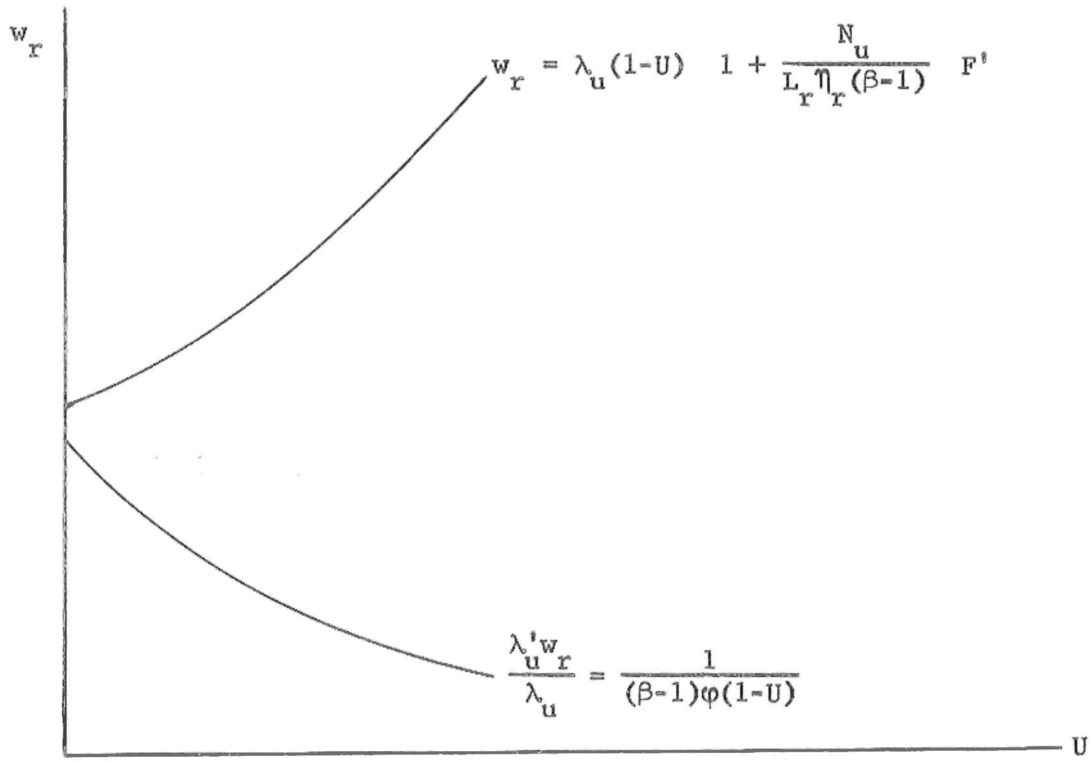


FIGURE 13b

Optimal Wages with Sharing

Optimality may require either a wage tax or a wage subsidy.[1]

$$(10.13) \qquad \frac{dF(\lambda(w^*)L_u) + G\left(L - \frac{L_u}{1-U}\right)}{d\tau} = \frac{dL_u}{d\tau}\left(F'\lambda - \frac{G'}{1-U}\right) - \frac{G'L_u}{(1-U)^2}\frac{dU}{d\tau} .$$

---

[1]From (10.7)

$$w^*\left(L^s - L_r\left(\frac{w_u}{\varphi\left(\frac{w_u}{w^*}\right)}\right)\right) = w_u L_u(w_u(1-\tau))$$

$$\frac{(1-\tau)dw_u}{w_u d(1-\tau)} = \frac{(1-\tau)w_u \frac{\partial L_u}{\partial w}}{-w^* \frac{\partial L_r}{\partial w_r}\left(\frac{1}{\varphi} - \frac{w_u}{\varphi^2}\frac{\varphi'}{w^*}\right) - \left(L_u + w_u \frac{\partial L_u}{\partial w_u}(1-\tau)\right)}$$

$$= \frac{-L_u \eta_u}{\eta_r(1-U)L_r\left(1 - \beta\right) - L_u(1 - \eta_u)} .$$

