A FURTHER NOTE ON LABOUR MIGRATION

by

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Massiel and Yotopoulos (M & Y), in their note \( \int_0^1 \int \), on Michalopoulos' migration article \( \int_0^b \int \), found that his conclusions were either trivial (if remittances by migrants were excluded) or wrong (if they were included). We agree. However, we would like to point out that although M & Y's algebra is correct, their geometrical presentation and some of the conclusions they claim to draw from diagrams and algebra are wrong. After documenting these points, we comment on an error made by Michalopoulos overlooked by M & Y.

We write M & Y's system with slightly simplified notation as:

\[ Q = f(S_1, U), \quad f_{u}, \quad f_{U}, \quad f_{G}, \quad (1) \]

\[ R = g(S_1, d), \quad g_{u}, \quad g_{d}, \quad (2) \]

\[ E = S_{2} - S_{1}, \quad (3) \]

\[ Y = Q - R \quad (4) \]

\[ L = S_{2} - U \quad \text{and} \quad (5) \]

\[ V = Y / L, \quad (6) \]

where \( Q = \text{domestic output}, \ 2 = \text{remittances}, \ E = \text{number of emigrants}, \ S_{2} = \text{skilled labour employed at home}, \ S = \text{total} \]
skilled labour, α = an unspecified shift parameter,
U = unskilled labour, Y = total income, L = total
domestically employed labour force, and V = income per
domestically employed worker.

This system can be solved for the six variables
Q, R, E, T, L and Y given preassigned values of U, B and B
Assuming that B is subject to policy control we can find
the value of B that will maximize V. The condition for
such a maximum is
\[
\frac{dV}{dB} = \frac{(S_h + U)(e_b - e_d) - (L + G)}{(S_h + U)^2} = 0
\]
where subscripts refer to partial derivatives. This
condition will hold when
\[
f_a - e_b = \frac{L + G}{S_h + U} \quad \text{or} \quad B - B = V
\]
where following M & Y, B is the marginal product
of skilled labour at home (f_a) and B is the marginal remittance of skilled emigrants (e_b).

It is useful at this juncture to examine the economic
meaning of equation (8) before looking at the geometry.
If one would-be migrant is forced to remain at home
domestic income is raised by the value of his marginal
product less the remittances he would have sent had he
emigrated. Thus the left hand side of (8) is the marginal
gain to the domestic economy of retaining one additional
skilled worker.

The equation states the familiar condition that an average is maximized when it is equated
to a marginal.

Our Fig. 1 correctly depicts the alternatives open
to such an economy. Skilled domestic labour is measured
...and average and marginal products on the Y-axis. Total skilled labour available is OS. The curve $N_h = (x_h)$ is the marginal product of skilled labour employed at home and the curve $Q/L$ is average domestic output per domestically employed (skilled and unskilled) worker. The curve (which seems to be the same as "V" in M & Y's Figure 1) reaches its maximum at E (corresponding to E in M & Y's Figure 1) where marginal and average domestic product are equal. V, income per domestically employed worker, differs from $Q/L$ by the amount of remittances received per domestically employed worker ($R/L$). The curve $R/L$ has been drawn under the assumption that the marginal remittance ($Mr$) is constant. Then the curve $V$ can be drawn by adding $N/L$ and $Q/L$. It reaches its maximum where it intersects the curve $M-M$ (drawn by subtracting the constant marginal remittance curve $Mr$ from the $N_h$ curve) at point F. Thus the optimum amount of domestic employment of skilled labour is OA and emigration is equal to AS. The marginal product of skilled labour at this point is AC, marginal remittance is FG (not OB as M & Y suggest), average domestic product is $AB$, average remittance is $AG$, and average income per domestically employed worker is maximised at level $AF$.

If there were no remittances, then the optimal employment of skilled labour would be $OD$; domestic product and income per domestically employed worker would be equal to $BE$ and marginal and average products would be equal. (This condition is obvious from equation 7). Thus the existence of remittances increases the optimal level of emigration.
Skilled Labour Employed at Home (Sh) \rightarrow \leftrightarrow \text{Skilled emigrants (S-Sh)}
M & Y claim to draw three comparative - static conclusions from the analysis, none of which is necessarily true.

