A TWO SECTOR MODEL OF MIGRATION WITH URBAN UNEMPLOYMENT IN DEVELOPING ECONOMIES

John R. Harris and Michael P. Todaro

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I. Introduction

Throughout many of the less developed economies of the world, especially those of Tropical Africa, a curious economic phenomenon is presently taking place. Despite the existence of positive marginal products in and significant levels of urban unemployment, agriculture/rural-urban labour migration not only continues to exist but, indeed, appears to be accelerating. Conventional economic models with their singular dependence on the achievement of a full employment equilibrium through appropriate wage and price adjustments are hard put to provide rational behavioural explanations for these sizable and growing levels of urban unemployment in the absence of absolute labour redundancy in the economy as a whole. Moreover, this lack of an adequate analytical model to account for the unemployment phenomenon often leads to rather amorphous explanations such as the "bright lights" of the city acting as a magnet to lure peasants into urban areas.

In this paper we shall diverge from the usual full employment flexible wage-price models of economic analysis by formulating a two-sector model of rural-urban migration which, among other things, recognizes the existence of a politically determined minimum urban wage substantially higher than agricultural earnings. We shall then consider the effect of this parameter on the rural individual's economic behaviour when the assumption of no agricultural labour surplus is made - i.e. that the agricultural marginal product is always positive and inversely related to the size of the rural labour force. The distinguishing feature of this model is that migration proceeds in response to urban-rural differences in the rate of expected earnings (defined below) with urban employment acting as an equilibrating force on such migration. We shall then use the overall model for the following purposes:

1. to demonstrate that given a politically determined fixed minimum wage at levels significantly exceeding rural marginal real products, the continued existence of rural-urban migration in spite of substantial overt urban unemployment represents an economically rational choice on the part of the individual migrant;
2. to consider the direct effects of agricultural price controls (e.g. marketing boards, fixed deliveries, etc.) on the level of urban unemployment, an aspect of agricultural pricing policies which has rarely been given its just recognition;

3. to show that the economists' standard policy prescription of generating urban employment opportunities through the use of "shadow prices" implemented by means of wage subsidies or direct government hiring will not necessarily lead to a welfare improvement and may, in fact, exacerbate the problem of urban unemployment;

4. to evaluate the welfare implications of alternative policies associated with various "back to the land" programs when it is recognized that the standard remedy suggested by economic theory - namely, full wage flexibility - is for all practical purposes politically infeasible. Special attention will be given here to the impact of migration cum unemployment on the welfare of the rural sector as a whole which gives rise to intersectoral compensation requirements; and, finally,

5. to argue that the optimal policy is, in fact, a "policy package" including both partial wage subsidies (or direct government employment) and measures to restrict free migration.

II. The Basic Model

The basic model which we shall employ can be described as a two-sector internal trade model with unemployment. The two sectors are the permanent urban and the rural. For analytical purposes we shall distinguish between sectors from the point of view of production and income. Specifically, it is assumed that the urban sector specializes in the production of a manufactured consumer good, part of which is exported to the rural sector in exchange for agricultural goods. The latter sector has a choice either of producing only a single agricultural good some of which is exported to the urban sector using all available labour or using only part of its labour to produce this good while exporting the remaining labour to the urban sector in return for wages paid in the form of the manufactured good. We are thus assuming that the typical migrant retains his ties to the rural sector and, therefore, the income that he earns as an urban
ker will be considered, from the standpoint of sectoral welfare, as accruing to the rural sector. However, this assumption is not at all necessary for our demonstration of the rationality of migration in the face of significant urban unemployment.

