ELASTICITY AND BUOYANCY OF FEDERAL TAXES IN PAKISTAN

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Introduction

This paper focuses on the revenue-expenditure activities of the federal government and evaluates the performance of the fiscal system on the basis of estimates of revenue productivity. This is measured by tax elasticity and tax buoyancy with the former taking account of the automatic response of revenue to income changes (i.e. revenue increase, excluding the effects of discretionary changes) and the latter measuring the total response of tax revenue to changes in income (i.e. including discretionary changes). A higher tax elasticity is always preferable in so far as it enables to plan for a higher level of current expenditure. However, for taxes with a lower elasticity, additional revenue may be generated only through discretionary changes, which is reflected in a higher buoyancy of the tax system.

In Pakistan, empirical investigations of tax elasticity and buoyancy have yielded varying results because of methodological problems. The elasticity coefficients for the years 1950-51 to 1957-58 obtained by Choudhry [1] on the basis of the "changing-base method" have limited policy relevance because of varying elasticity estimates for the different years. The "dummy variable method" used by Khan [4] to eliminate the effect of discretionary changes took care of only those changes for which a dummy variable was used. But since the tax structure underwent frequent changes during this period, not all the discretionary changes were accounted for, which casts serious

doubt on the reliability of the elasticity estimates. Jeetun [3] using the Prest formula to adjust for the revenue effects of discretionary tax measures calculated elasticities of Pakistan's tax system for the period 1960-61 to 1975-76. This result, were very different from those obtained by Khan even though the data set was the same.

Since revenue growth stems both from the growth in base as well as from new tax legislation, the built-in elasticity of the tax system has to be first adjusted for all discretionary tax measures. In a situation involving frequent changes in the tax structure the most relevent and practically useful techniques for estimating elasticity of taxes are (i) the Divisia Index method, and (ii) the Proportional Adjustment method.

In this paper the elasticity and buoyancy of Pakistan's federal tax system have been estimated for the years 1971-72 to 1982-83 by using the Divisia Index method and the Proportional Adjustment method. The size of the elasticity estimates of the total tax as well as of its components have been analysed and compared with the buoyancy measures. The variance between the buoyancy and elasticity estimates for individual taxes has also been explained. Finally, the estimates of buoyancies with respect to individual taxes are compared with the corresponding expenditure buoyancies to evaluate the revenue performance of the fiscal system. All the data have been taken from [6], [10], [11] and [12].

Measurement of Elasticity

Two methods, viz. the Divisia Index method and the Proportional Adjustment method have been used to isolate the effect of discretionary changes on aggregate tax revenue. The first method utilizes time-series data on the various taxes and is particularly helpful where reliable information about discretionary changes is not available. On the other hand, when data on discretionary change are available the Proportional Adjustment method yields better estimates of discretionary change [2]. In Pakistan, however, precise data on the discretionary changes introduced so far are not always available, given the complexity of the tax rate structure (in particular that pertaining to international trade) which is made even more intricate by changes in the tax base.[1] By using the two methods together the effect of 'non-tangible' factors like tax evasion, tax exempting investments etc. on the tax effort can also be accounted for.[2]

Divisia Index (DI) Method

The (DI) method of measuring the revenue effects of discretionary changes is widely used for measuring technical change and isolating the effect of exogenous factor on a variable. In this paper the DIM has been used to estimate tax elasticity by isolating the automatic growth of revenue from its total growth. Following Chaudhry [2] we estimate tax elasticity as follows:

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[1]The estimates of discretionary changes are available only from *Budget Speeches* [9] and *Annual Budget Statement* [8] from 1971-72 to 1982-83.

[2]For a detailed exposition of the two methods see Choudhry [2], Mansfield [5], and Prest [13].

