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Discussion Paper No. 23

Elimination of Management Bias from Production

Functions Fitted to Cross-Section Data:

A Model and an Application
to African Agriculture

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This paper presents a method for eliminating management bias from production functions fitted to cross-section data on multi-product enterprises. The method is applied to a sample of peasant farms in Rhodesia. The estimates are used to calculate marginal productivities, to examine the efficiency of allocation in the sample, to assess the relative importance of factors in leading to increases in output, and to examine the characteristics of better than average managers.

A. PROBLEMS OF STATISTICAL ESTIMATION

There is extensive literature relating to the problems of estimating the parameters in a production function.¹ Without trying to survey this literature here, we shall note briefly a few points that are germane to the present paper.

1. Identifiability

Consider the Cobb-Douglas function, written in logarithmic form,

$$Y_j = \sum \alpha_k X_{kj} + u_j \quad (1)$$

where Y_j = log of output of firm j ,

X_{kj} = log of input k used by firm j ,

u_j = a stochastic term,

α_k = the elasticity of production of input k .

One can estimate the coefficients in equation (1) by conducting an experiment in which arbitrary sets of values are assigned to the X_{kj} . Provided there is sufficient independent variation in the inputs, consistent estimates can be obtained from

¹For a survey of the literature, see A.A. Walters, "Production and Cost Functions: An Econometric Survey," Econometrica, Vol. 31, pp. 1-66, 1963; see also E. H. Phelps-Brown, "The Meaning of the Fitted Cobb-Douglas Function," Quarterly Journal of Economics, Vol. 71, pp. 546-560, 1957.

ordinary single-equation least-squares. However, such an experiment is frequently impossible or very expensive. Instead, the investigator collects data on a number of firms operating with different sets of input values.

If the input combinations were generated by a stochastic process that led to independent variation among firms in the X_{kj} , consistent estimates of the a_k could still be obtained by least-squares. But firms do not select input levels randomly; rather, they choose inputs according to some set of decision rules. In this case, the production function must be viewed as part of a larger system of equations in which output and inputs are jointly determined. It is then possible that there is little or no interfirm variation in the X_{kj} . If all firms use the same decision rule, they may tend to produce at the same point on the production function.

Suppose that each firm chooses inputs so as to maximize profits. Then

$$\frac{\partial y_j}{\partial x_{kj}} = \frac{a_k y_j}{x_{kj}} = P_{kj} \quad (2)$$

where y_j = output of firm j ,

x_{kj} = input of factor k used by firm j ,

P_{kj} = the price of input k to firm j , divided by the price of output.

With competitive pricing in factor markets each input is priced the same to all firms, so that (2), written logarithmically, and with an error term added, becomes

$$X_{kj} = -\log(P_k) + Y_j + \log(a_j) + w_{kj} \quad (3)$$

where w_{kj} is the error term. Then (1) is unidentifiable.¹

Now suppose that firm j determines its level of input k according to the following decision rule,

$$X_{kj} = -\log(P_k) + Y_j + \log(a_k) + \log(V_{kj}) + w_{kj} \quad (4)$$

where V_{kj} is a multiplicative constant that firm j associates with input k . Equation (4) may obtain instead of (3) because of differences among firms in attitudes toward risk, differences in the values of fixed factors, or differences in the elasticity of supply of input k among firms. With restricted profit maximization according to (4), firms will tend to operate on different points on the production function, and (1) is identifiable.

Unidentifiability is not a problem if some factors (soil type, management) are specific to the firm or if variable inputs have different elasticities of supply to different firms. For example, the elasticity of supply of fertilizer to a firm may depend on the firm's liquidity and credit position. In the present study, competitive factor pricing surely does not obtain.

2. Simultaneous Equation Bias

Even if the production function is identifiable, it does not follow that single-equation least-squares will yield consistent estimates of the coefficients. Even if (4) holds,

¹In this situation consistent estimates of the production coefficients can be obtained from

$$a_k = \prod_{j=1}^n w_{kj}^{1/n},$$

where w_{kj} = the share of output paid to factor k by firm j .