First, they claim "that the optimal level of migration is an increasing function of the level of marginal remittances. (p. 333). While this result is the most plausible empirically, it need not be true. This case can be analyzed by considering a change in a such that $\delta_0 > 0$ -- the marginal remittance rate is increased. Assuming that, condition (7) has been solved for the optimal $S_0 = S^*$, it can be implicitly differentiated to obtain the result

$$\frac{dS^*}{d\delta} = \frac{\delta + \delta_0 (\delta + U)}{d^2\delta/dS^2}$$

The second order condition for a maximum requires that $d^2\delta/dS^2 < 0$. Hence $\frac{dS^*}{d\delta} > 0$ as $\frac{1}{(\delta + U)} \delta + \delta_0 > 0$.

Since $\delta_0 > 0$, $\frac{dS^*}{d\delta} < 0$ if $\delta_0 > 0$. This is the result proposed by M & Y -- increased marginal remittance leads to more migration. However, it is clear that if $\delta_0$ is sufficiently negative -- that is that the average remittance falls even though marginal rises -- higher marginal remittances will be associated with a smaller optimum level of migration. This is quite clear from our Figure 1. Higher marginal remittances at the previous optimum ($S^*$) lower the $M_1 - M_2$ curve at that point. If $V$ remains constant or shifts up, the maximum point on $F$ must move to the left. However, if $V$ shifts down (because the $R/L$ shifts down) the maximum point can move to the right. This will occur only if the $M_2$ curve "twists" so that marginal remittances fall at lower levels while they rise at higher levels of migration.
M & T's second proposition is "if the marginal remittance curve is horizontal, the optimal level of skilled employment will be independent of the size of the total labour force, as can be seen from equation (7) or Figure 1". (p.333). This result cannot be seen from either their equation (7) or our equation (8); nor from their Figure 1 as asserted. In fact the statement is wrong.

There are several ways in which the labour force can be changed.

1. Assume the total labour force is changed by altering the value of U. Implicitly differentiating the condition for maximum V (7), simplifying and assuming that the second order condition for maximum V holds (d²V/dS² < 0), we find

\[ \frac{dS^*}{dU} < 0 \]  as \[ V + f_{S,U}(S + U) > f_u \] (10)

where \( S^* \) is the level of skilled employment at which \( V \) is maximized. Hence, the slope of the marginal remittance curve does not affect the direction of change of the optimal skilled labour force employed at home when the unskilled labour force changes. However, the optimum home employed skilled labour force does depend on the number of unskilled persons employed, independent of that slope, contra the statement cited above.

It is worth providing some interpretation of (10). Since in general \( f_u > V \) (that is, the marginal product of unskilled labour will be less than the average product of skilled and unskilled labour plus average remittances), and \( f_{S,U} > 0 \) (that is, skilled and unskilled labour are complementary inputs) it follows that \( \frac{dS}{dU} > 0 \). Thus, the effect of an increase in the total unskilled labour force will be to reduce the optimal amount of emigration.
That is, an increase in the skilled labour force will result in a smaller amount of skilled labour being employed at home. This is obvious from the diagram. The increase in the skilled labour force will leave the $M - M'$ curve unaffected (assuming a constant marginal remittance rate), but $dU$ and hence $V$ will shift up for each amount of skilled labour employed because of a higher number of emigrants, and the intersection of the two will shift to the left.

2. What happens when only the amount of skilled labour increases? Implicitly differentiating (11), we find

$$\frac{dS}{dN} = -\frac{g_S}{dU} \leq 0. \quad (12)$$

That is, an increase in the skilled labour force will result in a smaller amount of skilled labour being employed at home. This is obvious from the diagram. The increase in the skilled labour force will leave the $M - M'$ curve unaffected (assuming a constant marginal remittance rate), but $dU$ and hence $V$ will shift up for each amount of skilled labour employed because of a higher number of emigrants, and the intersection of the two will shift to the left.