The starting point is an aggregate tax function

$$T(t) = f \left[x_i(t), \ldots x_k(t); t \right] \qquad (1)$$

where T denotes the aggregate tax yield, x_i denotes the proxy tax base for the K catagories of taxes and the time variable t is a proxy for discretionary tax measures. The effect of discretionary tax changes at time t are obtained by taking the logarithm of the tax function; differentiating with respect to time, and rearranging the terms we get the following equation:

$$\frac{f_t(t)}{f(t)} = \frac{\dot{T}(t)}{T(t)} - \sum_{i=1}^{k} \frac{f_i(t) \, x_i(t)}{f(t)} \frac{\dot{x}_i(t)}{x_i(t)} \qquad \ldots \ldots (2)$$

Setting $\dfrac{f_i(t) \, x_i(t)}{f(t)} = \beta_i(t)$

and $\dfrac{f_t(t)}{f(t)} = \dfrac{\dot{D}(t)}{D(t)}$

where D(t) is the Divisia Index of discretionary tax changes, equation (2) can be rewritten as:

$$\frac{\dot{D}(t)}{D(t)} = \frac{\dot{T}(t)}{T(t)} - \sum_{i=1}^{k} \beta_i(t) \frac{\dot{x}_i(t)}{x_i(t)} \qquad \ldots \ldots (3)$$

Integrating equation (3) over the interval (o,n) we get the index of discretionary tax revenue

$$\frac{D(n)}{D(o)} = \left[\frac{T(n)}{T(o)} \right] \exp \left(- \sum_{i=1}^{k} \int_o^n \beta_i(t) \frac{\dot{x}_i(t)}{x_i(t)} \, dt \right) \ldots \ldots (4)$$

When equation (4) is normalized by setting D(o) = 1 D(n) can be viewed as the index of revenue growth stemming from discretionary tax measures at time (n). Replacing $\beta_i(t)$ by a $\hat{\beta}_i(t)$, which is a form of the weighted average of $\beta_i(t)$, where weights are the ratios of the instantaneous rates of growth of the bases to their average rates of growth in the

time interval (o,n), and taking the logarithm we get

$$\log D(n) = \log\left[\frac{T(n)}{T(o)}\right] - \sum_{i=1}^{k} \hat{\beta}_i \log\left[\frac{x_i(n)}{x_i(o)}\right] \quad \dots\dots\dots\dots(5)$$

Where $\log D(n)$ is the index of discretionary tax measures. To obtain the elasticity estimates the index of discretionary tax measures is adjusted by using the following formula.

$$r = b - \frac{\log D(n)}{\log x(n)/x(o)} \quad \dots\dots\dots\dots\dots\dots\dots(6)$$

where r = Tax elasticity

b = Tax buoyancy, obtained by regressing actual tax reve-
nue on GDP using logarithmic form of the equation.

$\log D(n)$ = Divisia Index of discretionary tax revenues.

$\log x(n)/x(o)$ = Index of automatic growth of proxy tax base.

Proportional Adjustment Method (PAM)

By this method elasticity has been calculated as in Mansfield [5] by first preparing a preliminary-series of adjusted tax yield and subtracting from the actual yield the estimated effect of discretionary tax changes. Next, the series is further adjusted by excluding the continuing impact of each discretionary change on future year's tax yields as given in equation (7).

$$T_{ij} = T_{j-1,j} \times \frac{T_{j-2,j-1}}{T_{j-1}} \times ---- \times \frac{T_{23}}{T_3} \times \frac{T_{12}}{T_2} \quad \dots\dots\dots\dots(7)$$

where

T_1, T_2 ---- T_n represent actual tax yields for a series of n years.

T_{ij} indicates the jth year's actual tax yield adjusted to the tax structure that existed in the year i.

In equation (7) the adjusted tax yield is obtained by multiplying the actual tax yield with a factor sequence, each element of which, represents the effect of automatic tax changes. Finally, T_{ij} is regressed on GDP to yield the elasticity estimates.

DISCRETIONARY AND AUTOMATIC GROWTH OF TAX REVENUE

To measure the revenue effect of discretionary changes on the different taxes we use the following equation which has been derived earlier.

$$\log D(n) = \log\left[\frac{T(n)}{T(o)}\right] - \sum_{i=1}^{k} \hat{\beta}_i \log\left[\frac{x_i(n)}{x_i(o)}\right] \quad \ldots\ldots\ldots\ldots\ldots (5')$$

where

$\hat{\beta}$ is the weighted average of β_i defined as $\dfrac{f_i(t)\, x_i(t)}{f(t)}$

The left-hand side of equation (5') measures discretionary tax growth. The first component of the right hand side measures total tax growth and the second represents automatic growth of taxes.