See Lawrence Klein, A Textbook of Econometrics, Evanston: Row Peterson, and Co., 1953, pp. 193-196; see also I Hoch, "Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function," Econometrica, Vol. 26, pp. 566-578, 1958.

the use of single-equation methods of estimation in (1) will result in what has been termed simultaneous-equation bias; the estimates will be inconsistent.¹ To see this note that the X_{kj} are statistically related to Y_j by equation (4); it follows from (4) together with (1) that the X_{kj} are functionally related to the u_j in (1), violating a condition for least-squares to yield maximum-likelihood estimates.

It has been shown that, if certain conditions are satisfied, simultaneous equation bias will not result.² Consider that equation (4) can be rewritten,

$$X_{kj} = -\log(P_k) + Y_j^! + \log(\alpha_k) + \log(V_{kj}) + w_{kj} \quad (5)$$

where $Y_j^! = Y_j - u_j$. Let us refer to $Y_j^!$ as the log of firm j 's anticipated output. It is clear from inspection of (5) and (1) that the X_{kj} are not functionally related to the u_j , the disturbance term in equation (1). In this case, simultaneous estimation is unnecessary.

Equation (5) may be expected to hold, rather than (4), if a firm's realized output differs from its anticipated output, and if the firm chooses inputs so as to maximize the latter. This may be the case if inputs are chosen before realized output is known, as in agriculture.

Equation (5) cannot be used if there is a high inter-correlation among the u_j . For example, if because of a below-average seasonal rainfall each firm's realized output is 30 per cent below its anticipated output, then simultaneous equation bias will still obtain, as is obvious from the definition of $Y_j^!$.

The case for single-equation estimation is strong if there

¹See I Hoch, op. cit.

²See I Hoch, "Estimation of Production Function Parameters Combining Time-Series and Cross-Section Data," Econometrica, Vol. 30, pp. 34-53, 1962.

is large interfirm variation in the u_j , and if this is not reflected in the firm's choice of inputs. Whether this condition is met in the present study is moot. The annual variation in amount and intraseasonal distribution of rainfall tends to create a large u_j . A shortage (say) of rainfall will affect the crop output of each farm adversely. But whether a farm's realized output falls short of its anticipated output by 10 per cent or 30 per cent will depend on how the farm allocates its resources. Some farmers may place greater weight than others on producing a specified level of output with high probability rather than simply maximizing the overall value of output. This may lead to differences among farms in the methods of cultivation (for example, the attention devoted to weeding). Although the u_j will still be intercorrelated, the degree of intercorrelation may be sufficiently low to justify the use of single-equation estimation; at least, this is assumed to be so in the present study.

3. Management Bias

Even if there is no simultaneous equation bias, there may still be a specification bias. Although the X_{kj} are not functionally related to the u_j , both the X_{kj} and the Y_j may be functionally related to a nonobservable input. An example of this is a situation in which both output and inputs are functionally related to the firm's management ability; this creates what is termed "management bias."¹

To see this, suppose that instead of (1), we have

$$Y_j = \log(A_0) + \log(A_j) + \sum \alpha_k X_{kj} + u_j \quad (6)$$

where A_j is a multiplicative index of farm efficiency, or

¹See I. Hoch, *ibid.*, Z. Griliches, "Specification Bias in Estimates of Production Functions," Journal of Farm Economics, Vol. 39, pp. 8-20, 1957; Y. Mundlak, "Empirical Production Functions Free of Management Bias," Journal of Farm Economics, Vol. 43, pp. 44-56, 1961.

management ability, and A_0 is a constant. From (6) it follows that better managers will obtain larger inputs, and from (5) it follows that better managers will also tend to use more of each input.¹ If differences in farm efficiency are not taken into account in estimating the coefficients in (6), the estimates will not be consistent.

This can be seen in Figure 1.² Firm 1 is operating on the production function AM, and firm 2 on BN. Written in logarithmic form, the functions differ only by the additive constant, $\log(A_1) - \log(A_2)$, which is equal to the distance BA in Figure 1. Because firm 1 is more efficient, it chooses to operate on point P, while firm 2 operates to the left of this, at point Q. If A_j is unobservable, ordinary least squares will yield estimates of the interfirm function, FH, whereas it is the intrafirm functions, AM and BN, that one is interested in.

