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THE 'O' LEVEL MATHEMATICS CURRICULUM FOR ZIMBABWE: DOES IT MEET THE EXPECTATIONS FOR QUALITY MATHEMATICS EDUCATION IN ZIMBABWE

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Abstract

This paper gives a critical analysis of the 'O' level mathematics syllabus for Zimbabwe. It argues that the present syllabus does not provide a balance between intuitive thinking and analytical thinking in Mathematics. It argues and justifies the retention of certain topics in the 'O' level syllabus and the introduction of new topics.

Introduction

Since independence Ministry of Education has achieved a quantitative expansion of the education system.

With this massive expansion came the introduction of new curricula, curricula meant to be in line with the new political social and economic order. The 'O' level mathematics curriculum was no exception. Since 1980 it has gone through some transformation. During a time when Ministry policy has turned to the qualitative aspects of education it seems reasonable to enquire into the suitability and quality of the various curricula, the assumption being that a well designed curriculum is a prerequisite to quality education. It is with this view in mind that an analysis of the present 'O' level mathematics syllabus is made.

The Tyler Model for curriculum planning and Kolyagin's criteria for content selection in mathematics are used to analyse the syllabus. A case is made for the introduction of new topics in the syllabus. The idea of a limit plays a central role in calculus and in mathematical analysis and it should be introduced to pupils early. It is important for both the learning of

subsequent mathematics and for mathematical growth. Algebraic structures play an important role in relating different branches of mathematics. Their unifying power makes a case for inclusion in the '0' level syllabus.

A Basis for the Analysis

a)   General criteria for selection of content

A critical analysis of any syllabus or part of it should take into account the principles that are involved in curriculum planning. Several educationists have written on this and several models have been suggested. Among them are Tyler (1949), Taba (1962), Lawton (1975) and Stenhouse's process model (1975).

Some curriculum models are quite explicit about how objectives and curriculum content should be derived but some are not. Tyler's objective model, which was later extended and refined by Taba, is useful for purposes of analysis and evaluation because it clearly specifies what factors have to be considered when selecting content. In this model Ralph Tyler suggests that we should consider both the needs of the learner, and those of society. He further goes on to suggest that when objectives and content have been derived from these two sources (with the help of subject specialist) these (the objectives and content) should then be passed through "psychological and philosophical check points". At the "psychological check point" one checks to see whether or not the content and objectives are in line with both the physical and cognitive development of the child. One also checks to see if the objectives and content are in line with currently accepted learning theories.

At the "philosophical check point" one asks questions about worth and nature of knowledge. In the case of mathematics one asks questions like, What are the essential features of mathematics? What knowledge contributes towards its growth? Is mathematics merely the accumulation of techniques and their application, or there is something more to it?

The Tyler model alone is not enough to help us do our analysis. The criteria given are too general. One needs to be more specific about what he means, for example, by pupil needs or needs of society.

This means that in addition to the Tyler model we need some other criteria that are

more specific. These two should complement each other rather than substitute each other.

There is so much literature on "criteria for content selection" in mathematics that one cannot possibly use all. For my purpose I will use the criteria given by the Soviet educationist Kolyagin Yu M. et al (1980).

(b)   Specific criteria for selection of content in mathematics

Kolyagin Yu et al give eight criteria for content. In summary form these are "instructural significance criterion, activity criterion, criterion of completeness, world outlook criterion, criterion of generality, criterion of breadth, criterion of development and
criterion of applicability".

Let us now look at what they say about each of these criteria. Kolyagin Yu M. et al say that for a topic to be in the "instructional significance" category it should be both a means for the subsequent study of mathematics and a subject of study itself. The "activity" category includes those topics which "operate actively over a prolonged period (throughout the study of a topic, section or course). Graphs of elementary functions, for instance, are used all through the teaching of algebra to school children".