The crucial assumption to be made in our model is that rural-urban migration will continue so long as the expected urban real income at the margin exceeds real agricultural product — i.e. prospective rural migrants behave as maximizers of expected utility. For analytical purposes, we shall assume that the total urban labour force consists of a permanent urban proletariat without ties to the rural sector plus the available supply of rural migrants. From this combined pool of urban labour, we assume that a periodic random job selection process exists whenever the number of available jobs is exceeded by the number of job seekers. Consequently, the expected urban wage will be defined as the fixed minimum wage (expressed in terms of manufactured goods) times the proportion of the urban labour force actually employed (see equation (6)). Finally, we assume perfectly competitive behaviour on the part of producers in both sectors with the further initial simplifying assumption that the price of agriculture (defined in terms of consumer goods) is determined directly by the relative quantities of the two goods produced.

Consider now the following mathematical and graphical formulation of the model.

**Agricultural Production Function:**

\[ X_A = f_A(N_A, L) \]

where,

- \( X_A \) is output of the agricultural good,
- \( N_A \) is the rural labour used to produce this output,
- \( L \) is the fixed availability of land, and \( f_A \) is the partial derivative of \( f_A \) with respect of \( N_A \).

**Manufacturing Production Function:**

\[ X_M = f_M(N_M, K) \]

where,

- \( X_M \) is output of the manufactured good,
- \( N_M \) is the urban labour used to produce this output,
- \( K \) is the fixed capital stock, and \( f_M \) is the partial derivative of \( f_M \) with respect of \( N_M \).
where,

- $X^*_m$ is the output of the manufactured consumer good,
- $N$ is the total labour (urban and rural) necessary to produce this output,
- $K$ is fixed capital stock, and $f'_L$ is the partial derivative of $f_L$ with respect to $N$.

**Price Determination:**

(3) \[ P_A = P \frac{f'(X^*_m)}{f'(X_A^*)} \quad \text{where} \]

where, $P_A$, the price of the agricultural good in terms of the consumer good, (i.e., the terms of trade) is a function of the relative outputs of agricultural and consumer goods when the consumer good serves as numeraire.

**Agricultural Real Wage Determination:**

(4) \[ W_A = P_A f'_L \]

where, $W_A$, the agricultural real wage, is equal to the value of labour's marginal product in agriculture expressed in terms of the consumer good.

**Manufacturing Real Wage:**

(5) \[ W = f'_L \Rightarrow \]

The real wage in manufacturing, expressed in terms of consumer goods, is equated with the marginal product of labour in manufacturing because of profit maximization on the part of perfectly competitive producers. However, this wage is constrained to be greater than or equal to the fixed minimum urban wage. In our analysis, we shall be dealing only with cases in which $f'_L = \bar{W}$ (i.e., there is never an excess demand for labour at the minimum wage).

**Urban Expected Wage**

(6) \[ \bar{W}^*_u = \bar{W}^* \cdot \frac{N}{N_u} \quad \text{where} \]

where the **expected real wage** in the urban sector, $\bar{W}^*_u$, is equal to the
minimum real wage $\bar{W}_u$, adjusted for the proportion of the total urban labour force (permanent urban plus migrants denoted as $N_u$) actually employed

$\frac{N_u}{N}$.

Only in the case of full employment in the urban sector is the expected wage equal to the minimum wage.

Labour Endowment:

(7) $N_A + N_u = \bar{N}_R + \bar{N}_u = \bar{N}$

that is, there is a labour constraint such that the sum of workers employed in the agricultural sector ($N_A$) plus the total urban labor force ($N_u$) must equal the sum of initial endowments of rural ($\bar{N}_R$) and permanent urban ($\bar{N}_u$) labour which in turn equals the total labour endowment ($\bar{N}$).

Equilibrium Condition:

(8) $\nu_u = \nu_A$.

Equation (3), an equilibrium condition, is derived from the hypothesis that migration to the urban area is a positive function of the urban-rural expected wage differential. This can be written formally as

(9) $\dot{N}_u = \phi \left( \frac{\bar{W}_u}{\bar{W}_A} - \frac{P \varepsilon^A}{P_A} \right) \phi > 0, \ \phi(0) = 0$

where $\dot{N}_u$ is a time derivative. Clearly then, migration will cease only when the expected income differential is zero, the condition posited in (3).