This problem was discussed independently by Y. Mundlak and I. Hoch.³ Both authors suggested that, to eliminate management bias, time series and cross-section data could be pooled, using analysis of covariance, to obtain consistent estimates of the coefficients in (6). Following Hoch (and using our notation), one can write,

$$Y_{jt} = \alpha_{00} + \alpha_{0j} + \bar{\alpha}_{0t} + \alpha_1 X_{1jt} + \dots + \alpha_p X_{pjt} + \epsilon_{jt} \quad (7)$$

where Y_{jt} = the log of output of firm j in year t ,

X_{kjt} = the log of input k used by firm j in year t ,

¹Because the cross partial derivative between the X_{kj} and $\log(A_j)$ will be positive.

²See Y. Mundlak, *ibid.*

³Y. Mundlak, *ibid.*; I. Hoch, *op. cit.*

$$\alpha_{oj} = \log(A_j), \quad \text{and}$$
$$\alpha_{oo} + \bar{\alpha}_{ot} = \log(A_o).$$

In (7), it is assumed that the $\alpha_1, \dots, \alpha_p$ are not functions of time, and that the "time" and "firm" coefficients, $\bar{\alpha}_{ot}$ and α_{oj} , respectively, are separable. Then interfarm differences that persist over the time period observed in the sample are assumed to reflect differences in the nonobservable variable, farm management.

4. Management Bias and Multi-Product Firms

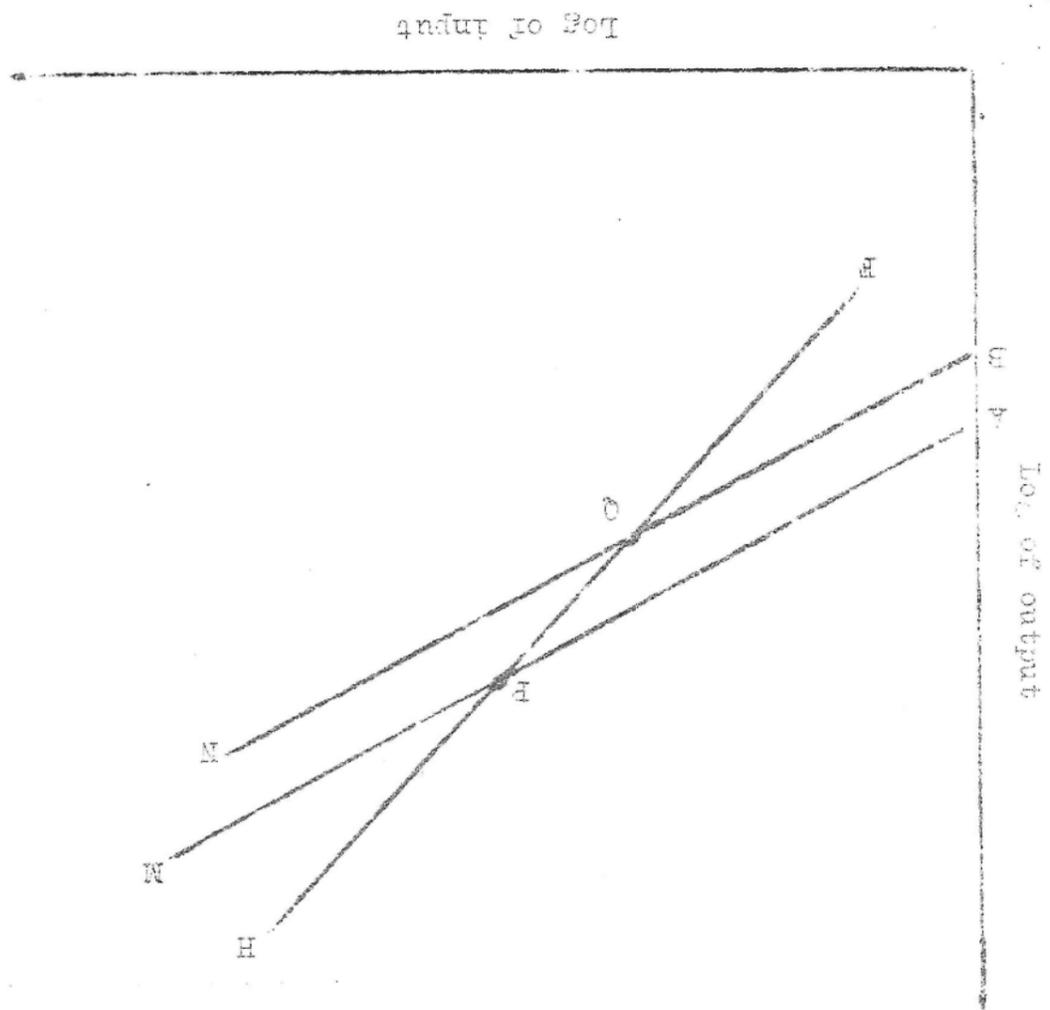
In the sample used here, it is reasonable to expect there to be interfarm differences in efficiency. It follows from the preceding discussion that ordinary least squares may yield inconsistent estimates of the production function coefficients. At the same time, because the data are for a single year only, the Hoch-Mundlak model cannot be used. However, the data relate to multi-product firms. With multi-product firms, if the production functions for different items are not interrelated, one can fit a function for each activity. By pooling product and firm data, and regarding each firm-product combination as a separate observation, analysis of covariance can be used, as above, to eliminate management bias if certain conditions are met.