Kolyagin Yu M. et al argue that school education involves three stages. These are knowledge, know-how (skill) and experience. Topics which permeate completely all the three stages are the ones which they include in the "completeness" category.

The "world outlook" and "applicability" criteria are very closely related. Topics which fall into the "world outlook" category should be those that clearly demonstrate that mathematics is a science, concerned with developing mathematical models of concrete reality; and those that fall into the "applicability" criteria should have a marked practical orientation. In other words they should have "direct everyday utility".

The "breadth", category includes those topics that make it possible to establish close links between and within subjects. For example the concept of a vector links geometric and algebraic concepts: the statement $a\underline{x} + a\underline{y} = a(\underline{x} + \underline{y})$ can be thought of

as an algebraic statement or it can be thought of as a geometric statement describing two similar triangles whose sides are in the ratio 1:a. The concept of a group is also another topic which falls into this category.

The criterion of "generality" includes material that constitute general statements about concrete facts being learnt. For example, from specific facts about triangles general statements of congruency or similarity can be made.

According to Kolyagin Yu M. et al, essential material (for content) should satisfy at least one of the criteria given above. The more criteria a topic satisfies the better it qualifies as content material. Ideally one would like all topics selected for the content of a given syllabus to satisfy all eight criteria.

One thing we can observe from the above criteria is that they answer the philosophical questions of "worthy", "what mathematical knowledge is important?" The questions of utility, relevance to real life and relevance to society are also considered.

Now let us briefly summarize what we have done up to this point. In trying to understand the mechanics of our analysis, we have given both general and specific criteria for content (selection). The general criteria are based on Tyler's objective model, and the specific criteria are given by Kolyagin M. Yu et al.

The rationale for having both general and specific criteria is that general curriculum theory should guide us in our analysis. But general theory alone is not enough, we need specific subject recommendations to operationalize it. On the other hand specific subject theory loses direction if it is not guided by general curriculum theory.

Let us now use (in a complementary manner) the two criteria given above to analyze the '0' level mathematics syllabus for Zimbabwe.

We will start by passing the content through a "psychological sieve". We have mentioned above that one of the things that we consider under psychological issues is cognitive development. Well known educationist in the field of cognitive development are Whitehead, Piaget and Bruner.

Whitehead gave his three stages of Romance, Precision and Generalization. Piaget

gave his four stages of sensory motor (approximately zero to two years), pre-operational (approximately two to seven years) concrete operations (seven to twelve) formal operations (approximately twelve years and above). On the other hand Bruner gives what he calls three levels of knowing as "enactive, iconic and symbolic." Now if we bear in mind that our children get to Form I at approximately the age of thirteen or fourteen then we can see that Piaget's theory of development is only useful to us in as far as it relates to late developers.

The implication of this is that if our '0' level syllabus is to be suitable to all pupils then we should have subject matter that is also suitable for the late developers, those who are still in the concrete operations stage. At this stage, as Piaget says, pupils have acquired the concept of conservation and reversibility and they have thus extended their use of symbols to assimilate past and present experience to future situations but they have not yet (fully) developed the ability to reason hypothetically and the ability to perform controlled experiments.

A look at our '0' level syllabus shows that most of the results or formulas have to be arrived at intuitively and then they are applied. Even important results like the pythagoras theorem and circle properties do not have to be proved formally, one just has to establish the results intuitively and then use them. This obviously accommodates the late developers, however, as I shall argue below there should be a balance between intuitive thinking and analytical thinking.

I however find Bruner's enactive, iconic and symbolic levels of knowing more appropriate to apply because, firstly, as Bruner says, these levels of knowing apply throughout life (although in different proportions), and secondly because the other theories of development like Piaget's seem to set limits on what children can do rather than, in Vygotsky's (1962) words, concentrate on what children do know and make progress from there". Bruner's theory is more positive in that it provides room for pupils to be stimulated into readiness. This thinking is also supported by Inhelder, B. (although an advocate of the Piagetian school of thought when she suggests ways of stimulating readiness in a memorandum, Bruner (1960:p. 40).