It is important to note that this assumes that a migrant gives up only his marginal product. Other assumptions could be made. Much of the literature has stressed that in peasant economies producers receive their average product which is higher than their marginal product. Indeed, this is at the heart of the well known Lewis and Fei-Ranis models. However, these models ignore the migration decision and seem to assume that migrants continue to receive their share of peasant production yet migrate only if jobs are actually available. At least in Kenya it appears that migrants continue to receive income from land after migration and commonly hire labour to work on their farms in their absence. There is also a considerable group of landless individuals who work on farms for wages. Thus it would appear that our assumption is not unreasonable. The analysis could easily be modified to make earnings foregone equal to average
product, however.

We thus have 3 equations in 3 unknowns (X, X, , X, , V, , V, , V, , N, , N, and F,) and given our production functions and the fixed minimum wage it is possible to solve for the equilibrium unemployment rate and, consequently, the equilibrium expected wage, relative output levels and terms of trade. Let us analyse the equilibrium and the overall adjustment process in terms of the following diagrammatic representations of the model.

Figure 1 brings together all 8 equations in terms of a four quadrant diagrammatic framework. The two production functions are shown in quadrants B and D. Given labor endowments as portrayed in quadrant C and these production functions, a hypothetical production possibility curve MZQT is generated in quadrant A. However, with a given and fixed urban real wage , we see from quadrant 5 that the maximum consumer goods output will be OE - i.e. that output at which . Moreover, if we assume that the permanent urban labour endowment, , cannot be transferred to the agricultural sector, then the maximum attainable level of agricultural output would be OR as shown in quadrants A and D. Consequently, the operative production possibility curve is ZZ'OE.

Now it is evident from this framework that the locus of full employment points is that shown by ZQ in quadrant A. And the only full employment point consistent with the prevailing minimum wage would be point Z. But is point Z an equilibrium point as defined in our model, that is, is the expected urban wage (in fact ) equal to the value of agricultural marginal product? Consider the transfer of an additional worker from the rural to the urban sector, i.e. rural employment falls to 1 in quadrant D while the urban labour supply rises to 1 in quadrant B. We see from quadrant (3) that, given , this migration will result in positive urban unemployment thereby lowering the expected wage , equal to the slope of AZ, below the minimum wage , equal to the slope of AZ (recall our random selection assumption). But this expected wage is equal to W = DE = VS consumption goods. The loss in agricultural output ( ) is , J = VZ. But given the terms of trade, assumed to be the slope of J derived from (3), these VZ agricultural goods are
equivalent to \( Z^T \) consumer good. Since \( Z^T < VS = VQ \) the migrant will gain income measured in terms of the consumer good by leaving the farm.

Alternatively, we see that his expected consumer goods earnings through migration (\( W_Q \)) is equivalent in value to \( QY \) agricultural goods at the terms of trade associated with point \( V \) on the actual production frontier. Since \( QY \) is greater than \( VZ \) the migrant also gains income measured in terms of the agricultural good. Consequently, we may conclude that there will be further migration and that equilibrium can be obtained only with urban unemployment \(^8\). Even though such an equilibrium, say at point \( H \) in quadrant \( A \), at which (3) is also satisfied, necessarily implies a suboptimum situation from the point of view of the economy as a whole, it does represent a rational, utility maximizing choice from the point of view of individual rural-urban migrants and, as will be demonstrated below, will likely represent a welfare improvement for the rural sector as a whole.