Since

$$w_u(1-U) = w^*$$

$$\frac{d \ln w_u}{d \ln(1-\tau)} = - \frac{d \ln (1-U)}{d \ln (1-\tau)}$$

$$\frac{dF + G}{d \ln(1-\tau)} = -L_u\eta_u\left(F'\lambda - \frac{G'}{1-U}\right)\left(1 + \frac{d \ln w_u}{d \ln(1-\tau)}\right) - \frac{d \ln w_u}{d \ln(1-\tau)}\frac{G'L_u}{(1-U)} = 0$$

(or $U = 0$) whence

$$\frac{1}{1-\tau} = \frac{\eta_u w_u\left(1 + \frac{d \ln w_u}{d \ln(1-\tau)}\right)}{\frac{w_r}{1-U}\left[\eta_u\left(1 + \frac{d \ln w_u}{d \ln(1-\tau)}\right) - \frac{d \ln w_u}{d \ln(1-\tau)}\right]}$$

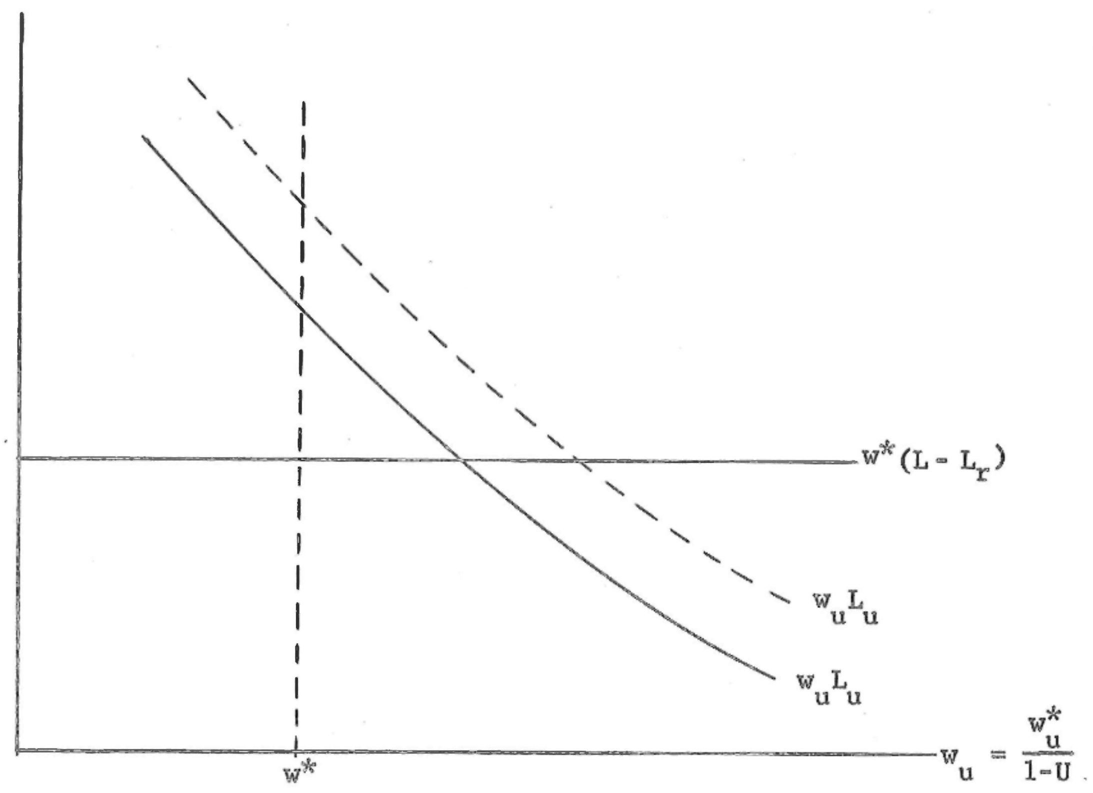


FIGURE 14a

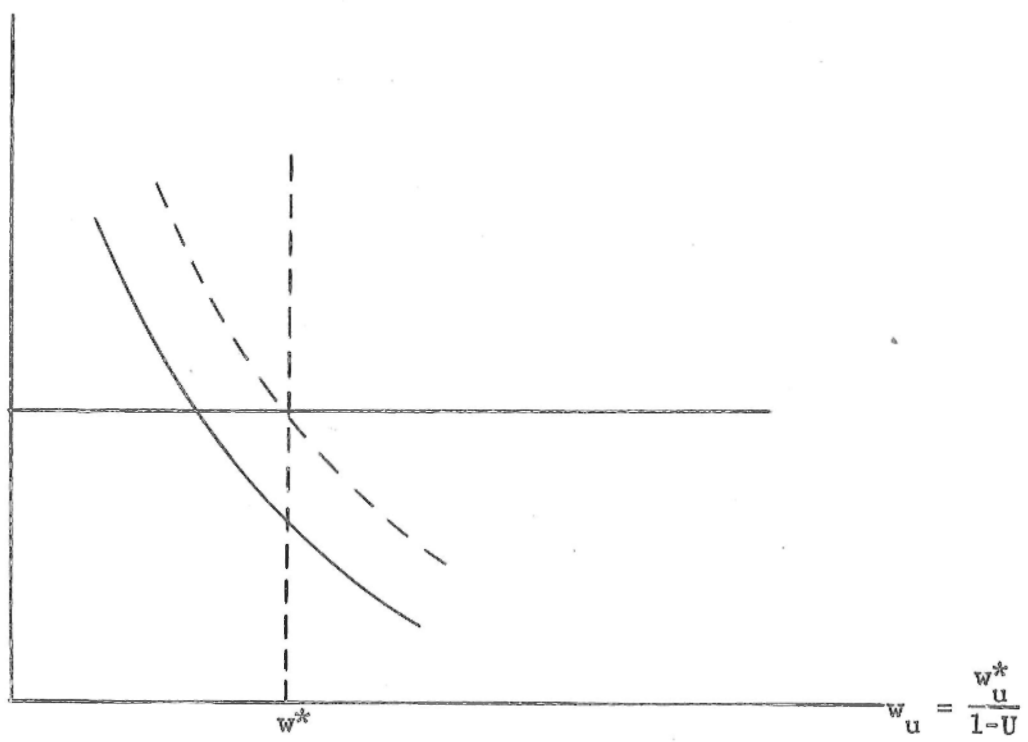


FIGURE 14b

Effect of Wage Subsidies

If $\beta = 1$, "normally" there is a small subsidy.

(b) At zero unemployment, there is an excess supply of individuals, so no static equilibrium exists (see above, p. 31). Then optimality requires imposing a wage subsidy to shift the $w_u L_u$ higher up until the equilibrium occurs at $U - 0$. If $\beta \leq 1$ further increases in the subsidy do not increase rural output and, by increasing U decrease urban output. Even if $\beta > 1$, provided $\varphi(1) = 1$, further increases in the subsidy decrease output.[1]

11. Concluding Comments on the Efficiency Wage Hypothesis

The arguments for the "efficiency" wage hypothesis are broader than just the simple dependence of productivity on nutrition. Indeed, if that were the primary explanation, one would expect that firms would provide housing and food for their workers, to ensure that the worker, and not other members of his family receive the benefits of the "high" wage, and that workers did not foolishly spend their money on unnutritious food. Even though food expenditures do increase with income, there is little evidence that the nutritional value of food expenditures increases; rather the worker may switch to more expensive but from a nutritional point of view, less satisfactory, foods. There is in addition the "incentive" effects of a higher wage; because they receive a higher wage, workers believe they should put more "effort" (conversely, they may feel little effort is commensurate with a low wage). Henry Ford is said to have based his policy of high wages on this argument.

There are two respects in which the efficiency wage hypothesis and the labor turnover hypothesis discussed in our earlier paper are closely linked. First, not only is "quitting" affected by the wage rate, but

[1]If $\varphi(1)$ is much greater than unity, if $\beta > 1$, further increases in unemployment by increasing the wage subsidy may be desirable.

"absenteeism" is also affected. The "cost" of being fired as a result of absenteeism (or other unproductive behavior like stealing) is greater the greater the unemployment rate, the urban rural wage differential, and the wage differential between the given firm and other firms in the urban sector. In this respect, the conditions for optimal efficiency depend on exactly the same factors that labor turnover does.