Since we are going to use Bruner's three levels of knowing in our analysis we should briefly say what each of them means.

Lawton (1973) explains Bruner's enactive, iconic and symbolic levels of knowing by

the use of an example. He says that a pupil can show his understanding of the principle of a balance beam in three ways. Firstly by actually manipulating himself, say, on a see-saw (enactive model), secondly, by using a model on which rings can be hung and balanced or by drawing the balance (iconic mode) or thirdly by means of language without diagrams (symbolic mode).

According to Bruner "The problem of learning is essentially how to find a kind of "best fit" between the structure of the task and the structure of the person's thinking". Readiness, he admits, is important but according to him what is more important is the mode of presentation (McIntosh J.A. 1971, p 70). The crucial question is, does the mode of presentation (which could be enactive, iconic or symbolic) match the learner's structure of thinking?

According to Brunerian theory it therefore means that the '0' level content we have can be learnt by our secondary school pupils. The most important thing is that the teacher should be able to translate the ideas into the language and concepts of the learner. Bruner (1962) further stresses that mathematics is a sequential subject, therefore any instruction should take this into consideration.

To conclude this section on cognitive development, this is what we have observed: first, that according to Piaget's stages of development the content is suitable for the pupils because most of them have already reached the stage of formal operation. However, research into Zimbabwean children, Orbell (1975), has shown that there are some late developers. On the other hand if we look at the syllabus we notice that major results, theorems and formulas do not require formal proofs, they have to be taught/learnt intuitively. To an informed teacher this provides amble opportunity for using Dienes' (1960) dynamic and constructivity principles. Concrete material and concrete situations can be used to construct mathematical concepts (before they are analyzed). This helps to accommodate the late developers. Finally, in this section; we have also found that according to Bruner the mode of presentation is the most important thing. He, like Inhelder, believes that pupils can be stimulated into readiness.

Recall that according to the Tyler model content has to be passed through both a "psychological sieve" and a "philosophical sieve". Some of the philosophical questions that are asked are; What knowledge is most worth? (This was asked by Herbert Spencer, quoted in Barrow (1976, p 38). What is knowledge? And in our context, what is the nature of mathematical knowledge? What are its essential elements? and what knowledge contributes towards mathematical growth?

These questions about worth are answered in Kolyagin Yu M. et al's eight criteria for content selection. To them (and also Dienes and Skemp) one important feature of mathematical knowledge is its evidence of structure. Piaget (1968) (quoted in Nyagura L (1982) defines structure as a "system which is a totality that has laws and properties that are characteristic of it as a totality".

For Bruner (again in Nyagura 1982), "to learn structure is to learn how concepts are related." The following statement shows how important the concept of structure is to Bruner, "Teaching specific topics or skills without making clear their context in the broad fundamental structure of field of knowledge is uneconomical.... Piaget on the other hand had this to say on mathematical structure "the question comes up whether to teach the structure, or to present the child with situations where he is active and creates the structure himself ... The goal in education is not to increase the amount of knowledge, but to create the possibilities for the child to invent and discover... Teaching means creating situations where structure can be discovered."

Now to make Piaget and Bruner's ideas on structure operational for content analysis one first of all has to answer the question "what topics in mathematics have the greatest power to relate other areas of mathematics? What topics give unity to mathematics?

One area of study which has unifying power is algebraic structure (groups, rings and fields). As Kinsella (1965) pointed out "any theorem that is proved about groups will also apply to any specific group whether the "elements" are numbers or geometric elements". This idea is also supported by the Cambridge conference 1963, when in its report on "Goals for School mathematics" it writes "We believe that these concepts; sets, functions, transformation groups and isomorphisms belong in the curriculum not because they are modern but because they are useful in organizing the material we want to present". In the present '0' level mathematics syllabus examples of groups could be drawn from arithmetic, matrices and geometric transformations like translations, rotations, and reflections. Besides relating different branches of mathematics, groups also help students to understand number properties better.