3. What will happen when the labour force expands through proportional expansion of both skilled and unskilled labour? That $S$ and $U$ rise in proportion can be described by

$$S = aU, \quad (13)$$

where '$a$' is some constant. Substituting this expression into the system, we again implicitly differentiate (11) and find that

$$\frac{dS}{dU} = 0 \Rightarrow (T_S - aS_u) (1 - \gamma) + V \gamma u + aS_u, \quad (14)$$

assuming a constant marginal remittance rate ($g_S = 0$), a comparison with (10) shows that while it remains likely that home employment of skilled labour will rise, it is somewhat
less likely than when unskilled labour alone increases because of the term $a_{2g}$. As in that case, the rise in unskilled labour will raise the marginal product of skilled labour, tending to lead to greater employment of skilled labour at home. However, this effect is counterbalanced to some extent by a rise in the $R_l$ curve, as occurred in case 2 when only skilled labour rose.

It is more interesting to ask whether, when skilled and unskilled labour forces expand proportionally, home skilled employment will rise more than in proportion to the growth of the labour force. The condition for a rise in the proportion of skilled to unskilled labour employed at home is found by implicitly differentiating (11), still assuming (12). After simplifying, the condition can be written as

$$
\frac{\partial \log U}{\partial \log L} = \frac{1}{3} - \frac{1}{N_0} \leq 0
$$

when $K_{np} = 0$. There is no simple interpretation of the condition but it is readily seen that it will depend heavily on the relative magnitudes of $f_{a_1} a_{12}$ and $f_{a_2} a_{21}$. The stronger the degree of diminishing returns to skilled labour, $(f_{a_1} a_{12} < 0$ and large in absolute value) and the less complementary are skilled and unskilled labour in production $(f_{a_2} a_{21} > 0$ but small), the greater will be the optimal proportion of skilled labour emigrating.

Of one thing we can be certain, however: the amount of skilled employment at home will in general not be independent of the growth of the labour force when the marginal remittance curve is held constant.

This third proposition that M & Y claim to derive from the model is that "the optimal level of migration will be less
if the objective is to maximize income per total population (including emigrants)." (footnote 2 p. 333). This proposition too is wrong. The opposite can be shown to be true.

we can formalize this version of the model with the following equations:
\[ Q = f(S_h, U) \quad (1) \]
\[ T = t(U) \quad (15) \]
\[ E = S + S_h \quad (3) \]
\[ Z = Q + T \quad (16) \]
\[ N = S + U \quad (17) \]
\[ W = Z/N = \frac{Z}{S + U} \quad (18) \]

where \( T \) is total earnings of emigrants and \( W \) is the total citizen population residing both at home and abroad. \( \bar{W} \) is then the average income of all citizens and the objective of the system is to maximize \( W \). Note that remittances in this model are merely transfers between individuals who are equally weighted in the implicit welfare function. The condition for a maximum of \( W \) is:
\[ \frac{dW}{dS_h} = \frac{1}{\bar{W}} (f^*_{S_h} - t_{S_h}) = 0, \quad (19) \]

\( f^*_{S_h} \) is starred to distinguish the optimal marginal product of skilled labour when \( W \) is maximized compared with when \( V \) is maximized. The condition is obvious — migration takes place until marginal earnings of skilled labour is equated between home and foreign employment.\(^*\). All maximization of \( W \) result in more or less migration than maximization of \( V \). We can find the answer by comparing the optima represented by equations (8) and (13) Rewriting,
\[ f^*_{S_h} - V - S_{Ph} = 0 \quad (8) \]
\[ f^*_{S_h} - t_{S_h} = 0 \quad (19) \]
and subtracting (19) from (8) we find
\[ (f_n^* - f_n^*) = V + (M_t - M_r) \] (20)

where Mt is the marginal earnings abroad of an emigrant.

The second term on the right hand side of (20) retained
\((Mt - Mr)\) is the marginal earnings of emigrants. Thus the
sign of \(f_n^* - f_n^*\) depends on whether or not the retained
earnings abroad of emigrants is greater or less than the
average income of all persons remaining at home. Recalling
that all emigrants are skilled workers while the income at
home is averaged over the mass of unskilled workers, it
seems plausible to impose the condition \(V - (M_t - M_r) < 0\).
This is indeed a rather weak behavioural constraint,
skilled workers will only migrate if their standard of
living (retained income) abroad is at least as high as
the average income of all residents of the home country.
If this condition is imposed, it follows that \(f_n^* < f_n^*\),
which implies that optimal migration will be greater if
the welfare (income) of emigrants is taken into account.