The results reported in Table 1 show that with a few exceptions almost all the gorwth in the various taxes stemmed from endogenous factors. This is reflected in the overwhelming contribution of automatic tax growth to total growth.

Table 1

Decomposition of Total Growth into Discretionary and Automatic Growth (1971-72 to 1982-83)

Tax Catagory	Total Growth of Tax Revenues	(Percentage per Annum) Growth in Tax Revenue due to	
		Discretionary change	Change in base
All taxes	2.154	-0.177	2.331
Indirect taxes	2.207	-0.171	2.378
Excise tax	1.807	-0.147	1.954
Sales tax	1.956	-0.126	2.082
Import duties	2.692	-0.109	2.801
Export duties	-0.869	3.341	-4.21
Direct taxes	1.955	-0.023	1.978
Corporation tax	2.805	0.249	2.556
Income tax	1.233	-0.205	1.028

On the other hand, the revenue effect of discretionary changes in the various taxes has either been extremely small, as in the case of corporation taxes or negative. The only exception is that of export duties in which case discretionary tax changes yielded substantial additional revenue, thanks both to upward revision of tax rates and to the introduction of some new taxes. This result would hold even when allowance is made for the over-estimation of discretionary growth by the Divisia Index Method arising from excessive statutory tax changes.[3] However, because of the adverse movement of automatic factors the total growth of export duties was negative.

The negative effect of discretionary changes on revenue, particularly in the case of indirect taxes, led to relatively lower values of tax revenue buoyancy and elasticity. These are analysed in greater detail in the next section.

[3] Due to this factor the efficiency of estimation of the DIM is not effected because during the Seventies, the discretionary changes for export duties were not many even though the impact on aggregate revenue was substantial.

ELASTICITY AND BUOYANCY OF TAX REVENUE

As explained above the elasticity estimates are obtained by eliminating the effects of discretionary changes on buoyancy estimates using a discretionary change Divisia Index transformation. It follows that the larger is the element of discretionary change in tax revenue the greater will be the variance between elasticity and byoyancy. In case discretionary changes add to total revenue the difference between the total and automatic growth of tax revenue will be positive and vice versa if new tax legislation reduces tax revenue.

The two elasticity estimates obtained by using the PAM and DIM and the buoyancy[4] estimates for various taxes given in Table II show that for total tax the elasticity was greater than unity and higher than buoyancy. This explains that while a growing proportion of incremental income has been transferred to the government in the form of tax revenues as tax base grows, the role of discretionary changes was not very substantial in raising additional revenue. In fact, discretionary changes had an adverse effect on the growth of aggregate tax revenue as reflected in the value of tax elasticity exceeding that of buoyancy. With the exception of corporation tax and export duties this was true for all the other components of total tax revenue.

[4] Buoyancy estimates are similar for the two methods.

Table II

Elasticities and Buoyancies of Federal Taxes

Tax	Elasticity			Buoyancy	
	DIM	PAM	R^2	Co-efficient	R^2
Total taxes	1.25	1.20 (36.25)	0.99	1.16 (43.86)	0.99
Indirect taxes	1.25	1.18 (21.76)	0.97	1.15 (24.08)	0.99
Excise tax	1.13	1.07 (24.73)	0.98	1.05 (21.96)	0.98
Sales tax	1.22	1.18 (22.17)	0.98	1.15 (27.73)	0.99
Import duty	1.54	1.56 (21.88)	0.98	1.48 (25.44)	0.98
Export duty	-2.39	-0.72 (3.46)	0.54	-0.64 (2.31)	0.35
Direct taxes	1.22	1.24 (10.99)	0.92	1.21 (11.07)	0.92
Corporation tax	1.75	1.73 (4.98)	0.88	1.88 (8.99)	0.89
Income tax	0.78	0.71 (4.98)	0.71	0.69 (5.49)	0.81

Note: Values in brackets are t-values.