So far we have assumed that an equilibrium solution to the equation system exists. We will make no attempt here to prove such existence but will remain content to assert that such is the case. However, we do want to investigate the stability of equilibrium in the model. To do so, let's differentiate \( \phi \) (equation (9)) with respect to \( N \) keeping in mind that \( dN = \frac{\delta N}{\delta T} \) according to (7). We obtain

\[
\frac{\delta \phi}{\delta N_u} = \phi' \left( - \frac{P_u}{(N_u)^2} + P_A f'' + \frac{P_A}{X_A} f_1 \right).
\]

Stability requires \( \frac{\delta \phi}{\delta N_u} < 0 \) which is satisfied if

\[
\frac{\delta P_A}{\delta X_A} < \frac{(N_u)^2}{(f_1')^2}.
\]

The right hand side of this inequality is unambiguously positive, hence our assumption that \( \frac{\delta P_A}{\delta X_A} < 0 \) will certainly insure stability and indeed is stronger than necessary. The adjustment mechanism may be made more clear by the phase diagram, Figure 2 in which the function \( \phi \) is plotted.
Its positive slope reflects the hypothesis that migration flows will increase with the magnitude of the urban-rural expected wage differential. In Figure 2, \( \phi \) is plotted under the assumption that \( \phi (0) = 0 \), hence the horizontal intercept is at the origin (in general the intercept would be \( a \)) and have further made the purely arbitrary assumption that \( \phi \) is a linear function.

The arrows show direction of adjustment in accordance with (10). If \( \frac{\partial N_u}{\partial K^P_A} > 0 \), then \( N > 0 \) but we know that if \( f^A > 0 \), the expected wage differential will decrease.

Additional migration, by increasing \( N_u \) without affecting \( N \), will reduce the expected urban real wage through increased unemployment while transfer of labour out of agriculture raises \( f^A \) and reduced agricultural output will also cause \( F^A \) to rise. Thus migration reduces the wage differential up to the point that it is zero and equilibrium is achieved when there is no further incentive for migration.

One other point might be raised at this juncture. So far we have assumed that the urban minimum wage is fixed in terms of the consumer good. What, if instead, the minimum wage were fixed in terms of the agricultural good?

We would then substitute for equation (5)

\[
(W^M = \frac{f^A}{F^A})
\]

Substituting (4) and (5') into (7) we get the equilibrium relationship
We can then imagine an economy starting initially at production point Z on the production possibilities frontier (Figure 1A) again assuming that $f'_{ny} > 0$ at that point. The adjustment process will again be reached through a simultaneous raising of $P_A$, $f'$ and lowering of $V_u$. As relative agricultural output falls, $P_A$ will rise. This in turn will cause output of the consumer good to fall since producers will produce only up to the point that $f'_{ny} = \frac{V_u}{V_n}$ rises in terms of consumer goods and $f'$ can be raised only through output restriction ($f'_{ny} < 0$). Therefore, in general we would find the equilibrium point lying southwest of Z and south of H. Output of both goods suffers. Whether this will cause more or less unemployment than in the initial case is indeterminate since $V_u$ falls.

Although our initial assumption is a bit easier to handle, the general conclusion remains unaffected if we make the minimum wage fixed in terms of the agricultural good. Equilibrium is only achievable with unemployment. Actual minimum wage setting is usually done with reference to some general cost of living index, and food is the largest single item in the budget of most urban workers [20], [21]. Hence, the second case may be somewhat more realistic. Note that in the first case the "true" real wage was reduced somewhat by the rising agricultural good price, while in the latter case it is increased by the falling price of the consumer good.

III. Policy Alternatives

A. Agricultural Price Fixing

One very common area of economic policy making in less developed countries relates to the question of the role of the agricultural sector in the overall development process. Policy measures in several countries have for numerous reasons been directed towards keeping agricultural prices from rising. Kenya and Ghana among others, have instituted government agencies to handle the collection and distribution of major crops at fixed
prices; India has strictly controlled inter-state trade in food crops. If the price of the agricultural good were fixed in terms of the price of the consumer good in our model (i.e. if $P_A$ were fixed), then it is clear that the entire adjustment of $P_A$ will have to be in the $f$ term. Equilibrium will occur only with a higher $f$ and lower $u$ than if $P_A$ were allowed to vary. Hence, the effect of such price policy will be to increase the equilibrium level of unemployment. Thus, although economists have widely recognized the potentially harmful effects of such policy on the growth of agricultural output, the effect on urban unemployment has not generally been considered.