Write

$$Y_{ij} = \alpha_{oo} + \alpha_{oj} + \bar{\alpha}_{oi} + \alpha_{oi} + \alpha_{1i} X_{1ij} + \dots + \alpha_{pi} X_{pij} + \epsilon \quad (8)$$

where α_{oo} and α_{oj} are as in (7) the general mean and the "farm" variable, respectively; $\bar{\alpha}_{oi}$ is a constant associated with crop i ; Y_{ij} is the log of output of crop i by firm j ; and X_{kij} is the log of input k used by firm j to produce crop i . In equation (8), the $\alpha_{1i}, \dots, \alpha_{pi}$ are the production elasticities associated with the independent variables in the production of crop i .

Fig. 1 - Interest and inflation production functions



As contrasted with equation (7), the production elasticities in (8) have a crop subscript. Although it may make economic sense to assume that the elasticity of production of input k is constant over time (especially if the period is relatively short), it makes much less sense to assume that input k 's elasticity of production is the same for all crops. Subscripting the elasticities creates no difficulty; but more degrees of freedom are used to estimate the coefficients in (8) than in (7). If there are n farms, m crops, and p_i inputs in the i^{th} production function (exclusive of management and the crop constant), the number of coefficients to be estimated in (8) is $n + m + \sum_{i=1}^m p_i$. The total number of observations is simply mn . Use of equation (8) requires the assumption of no interaction between farm efficiency and crop.

The term "management" includes both a technical efficiency component (output per unit of input) and an economic efficiency component (efficiency in the allocation of resources). The α_{0j} in equation (8) correspond to the technical efficiency component only. The α_{0j} also include any factors that affect a farm's technical efficiency but that do not appear explicitly as arguments in the production function.

It is difficult to know whether there is an interaction between the α_{0j} and the crops. But farming in Darwin requires no particularized skills that would enable a farmer to become significantly more efficient in producing one crop rather than another. Techniques are straightforward so that a farmer with better than average ability is likely to be more efficient in crop production generally.

If farm efficiency is crop oriented, so that there is a farm-crop interaction effect, a farmer may exploit his relative advantage. In this case, two farms with the same level of

efficiency as measured by α_{0j} , can then be expected to choose to allocate inputs differently; in this case, the usefulness of the model is greatly reduced.

B. THE SAMPLE

The sample consists of 20 peasant farms from the Mt. Darwin district of Rhodesia. The farms averaged 219 acres, held under freehold tenure, with an average of 23 acres cultivated. The farms were surveyed during the 1961-62 crop year by the District Agricultural Officer and his assistants. Each farm was visited several times per week throughout the year.

Income earned on the farm is principally derived from the production of three crops: corn, millet, and peanuts. We shall refer to a farm-crop combination as an observation. For each observation, we have data on output, acreage, soil type, chemical fertilizer, organic manure, and labor. We shall discuss each variable briefly.