Secondly, at sufficiently high urban wages, there may be a marked change in individual's behavior: workers may essentially sever their connections with the rural sector and become part of the "urban proletariat." Labor turnover will then no longer be affected by further increases in the urban rural wage differential (although it still will depend on the unemployment rate and intra-urban wage differentials.) This introduces a non-convexity into the firm behavior with many of the same implications that the non-convexity in the "efficiency wage" function discussed above. (See Figure 15.)

The major conclusions to be drawn from our analysis of the efficiency wage hypothesis are the following:

(1) If efficiency in the rural sector is not affected by income levels there and if the expected urban wage equals the rural wage, then if the marginal productivity of labor in the rural sector is constant, the competitive equilibrium is identical to the optimal allocation, even though there is unemployment. If there is diminishing returns in the rural sector, the efficiency wage is still the optimal wage, but the competitive allocation will involve too few workers in the urban sector and too high an unemployment rate. The shadow price of labor will in general lie between the rural and urban wages.

(2) Unlike the labor turnover model, provided there is no sharing, the wage subsidy is not shifted and a wage subsidy will increase national income.

(3) If workers share their wage income with non-workers (or rural workers), the competitive wage will be higher (than if they do not share). As a polar case, if they share their income equally with all the unemployed, the competitive wage will be the efficiency wage divided by $1-U$.