These concepts (groups rings and fields) are definitely not beyond the cognitive

levels of '0' level students. In fact the Cambridge Conference argued that these concepts "can be introduced in rudimentary form to very young children and repeatedly applied until a sophisticated comprehension is built up."

Recommendations

To conclude this section I suggest that the idea of a group, and the idea of a function (not just drawing graphs) and their applications to other areas of mathematics as well as to real life situations be included in the '0' level mathematics syllabus. These concepts could be introduced and developed through an intuitive practical approach. These topics also satisfy some of Kolyagin Yu M. et al's criteria for content. For example they satisfy the "instructional significance" criterion because they (groups and functions) are both a means for the subsequent study of mathematics and a subject of study in themselves. The "activity", "generality" and "applicability" criteria are also satisfied. Finally, these topics are also important for the development or growth of mathematics. For example in algebraic topology, new structures are generated by throwing (or superimposing) a group structure on given topologies. In general new mathematical structures can be developed through an interaction of group structures with existing mathematical systems or structures.

Also related to the idea of mathematical growth and subsequent mathematical use is the concept of a limit. The idea of a limit should be introduced to pupils early. One does not necessarily need to do rigorous theory of limits at '0' level. A concrete and intuitive approach can be used. For example pupils can investigate limits of functions using calculators. Some limits can be illustrated using concrete examples.

Here is an example that could be used to show that
$$\tfrac{1}{2} + \tfrac{1}{4} + \; + \ldots\ldots\ldots\ldots = 1$$

$$\tfrac{1}{2} + \tfrac{1}{4} +$$

0   ½   ½ + ¼   1

In the diagram AB = 1 unit
We half the unit interval AB, half the resulting sub-interval on the right and repeat this process endlessly. If we add the resulting lengths we get $1/2 + 1/4 + 1/8 + 1/16 \ldots = 1$ and pupils can see that with each division we get nearer and nearer to one but we never exceed it. Hence $1/2 + 1/4 + 1/8 + 1/16 \ldots = 1$

The idea of a limit is central to both the learning of subsequent mathematics and to mathematical growth because it is the basic idea (together with continuity) on which calculus is built, and calculus is an important tool for mathematical analysis.

Estimation of area under a curve by counting squares and by dividing into trapezia is included in the '0' level syllabus. This is good but it should be explicitly stated that both larger (outside) and smaller (inside) areas are considered, with larger areas progressively getting smaller, and smaller areas progressively getting larger. This is a very simple and practical idea because all that the student need to do is to progressively divide the squares into smaller and smaller squares. This again gives the pupils an intuitive idea of a limit, and the fact that a limit point can be approached both from the top and below. But even more important is the fact that we will be preparing the pupils for "integration". This helps to de-mystify concepts like upper bound, lower bound, upper Reiman sums and lower Reiman sums which students find difficult when they come to college. Although these terms are not used at '0' level, the practical activity is very useful.

Furthermore, where possible, students should be asked to give the physical meaning of the area under the curve. This remark equally applies to gradient.

In the statistics section, an intuitive idea of correlation can be introduced. For example pupils could examine tables of two sets of data to see if they are correlated or not. They can also exhibit the data diagrammatically using scatter graphs. Although at this stage there is no actual measure of correlation which is given, pupils should be able to say whether two sets of data are (roughly) correlated or not. The inclusion of this concept in the syllabus is in line with Kolyagin et al's applicability criteria.