Output is measured in physical units -- pounds harvested. There is frequently some difference in crop quality from one farm to another, but such differences tend to be of little importance. Therefore output of each crop can be regarded as a homogeneous variable. For comparability among crops, output is weighted by the average price received for the crop. No distinction is made between marketed output and that consumed on the farm.

Land is measured in acres planted to each crop. There is a difference among types of soil in the area, however, with some types being more fertile than others. To complicate the matter, the relative fertilities of different kinds of soil

are not uniform among crops. Thus one kind of soil may result in a higher corn yield but a lower peanut yield than another soil. On balance, however, two types of soil appeared to be significantly more fertile than the other two types, so we used the arbitrary procedure of classifying soils into these two broad groups. The small sample prevents a finer soil breakdown.

We defined a dummy variable for soil type, taking on the value unity for "good" soil and zero otherwise. Soil type then enters the production function as a shift variable, implying that the absolute difference in the log of output between good and bad soil is independent of the quantities of other inputs used.

Organic manure and chemical fertilizer are used only in the production of corn. Manure is measured in tons of compost applied. As some farms used no manure, a constant was added to the manure variable before taking logs. The constant chosen was 100.

Only 6 of the 20 farms used chemical fertilizer. For this reason we decided against using the value of fertilizer as a variable in the production function. Instead a dummy variable was employed to distinguish fertilizer use from non-use.

All farms in the sample had plows and cultivators, and greater than two-thirds of the farms had mechanical planters, harrows, and "scotch carts" (ox-drawn carts). Only a few farms had other types of equipment. As an index of fixed capital, the value of farm implements, at undepreciated replacement cost was used. This index is subject to criticism, but is the best we have. It omits the services of draft animals, as well as investment in land improvement and soil conservation.

It also provides no information on the extent to which capital was used for one crop rather than another; we must regard capital as a joint input available for all crops.

Family labor constitutes the principal component of the farm labor force; in addition, laborers from a nearby reserve were frequently employed. There is also what we can term "social" labor -- that is, labor performed jointly by members of the farm family and their friends, typically combining work in the fields with the consumption of beer.

Labor was measured as the number of hours spent weeding each crop. Social labor and labor performed by children were weighted by one-half.

C. EMPIRICAL RESULTS

1. Estimates of the Coefficients

A Cobb-Douglas function was used to relate the output of each crop to the set of observed inputs used in producing the crop. The function, in logarithmic form, is written,

$$Y_{ij} = b_{oi} + \sum_{k=1}^6 b_{ki} X_{kij} + v_{ij} \quad (9)$$

where Y = log of output,

X_1 = log of land,

X_2 = log of labor,

X_3 = fertilizer dummy variable,

X_4 = log of manure - plus - 100,

X_5 = log of fixed capital,

X_6 = soil type dummy variable,

and i denotes the crop and j the farm.

The coefficients in equation (9) were estimated using analysis of covariance, as described above. However, a computational difficulty was encountered. Note that a farm's soil type and the value of its capital stock are the same for all three crops. As a result the intrafarm matrix has two rows of constants and is accordingly singular. In principle, one can write a computer program that takes this singularity into account, and that permits estimation of the entire set of coefficients. However, it was found much simpler (and cheaper) in practice to use a somewhat less (statistically) satisfactory procedure. This procedure involves using least-squares to obtain estimates of the coefficients in each (interfarm) production function individually. It is assumed that farm management is uncorrelated with soil type and with fixed capital (on the grounds that these variables are determined exogenously), so that the interfarm and intrafarm coefficients of these variables are identical. Then a new set of variables is defined,

$$Z_{ij} = Y_{ij} - \hat{b}_{5i} X_{5ij} - \hat{b}_{6i} X_{6ij} \quad (10)$$

where Z_{ij} is an estimate of the log of output of crop i obtained by farm j , net of farm j 's soil type and capital stock, and \hat{b}_{5i} and \hat{b}_{6i} are least-squares estimates of the coefficients of capital and soil type, respectively. It follows that Z_{ij} can be substituted for output in the crop production functions, and analysis of covariance used to obtain intrafarm estimates of the remaining coefficients.