In our model, if $P_A$ is fixed there is a determinate level of unemployment and agricultural output, but the markets for goods would not clear at this price since the fixed $P_A$ would not be consistent with the relative quantities of the two goods produced at the "equilibrium". With our extremely simple price determining equation (3) there would be excess demand for the agricultural good and excess supply of the consumer good.

In terms of the model there would in general not be an equilibrium consistent with both $u$ and $P_A$ fixed. One way in which equilibrium could be attained would be obtaining agricultural goods from outside the economy such as through the U.S. and from the U.S.

The point that we wish to make is that the provision of such aid may serve to allow pursuit of a policy of keeping food prices low and this in turn leads to higher levels of open urban unemployment than would otherwise persist. Thus the gains in real income accruing to the recipient country are at least partially offset by unemployment.

B. Planning in Terms of Shadow Prices: A Parable of Incomplete Analysis.

Suppose that there is an economy that behaves according to our model and that the government of the country becomes concerned about the high level of urban unemployment associated with its equilibrium at point $H$ (Figure 1a). It, therefore, contracts for the services of a high powered economist to come and reconsider policies to solve the problem. The economist arrives and soon confirms the fact that there is indeed a lot of open urban unemployment. He does a quick calculation on the back of an envelope and finds that the minimum wage is four times the marginal
product in agriculture (which is positive, however). He is also told by the Minister of Economic Planning that it is politically impossible to lower the minimum wage in real terms so that he had better think of some other solution.

Our economist, being familiar with the current development literature, announces that this is a clear case of money costs diverging from social costs and that the government should use a shadow price for labor in planning output of the manufactured good. The Minister of Economic Development is a bit puzzled at first, but soon the visiting economist convinces him of the wisdom of pricing factors in terms of opportunity cost. The Minister then asks what shadow price they should use. The Economist gains quick access to a computer and improvises a linear programming model of the economy. In the first version, he uses a general labor constraint and finds that the associated dual variable has a value of zero. He then runs a second version with separate urban and rural labor constraints and finds associated dual variables of zero and a positive, but low, value respectively. Having just told the Minister that the shadow price to be used is just the value of the dual variable, he is a bit embarrassed when asked which of the dual values to use. Although his first instinct is to say that the appropriate shadow price is zero since the urban unemployed comprise a pool of redundant labor which can be tapped, he then remembers reading Little's article which shows that even with a marginal product of zero there is a positive opportunity cost of hiring additional labor since total consumption will rise. After a bit of head scratching our expert becomes somewhat conservative and says that the shadow price to use is the marginal product in agriculture. He claims that if a wage subsidy is paid to private producers that the effective cost of hiring labour is equal to this shadow price and if government enterprises hire labour as long as it is profitable using the shadow price, the economy will move to an efficient production point since this will equate marginal products in the two sectors. A quick calculation using the manufacturing production function and unemployment data suggests that such a scheme should expand industrial employment by enough to eliminate existing unemployment.
The Minister quickly replies that this will cause budgetary problems since the public corporations will run deficits and substantial amounts of subsidy will have to be paid to private firms. There will probably be complaints from parliament that the public firms are inefficient as evidenced by their deficits and that windfall profits are being given to capitalists. The Economist ponders this for a moment and explains that parliament has to be educated on this issue and volunteers to address a special session.

Parliament is convened. The Economist explains the issues just as he has to the Minister, and is given a five-minute standing ovation at the end of the presentation. One member rises to propose a motion that minimum wage legislation be repealed so as to allow wages to reflect social costs. He argues that this would avoid budgetary complications involved in actually implementing planning using shadow prices. The motion is soundly defeated.

A second member then rises to propose a motion that a wage subsidy be paid to employers so as to equate social and private costs of hiring labor. A good debate ensues with arguments about subsidy to capitalists and need to keep on good terms with aid-giving nations. The latter argument carries the day and the motion is passed unanimously.