At this point I would like to make it clear what my ideological stand is on mathematics learning/teaching: I strongly believe that there is value in learning via intuitive thinking. However, I also believe that in the learning/teaching of mathematics there should be a balance between intuitive thinking and analytical thinking. For example those topics which we consider to be sufficiently simple for the '0' level students should be thoroughly investigated through analytic processes of thinking. As one introduces the topic one may start by appealing to the pupils' intuition but as the subject unfolds analytic thinking should be brought to bear on the subject. However, higher concepts (like groups and limits discussed above) which we consider to be central or important in mathematics can start and end at an intuitive level at '0' level. The rationale behind this is that since these concepts are important we want them to

be introduced early so that pupils can gain facility in using them, but again treating them analytically can prove to be too difficult for the pupils at this stage.

I have talked about balancing intuitive thinking and analytic think in mathematics. This is one thing which I think is missing in our '0' level syllabus, there does not seem to be balance between intuition and rigour. A look at the syllabus shows that major results like pythagoras theorem and circle properties are to be derived intuitively and then used, no formal proofs are required. This is a weakness because to my mind, pupils should at some stage be required to question what they "see" intuitively. They should also be helped to appreciate the need for proving things because this is central to the learning of mathematics.

Furthermore some people do mathematics because they see the beauty of mathematical reasoning; "formal mathematical proof" is one area in which this beauty is manifested. Therefore to exclude formal proofs from the syllabus would be to deprive pupils of an opportunity to also experience this beauty and pleasure.

Kolyagin Yu M. et al have "world outlook" and "applicability" criteria. These criteria include what others call "everyday utilitarian value." As long as the topics number and consumer arithmetic are taught well the syllabus relates mathematics to everyday use. What is required in the process of teaching/learning is to relate theory to practice. For example pupils should not learn about mathematics of finance like profit, loss, interest, commission and discount theoretically without relating it to the actual transactions that take place in the community (for example in local banks or in the local post office). This concept of integrating theory with practice is supported by other educationists. Amongst them are Dewey, J (1971), Freire Pauls (1973), Gray, G.J. (1968) Mao (1979) and Marx and Engels (1979).

According to Marx (quoted in Mao (1979). "Practice is the basis and purpose of the cognitive process. ...knowledge can only be truly attained in man's relation to his environment ..." According to Friere "man's activity consists of action and reflection: it is praxis ... Theory and practice" on the other hand Gray argues that "all happiness is related more closely to that kind of activity which understands how to relate the head to the hand, theory to practice ..."

The point I am making here is that our '0' level syllabus has some topics that have everyday utilitarian value but this alone is not enough, a correct pedagogical approach is also necessary.

Since most financial institutions in this country charge compound interest instead of simple interest it would also be better to include simple problems of compound interest in the syllabus.

Conclusion

Inclusion of the topics; graphs, statistics and linear programming in the syllabus is good because these topics have applications in industry and commerce. I however feel that in this computer age it is necessary to include simple computing courses like, algorithms and flowcharts, topics which can be done even if one does not have a computer. Pupils should be taught how to use a calculator both as learning aid and as a machine to help them in performing routine calculations. The inclusion of the topics, approximations and estimates, in the syllabus is a welcome thing because as people rely more and more on electronic devices for their calculations there is need for them to have skills in approximating and estimating so that they can check on the "reasonableness" of the answers they get using the calculators.

Considering that this is a four year syllabus the content that is given in the '0' level syllabus can be covered within the given time. In fact the topics I have suggested for inclusion can be added to the syllabus without having any problems with time.

To summarize I have started by setting up a theoretical framework on which I was going to base my analysis. To this end I have used Tyler's curriculum model, and the content criteria given by Kolyagin Yu M. et al. I have then used this theoretical framework to analyze the '0' level mathematics syllabus for Zimbabwe. In the process of doing this extensive use has been made of the ideas of Bruner and Piaget especially on cognitive development and the idea of mathematical structure. I have argued that although the content is sufficiently related to the pupils everyday lives and to the needs of society, there is no balance between intuitive and analytic thinking. Finally I have suggested some topics (because of the role they play in both subsequent learning of mathematics and mathematical growth) for inclusion in the syllabus.

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