The estimated interfarm and intrafarm coefficients, together with their standard errors, are presented in Table 1 and 2, respectively. We tested the hypothesis of no farm effect (the $a_{0j} = 0$ for all j). An F value of 2.103 was obtained which, with 19 and 30 degrees of freedom, is significant at the

Table 1

ESTIMATED INTERFARM REGRESSION COEFFICIENTS^a

Input	Crop		
	Corn	Peanuts	Millet
Land	.588 (.465)	.755 ^b (.278)	.801 ^b (.202)
Weeding	.176 (.138)	.079 (.143)	.305 ^b (.135)
Fixed capital	.050 (.137)	-.151 (.180)	.173 (.162)
Soil type	.122 (.084)	.157 (.118)	.446 ^b (.111)
Fertilizer	.247 ^b (.087)
Manure	1.004 (.969)
Multiple correlation coefficient	.821	.656	.822

Notes:

... Indicates input not used in producing this crop.

^aRegression coefficients are stated first, followed by the respective standard errors in parenthesis.

^bDenotes significance at 5 per cent level, using one-tail test.

Table 2

ESTIMATED INTRAFARM REGRESSION COEFFICIENTS^a

Input	Crop		
	Corn	Peanuts	Millet
Land	.820 (.1629)	.803 ^b (.271)	.773 ^b (.214)
Weeding	.060 (.161)	.065 (.140)	.296 ^b (.154)
Fertilizer	.195 (.111)
Manure	.473 (1.126)

Notes:

... Indicates input not used in producing this crop.

^aRegression coefficients are stated first, followed by respective standard errors in parenthesis.

^bDenotes significance at 5 per cent level, using one-tail test.

5 percent level. The data are therefore inconsistent with the hypothesis of equal farm efficiency.

The proportion of the variance in crop output explained by the observed independent variables is small. The coefficients of multiple correlation in the interfarm production functions range from .656 to .882. Using a table of values of the correlation coefficient for the null hypothesis of no correlation, significance at the one percent level corresponds to a correlation coefficient of .561. All of the regressions are significant at this level. Nevertheless, for the peanuts regression, less than half of the interfarm output variance is explained by the set of independent variables.

Some variables were not statistically significant at the 5 per cent level in either interfarm ^{or intrafarm} functions. Accordingly the marginal productivities and elasticities of these variables should be interpreted with caution. Other variables were statistically significant in the interfarm function but not in the intrafarm function.

As evidenced by the t-ratios, the factors that are most important in explaining interfarm differences in output are fertilizer in the corn function; land and (although not significant) soil type in the peanuts function; land, labor and soil type in the millet function. It is likely that multicollinearity is at least partly responsible for the low levels of statistical significance, especially in the intrafarm functions. For this type of exercise, 20 farms is a rather small sample.

2. Elasticities of Production

Ordinarily, in fitting a Cobb-Douglas function, the coefficients equal the elasticities of production of the respective inputs. One feature of the Cobb-Douglas function is that

these elasticities are independent of factor ratios. In the function used here, the regression coefficients for land, capital, and labor are equal to the production elasticities, but for the remaining variables this is not the case. The elasticity of manure is obtained by multiplying the regression coefficients by $\frac{M-100}{M}$ where M = the value of manure plus 100, calculated at the geometric mean. For fertilizer which enters the production function as a shift factor, the elasticity of production equals the regression coefficient multiplied by the value of the variable; the elasticity was calculated at the arithmetic mean. The estimated intrafarm elasticities are presented in Table 3. The table also contains the sums of the estimated elasticities of the variables in each function, excluding soil which is regarded as a shift variable.

To test for returns to scale, a two-tail t-test was used; the null hypothesis was that the elasticities sum to unity for each crop.¹ None of the sums is significantly different from zero. The data are thus compatible with constant returns to scale in producing each crop.

3. Marginal Productivities

From the estimated elasticities one can obtain a set of estimated marginal productivities. The marginal productivity of factor k in producing crop i is denoted by f_{ki} and is given by

$$f_{ki} = E_{ki} \frac{y_i}{x_{ki}} \quad (11)$$

¹In calculating the t-ratios, the variance of the sum of the estimated elasticities includes the appropriate terms from the inverse of the moments matrix.