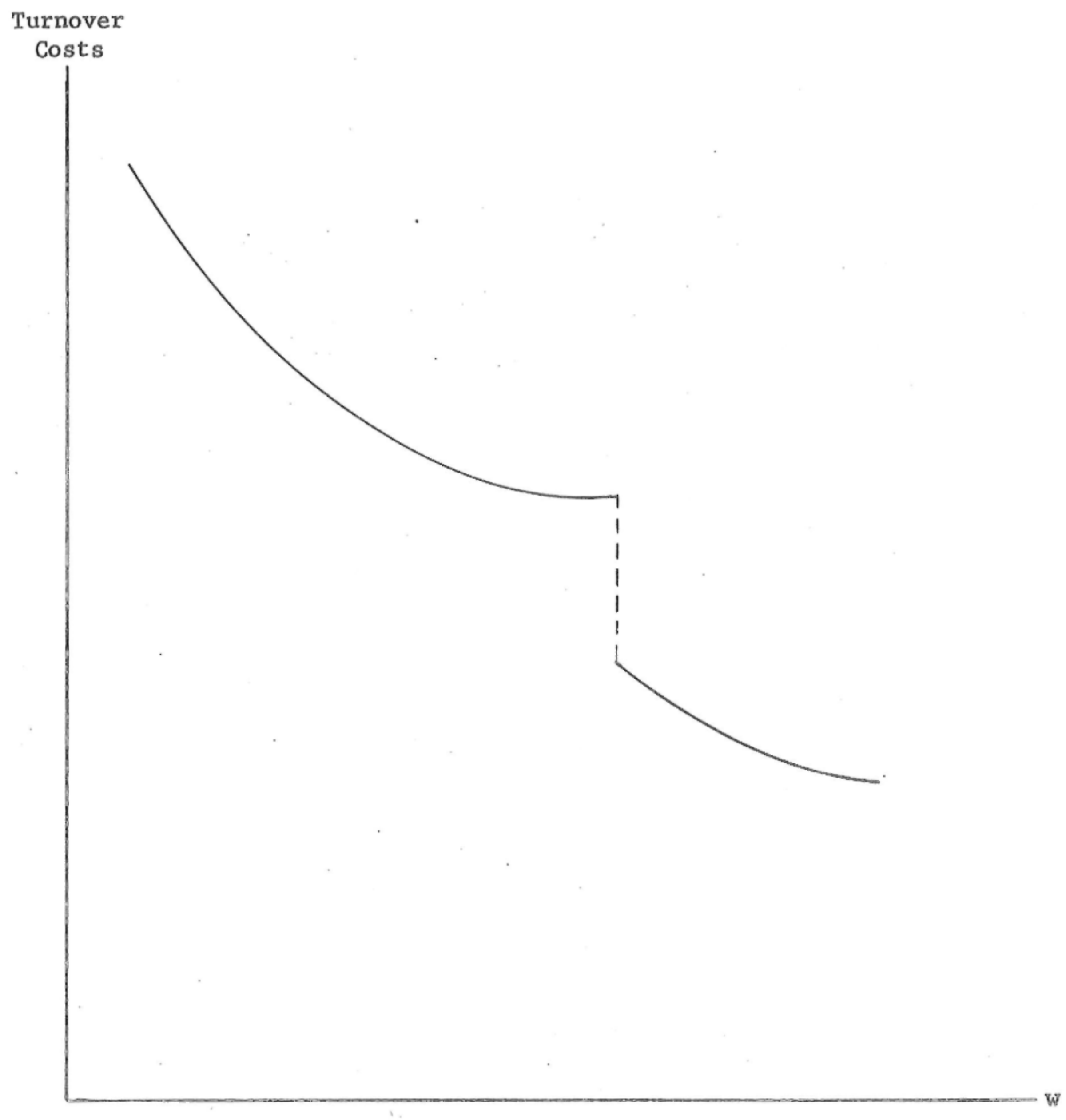


FIGURE 15

Nonconvexity in Turnover Cost Function

(4) If there is sharing, a wage subsidy may <u>increase</u> the unemployment rate and reduce rational output; in some circumstances, a wage tax is desirable, in others a wage subsidy is called for.

(5) If the efficiency in the rural sector is affected by income levels there, there will be a trade-off between employment and output in the rural sector, and between output and income distribution.[1,2] These trade-offs are illustrated by the following special cases:

(a) If land is owned by a "landlord" class, which maximizes its profits (rents), then there will be true unemployment, even though those who work receive a positive wage (and have a positive marginal product equal to that wage). The competitive wage is the rural efficiency wage.

(b) If workers receive their average product, and all peasants are treated identically, then the social marginal product of a laborer in the rural sector is negative. The competitive wage in the urban sector is optimal, but optimality requires hiring workers in the urban sector at least to the point where the social marginal product of workers in the rural sector is positive.

(c) If families maximize their family income, there will again be true surplus laborers. Those who work receive their average product which is equal to the agricultural "efficiency wage"; those who do not receive nothing.

(d) If families maximize their family utility, described by an additive utility function with diminishing marginal utility, the family will again be divided into two groups--hard workers and easy workers. Hard workers

---

[1]In conventional economic models, there is not a tradeoff between employment and output as long as workers can be put to work without doing harm to the output of other workers.

[2]This should be distinguished from the conventional models, which allow for a trade-off between employment (output, income distribution) and economic growth. In the long run, these models yield the result that if savings are dependent on the distribution of income, hiring more workers today will result in greater output today and more equality today (smaller profits), but less employment and output in the future. Here however we are concerned completely with static results: at each moment of time there is this trade-off.

will receive less than the efficiency wage; but an income greater than the wage of the point of inflection of the efficiency curve; easy workers will receive a positive wage in the convex region of the efficiency curve. The "hard workers" receive a share of the rent more than proportionate to their numbers but less than proportionate to their contribution to output. The size of the wage "gap" increases as the number of workers in the rural sector increases. The social marginal productivity of workers in the rural sector is negative (even though all individuals receive some wages and do some work).

It is optimal to have a sufficiently large wage subsidy to lead to full employment in the rural sector, but not to full employment in the urban sector.

The results of this model stand in marked contrast to those of the labor turnover model. There were noted a presumption that a wage subsidy would be shifted, would increase urban unemployment and reduce national output. Here, we note that except in one case, although a potentially an important one (where workers share their income with non-workers), a subsidy is desirable. Other implications of this model differ markedly from those of the labor turnover model. The efficiency wage model predicts that the urban real wage remain roughly constant;in the labor turnover model, if the unemployment rate were constant, the urban real wage would move in the same direction as the rural wage. Accordingly, if the economy is successful in its development program and the rural wage beings to rise, the urban wage will rise as well.

In the final part of this paper, we explore too other models of wage determination: a Cambridge-type Distribution Model and the Rigid Wage Hypothesis.