A number of other observations can be made on the basis of the figures in Table II. First, the low buoyancies of the income tax and export duties has adversely effected the overall buoyancy of the total tax whose other components are fairly buoyant. Second, the higher buoyancy of direct taxes despite the less than unity income tax buoyancy was due to the extremely high buoyancy of the corporation tax which was also the largest contributor to the overall tax buoyancy. Third, the negative buoyancy of export duties shows that the effect of automatic growth factors was negative and exceeded the effect caused by positive discretionary changes. This is also reflected in the higher buoyancy of export duties as compared with their elasticity. Fourth, the contribution of additional revenue to total tax generated through discretionary changes (through increased tax rates and the broadening of the tax base) was not enough to offset the decline in revenue due to export duties and the adverse effect on revenue of an income tax elasticity of less than unity (which implies an adverse effect on the income tax ratio over time). In other words, the rise in the tax ratio during the years from 1971-72 to 1979-80 would have been faster had it not been for the negative influence of the discretionary changes (see [7]). Fifth, the import duties, which are the largest source of tax revenue in Pakistan, had a high elasticity of over 1.5. This points to a weakness in the overall tax structure in so far as there is an excessive dependence on an unstable source of revenue which contributed substantially to revenue growth.

DECOMPOSITION OF ELASTICITIES

To obtain a closer picture of the performance of the fiscal system, tax elasticity can be evaluated as the weighted average of the elasticities of its various components viz. the tax elasticity to the base and base elasticity to income.

$$\left[\frac{\Delta T_K}{\Delta Y} \cdot \frac{Y}{T_k} \right] = \left[\frac{\Delta T_k}{\Delta \beta_k} \cdot \frac{\beta_k}{T_k} \right] \left[\frac{\Delta B_k}{\Delta Y} \cdot \frac{Y}{B_k} \right] \quad \dots\dots\dots\dots\dots (8)$$

where

T = tax revenue

Y = GDP

B_k = is proxy base related to individual categories of taxes.

The decomposition of tax elasticities enables to identify the dynamic as well as the lagging components of the tax system. The division of each elasticity estimate into its two elements, viz. tax elasticity to base and base elasticity to GDP also has important policy implications. In so far as the tax to base element is controlled by the government it can be influenced to impart greater elasticity to the particular tax.

The results of decomposition for five major types of taxes are given in Table III.[5]

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[5]This equality is strictly true when equation (8) is perfectly estimated, i.e. when the R^2 level of each component of the equation is 1.00. Since the elasticities of decomposed elements were obtained separately there are differences in the overall elasticities of taxes and the product of the decomposed elements due to this factor as follows: income tax (0.78 vs 0.64), corporation tax (1.75 vs 1.91), import duty (1.54 vs 1.63), export duty (-2.39 vs 1.29), Excise tax (1.13 vs 1.22).

Table III

Tax	Related base	Tax-to-GDP elasticity	Base-to-GDP elasticity	Tax-to-Base elasticity
Income tax	Value-added in non-agriculture sector	0.78	1.15	0.56
Corporation tax	Value-added in large scale manufacturing sector	1.75	1.09	1.75
Excise tax	Value-added in large scale manufacturing sector	1.23	1.09	1.12
Import duty	Value of imports	1.54	1.37	1.19
Export duty	Value of exports	-2.39	0.90	-1.44

Table III shows that the low elasticity of income tax stemmed primarily from the low value of the tax to base elasticity since the base to GDP elasticity of 1.15 was fairly high pointing to a relatively faster income base growth with respect to GDP. The impact on the income tax elasticity would have been positive and substantial if tax revenue from this source had grown at least in the same proportion as the base. For a variety of reasons, which may include loss of revenue due to tax evasion, the growth in tax collection was less than proportionate to the expanding base which is reflected in the lower tax to base elasticity.

In the case of corporation tax the expansion in the tax base was only slightly faster than the growth of GDP, and almost all the contribution to the higher tax to GDP elasticity was provided by the tax to base elasticity, which reflects an upward revision in the tax rates for these commodities. For excise taxes both components contributed almost equally to the tax to

GDP elasticity. For import duties the base to GDP elasticity was the major contributer to the higher total elasticity even though the tax to base elasticity was also quite substantial. The negative elasticity of export duty was due both to a relatively slower growth of the export base as well to the adverse effect of the negative tax to base elasticity.