Table 3

ESTIMATED ELASTICITIES OF PRODUCTION

Input	Crop		
	Corn	Peanuts	Millet
Land	.820	.803	.773
Weeding	.060	.065	.296
Fixed capital	.050	0	.173
Fertilizer	.059
Manure	.106
Total	1.095	.868	1.242
Soil type	.073	.094	.268

Notes:

... Indicates input not used in producing this crop.

where E_{ki} = the elasticity of factor k in producing crop i,
 y_i = the output of crop i, and
 x_{ki} = the input of input k used in producing crop i.

The estimated marginal productivities were calculated at the means of the variables y_i and x_{ki} and consequently relate to the "average" farm.¹ These figures appear in Table 4.

To test for the significance of the difference among the marginal productivities of each factor in different uses, an F-test was used. Carter and Hartley have shown that an estimate of the variance of a marginal productivity estimated from a Cobb-Douglas function is given by²

$$\text{var}(\hat{f}_{ki}) = \left(\frac{y_i}{x_{ki}}\right)^2 \left[\text{var}(E_{ki}) + \frac{(S_i)^2 (E_{ki})^2}{n} \right] \quad (12)$$

where $(S_i)^2$ = the "unexplained" variance in $\log(y_i)$,
 n = the number of observations,

and where y_i and x_{ki} are chosen at their geometric means.

Equation (12) was used to calculate the estimated variances of the marginal productivities of land and weeding for each crop. These were used to test the hypothesis that each factor's marginal productivity is the same in all uses. For both land and labor, the F ratio was significant at the one per cent level, providing evidence that the marginal productivity of each input differs among crops.

4. Allocative Efficiency

One can calculate the gain achievable from reallocating inputs more efficiently. This gain is simply the difference

¹The geometric mean was used for logged variables and the arithmetic mean for the remaining variables.

²H. O. Carter and H. O. Hartley, "A Variance Formula for Marginal Productivity Estimates Using the Cobb-Douglas Function," Econometrica, Vol. 26, No. 2, April 1958, pp. 306-313.

Table 4
ESTIMATED MARGINAL VALUE PRODUCTIVITIES
(dollars per unit of measure)

Input	Crop		
	Corn	Peanuts	Millet
Land (acres)	21.59	27.99	22.21
Weeding (hour)	.028	.039	.134
Fixed capital (dollar cost)	.099	0	.062
Soil type (per acre)	3.09	5.00	10.70
Fertilizer (dollar cost)	2.9
Manure (tons)	1.42

Note:

... Indicates input not used in producing this crop.

between the optimal output (output achieved when the marginal productivity of each factor is equated in all uses) and actual output. The potential gain was calculated at the geometric means of output, labor, and land, ^{and} was found to be approximately \$25 - 30, or 4 - 5 percent of the value of actual output. The actual scope for reallocation is almost certainly less than this, due to constraints on resource use (such as the need for crop rotation), and due to discrepancies between long-term and short-term optimization and between anticipated and realized marginal productivities. Moreover, the figures are based on estimated marginal productivities which can be expected to differ from actual marginal productivities.

However, in examining allocative efficiency, not only the marginal productivity of each input on the average farm, but the dispersion of individual marginal productivities around this average is relevant. Efficient allocation on the average farm is a necessary but not a sufficient condition for efficiency on individual farms. Although the results here provide little evidence of potential gains from reallocation on the average farm, there may be considerable scope for gain to individual farmers.

5. Returns to Resources

Output on farms using fertilizer is \$75.59 higher than on farms using no fertilizer. The mean expenditure on fertilizer was \$7.71. As only 30 percent of the farms used any fertilizer, then, of those farms that did use it, the mean expenditure was \$25.70. Dividing this figure into the marginal productivity figure, we obtain an estimated "average-marginal" product per dollar of fertilizer used of \$2.94. There is thus scope for increased fertilizer use.

The marginal productivity of organic manure is \$1.42 per ton. An average of 9.3 hours was used to apply a ton of manure. Consequently the return to this labor figures out to be 15 cents per hour.