This result sheds some light on the question of whether
developing countries need to restrict free emigration. The
answer depends on whether or not welfare of citizens living
abroad enters the country's welfare function. If it does,
and migration of skilled labour takes place so long as
earnings are higher abroad, equation (19) demonstrates
that free migration will maximize such a welfare function.
On the other hand, according to condition (20) free migration
will result in average income of residents of the country
being lower than it would be if migration were restricted.
Thus if only income of residents is important to the
government, some restriction of migration will be desirable.
This is, of course, a very simple model. A complete evaluation of the desirability of migration restriction would have to include considerations of the effects of emigration on capital accumulation and appropriate allocation of investment between physical and human capital, as well as on intangible but important considerations such as respect for liberty of the individual.

Finally, we would like to point out that Michalopoulos' conclusion on the optimal size of population when the marginal product of labour abroad exceeds the maximum average product of labour at home is incorrect. Our Figure 2 reproduces his Figure 2, in which \( AP \) is the average product of labour (or average per capita income if we assume that the labour/population ratio is fixed) in country B, \( MP \) is the marginal product of labour in country B, and \( YY \) is the marginal product of labour in country A and labour is assumed to be homogeneous. According to Michalopoulos, "if there were no restrictions to migration, there would be an incentive for B's workers to emigrate until only 0.7 remained in B" (p. 136). This is clearly incorrect, for while the 'marginal product' of labour in B equals the marginal product in A, it is impossible (without subsidies from abroad, which are excluded from the analysis) that labourers in B will be paid this marginal product. Indeed, if the whole domestic product of B is paid to labour, the average earnings of labourers will come to only \( JM \), which is less than what they could earn abroad. In the circumstances shown, the optimal population in B will be nil - the case of Antarctica.

Perhaps it is worthwhile commenting a little more fully upon the use of the diagram, since similar confusions can arise in the analysis of other problems. There are two
basic arguments which might be used to justify the upward sloping marginal and average product segments. First, a variable proportions argument that the amount of land, capital, technology, etc. is held fixed and that if very small amounts of labour are applied to this fixed quantity of complementary resources, the marginal (and average) products of labour will rise over some range. On the assumption of a constant returns to scale production function, the marginal products of the fixed factors will be negative in this range. Insofar as it is possible for the economy to apply its labour to only some of these resources (which seems empirically reasonable since most countries of the world, including even the poorest, contain some unused land, mineral deposits, etc.), the relevant average product of labour will be given by NR (corresponding to the highest average product of labour) up to labour input OS, beyond which the curves are as shown by Michalopoulos.
Second, what if the upward rising segments of the marginal and average product curves are explained by "increasing returns to the size of the economy"? There are several stories which could be told to account for this. However, they all have in common the notion that output per bundle of resources will rise as the economy expands. If the economy consists of atomistic decision-making units, no one will find it worthwhile to take this relationship into account in making his decisions. The relationship will be external to firms (and labourers), but internal to the economy. (The common analogy would be the Marshallian analysis of economies external to the firm, but internal to the industry).

In this case, the MP curve will indicate the increment in output which will be associated with increasing the labour force by one unit (where the labour force is exogenously determined), when the amounts of all other factors adjust to the new amount of labour according to the "economic laws" of the economy under consideration. Its relationship to any observed locus of output/labour points will depend upon the nature of these "laws". The curve is only of practical relevance if it becomes a central decision-maker who can control the labour supply (through birth control and health programs, immigration and emigration control, etc.).

**FOOT NOTES**

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4. The relationship between this curve and the AF curve and the "operating" MP and AP curves (shorn of the externality element) will be analogous to the relationships among long run marginal and average cost, and short run marginal and average cost, in the conventional analysis of the cost curves of the firm.

5. For the policy implications of maximizing an objective function similar to that proposed by Michalopoulos, under this type of production function, fixed (non-labour) factor prices, and additional assumptions about labour supply (supply of effort by residents, degree discrimination possible in payment of immigrants, etc.), see Bovey D. Domar, "The Soviet Collective Farm as a Producer Co-operative", American Economic Review, Vol LVI, No. 4, Part I, September, 1966, pp. 734-57.