REVENUE PRODUCTIVITY

A comparison of tax and expenditure buoyancies in Table IV shows that during the years 1971-72 to 1982-83 revenue productivity was not a major problem in Pakistan in so far as the expenditure buoyancies have been lower than revenue buoyancies This is an interesting result as it implies that some measure of success was attained in limiting the growth of current expenditure below that of tax proceeds. The higher overall expenditure buoyancy was therefore, attributable to the faster growth of capital expenditure. This is substantiated by the fact that capital expenditure buoyancy not only exceeded that of current expenditure but of aggregate tax as well.

Table IV

Buoyancies of Revenues and Expenditure of the Federal Government

Dependent Variable and Base	Buoyancy	T-Value	R^2
Tax revenue-GDP	1.16	43.86	0.99
Total expenditure-GDP	1.10	57.45	0.99
Capital expenditure-GDP	1.17	46.76	0.99
Current expenditure-GDP	0.99	18.19	0.97

CONCLUSION

The DIM was used to assess the impact of discretionary changes on tax revenue and the results were compared with those obtained from using the PAM. Analysing the two estimates of elasticity it was found that they did not vary substantially and gave similar trends. The little difference can be explained by the fact that the DIM accounts for the 'production effects' of tax changes through tax base adjustment, while the PAM adjusts for tax yields against the base year tax-rate structure only. Second, the PAM uses estimates of discretionary tax changes obtained from annual Budget speeches, which can vary from the actual figures due to a number of reasons, like tax evasion, tax exempting investments, etc. It was noted however, that during the Seventies, with the exception of export duties the effect of these factors was only minimal.

The decomposition of elasticity into its various components showed an expansion in the tax base for nearly all the categories of taxes combined with positive discretionary changes. The low income tax elasticity with respect to GDP points to the need for a stronger effort in tax collection at existing rates and in preventing tax evasion as well as in minimizing tax exemptions.

It was found that unlike most other developing countries the built-in elasticity of Pakistan's tax system was greater than unity. But for a low buoyancy the tax elasticity would have been even higher. Tax buoyancy exceeded that of expenditure thanks to the fact that the growth of current expenditure was limited below that of tax proceeds.

REFERENCES

1. Chaudhry, A. H. M. N. "The Predictibility and the Flexibility of Tax Revenues in Pakistan". Pakistan Development Reviw. Summer 1962.

2. Choudhry, N. N. "Measuring the Elasticity of Tax Revenues: A Divisia Index Approach". IMF Staff Papers. Vol. 26, No. 1. March 1979.

3. Jeetun, Azad, "Buoyancy and Elasticity of Taxes in Pakistan". Karachi: Applied Economics Research Institute, 1978. (Research Report No. 11)

4. Khan, M. Zubair. "The Responsiveness of Tax Yield to Increases in National Income". Pakistan Development Review. Winter 1973.

5. Mansfield, C. Y. "Elasticity and Buoyancy of a Tax System: A Method Applied to Paraguay". IMF Staff Papers. Vol. 19, No. 2. July 1972.

6. Naqvi, Syed Nawab Haider, et al. The P.I.D.E. Macro-econometric Model of Pakistan's Economy. Islamabad: Pakistan Institute of Development Economics. 1983.

7. Naqvi, Syed Nawab Haider, and Khwaja Sarmad. Pakistan's Economy through the Seventies. Islamabad: Pakistan Institute of Development ment Economics. 1984.

8. Pakistan. Finance Division. Economic Adviser's Wing. "Annual Budget Statement". (Various issues)

9. Pakistan. Finance Division. Economic Adviser's Wing. Budget Speech. Islamabad. (Various issues)

10. Pakistan. Finance Division. Economic Adviser's Wing. Pakistan Economic Survey. Islamabad. (Various issues)

11. Pakistan. Finance Division. Economic Adviser's Wing. Public Finance Statistics. Islamabad. (Various issues)

12. Pakistan. Finance Division. Economic Adviser's Wing. Taxation structure of Pakistan. Islamabad. (Various issues)

13. Prest, A. R. "The Sensitivity of the Yield of Personal Income in the United Kingdom". Economic Journal. Vol. 72, 1962.