Our measure of the capital stock is based on a gross capital concept. The gross rate of return, based on this value of the stock, is 16 percent (obtained by summing the returns for the three crops). If we assume an average life of equipment of 10 years, and assume that the net stock is 75 percent the value of the gross stock, the net rate of return figures out to be 8 percent. Although these figures must be treated with caution, the results do suggest little scope for investment in fixed capital.

The marginal productivity of labor in producing millet is 13.4 cents, but only 3.9 and 2.8 cents for peanuts and corn, respectively. With the possible exception of millet, there is little scope for raising output through increased use of labor. Labor is genuinely scarce in the area; there is no evidence of disguised unemployment or underemployment. The family works long hours on the farm, throughout the year. Any increase in labor input would require drawing more heavily on hired workers.

The marginal productivity of land ranges from \$21.59 to \$27.99. There is some opportunity for the farmer to increase his cultivated acreage, although this would involve clearing large trees from the land. Given the high return to arable land, one would expect the farmer to bring more acres under cultivation, and either spread his labor and capital more thinly over this larger acreage or use more hired labor.

6. Management.

The management index includes the contribution of unobserved variables that constitute what can be regarded as a "farm" effect. Although differences among farms in soil type and in observable inputs have been netted out, there likely remain other factors that account for interfarm differences that one would ideally wish to isolate from management. For example, investment in land improvement, selection of seeds, or use of pesticides, will be reflected in a farm's management index, as measured by the a_{oj} . The a_{oj} measure output per unit of observed input, where the inputs are combined multiplicatively and weighted by the coefficients in the intrafarm production functions.

Setting the average a_{oj} equal to zero, the estimated a_{oj} ranged from $-.0310$ to $.300$. Taking antilogs of these figures, the best farm could, with given inputs, obtain just twice the output of the average farm which, in turn, could obtain twice as much output as the worst farm.

It is of interest to examine the characteristics of good managers and, in particular, the tendency for better than average managers to use more (or less) of any of the productive inputs. Management (as estimated by the a_{oj}) was regressed against each factor input in each of the three production functions. Table 5 shows the estimated simple correlation coefficients between management and the factor inputs. None of the coefficients is significant at the 5 per cent level. The results provide some evidence that better managers use more fertilizer and spend more time weeding corn.

Sixteen of the twenty farms were settled within four years of the year in which the survey was conducted. It is therefore of interest to examine the relationship between the

Table 5

SIMPLE COEFFICIENTS OF CORRELATION BETWEEN
MANAGEMENT AND FACTOR INPUTS

Inputs	All Crops	Crop		
		Corn	Peanuts	Millet
Fixed capital	-.10	n.a.	n.a.	n.a.
Soil type	.02	n.a.	n.a.	n.a.
Land	-.03	-.08	-.04	.08
Weeding	.16	.27	-.01	.05
Fertilizer	.21	.21
Manure	.10	.10

Notes:

- ... Indicates input not used for producing that crop
- n.a. indicates not available.

management index and years in the area. The farmers had been drawn from other agricultural areas and from other sectors of the economy. For these farmers, this was their first opportunity to own their own farm, to cultivate a relatively large acreage, and to farm commercially. One might expect there to be a "learning" factor -- a farmer may learn new techniques as a result of (1) exposure to the new form of farming, (2) agricultural extension services, and (3) trial and error. In this case, those who have been in the area longest would tend to be the best managers.

On the other hand, the fertility of virgin soil is typically high. Therefore, the longer the land is cultivated, unless adequate soil conservation measures are undertaken, the greater the reduction in soil fertility. For this reason, one might expect those recently settled in the area to obtain larger output per unit of input.

We calculated the mean management index for groups of farmers arranged by length of tenure on their farm (see Table 6). The results are inconclusive. However, with the exception of the first group, there is little suggestion of a systematic pattern. An F test shows no significant difference among the means. The first mean is considerably lower than the others, suggesting that it may take more than a year for a farmer to adjust to the new routine.

Table 6

MEAN MANAGEMENT INDEX BY YEARS ON FARM

	Years on Farm				
	1	2	3	4	Over 4
Mean management index	49	132	141	107	123
Number of Farms in group	4	4	5	3	4