Then the Finance Minister rises and says that he wants to ask the Economist a question. How could the subsidy be financed? The Economist answers that they should raise the revenue through non-distorting lump-sum taxes. Suddenly the Finance Minister jumps up again and says that this would involve redistributing income in favor of industrial workers at the expense of peasants since they are the principal candidates for taxation. He is told that if that is undesirable they might reconsider repealing minimum wage legislation. The Finance Minister then tables a resolution calling for a tax on land to be used to pay the wage subsidy. The motion carries unanimously. The Economist is thanked profusely on behalf of the Parliament and Cabinet, collects his fee, and heads for the airport confident that he had indeed solved the unemployment problem.

A few months later the Finance Minister asks to address a special session of Parliament. He announces that the new policy has failed
miserably. Since inauguration of the wage subsidy, employment in the consumer goods industry has increased by somewhat more than the initial amount of unemployment. Nevertheless, there remains alarmingly high unemployment and agricultural output has dropped substantially. Food prices have risen. Furthermore, in order to finance the wage subsidy the land tax has caused rural unrest and tax collectors are being massacred. A telegram is immediately dispatched summoning the Economist to return and explain what has happened.

He arrives on the next plane and goes directly into session with the Finance Minister and the new Minister of Economic Planning (the first one had been sacked). After reviewing the available facts, he begins to realize that both he and the literature on shadow-pricing failed to anticipate the indirect effects of the policy he had recommended. Specifically, he did not take account the fact that total migration to the urban areas would increase in response to the wage subsidy. He asks for a couple of weeks to analyse the situation in more detail.

At the end of the period he returns, smiles a bit nervously, and assures the Ministers that he now has the answer. He pulls out of his brief case a document that is identical to Part II of this paper. You see, he continues, I have found that individuals in this economy migrate up to the point that their earnings in agricultural employment are equated with their expected urban earnings. As industrial employment increased as a result of the wage subsidy, unemployment was initially reduced. But the lower unemployment rate increased expected urban income which in turn induced additional rural-urban migration. Figure 3 will make clear what happened (Fig. 3 corresponds to Fig. 1 A).

When I arrived the economy was producing at point H. Because of the minimum wage, output of the consumer good was restricted to the quantity OX_H. If individuals did not migrate in response to expected wage differentials, the economy could have produced at point Z, but instead migration with unemployment reduced agricultural output to the level OQ. The standard theory of shadow pricing led me to believe that through wage subsidies the economy would have moved close to point N on the
production possibilities frontier at which the output of manufactured good would have increased by $HN$ with agricultural output constant. Indeed, with further adjustment of the subsidy to reflect changing marginal productivities and prices, the economy could have reached $L$, the optimum position. Since all individuals in this economy have identical homothetic preference maps and this government has an individualistic social welfare function, we can draw social indifference curves. The economy would have moved, therefore, from a welfare level of $V_2$ to a higher level, $V_4$.

Instead, the wage subsidy caused the economy to move to point $J$. The price of the agricultural good (equal to the slope of the indifference curve at the production point) increased. At point $J$, $\frac{P_A}{P_A} = \frac{V_4}{H}$ and migration ceased. It is clear that the subsidy had to cause the economy to move northwest of $H$ since additional urban employment would cause more migration, hence agricultural output would have to decrease. If the equilibrium were east of $U_2$, the economy would have been better off than at $H$. As it happened, the equilibrium at $J$ has a welfare value $U_2$, which represents a worsening of welfare. Had the subsidy been increased to the point that unemployment were eliminated the economy would have moved to $K$, the only efficient production point at which $P_A = P_A$. It is clear that such subsidy would carry sectoral reallocation too far since consumer goods output in excess of $X^*$ can only occur with $P_A < P_A$, hence the marginal product in manufacturing will always have to be lower than the value of the marginal product in agriculture. Such a policy would increase welfare if it lay on the segment $FN$ of the production possibilities frontier. The standard theory led me awry because it failed to consider the effect of the subsidy on migration.


The new Minister for Economic Planning looks at Figure 2 and says, "If we would just restrict migration so as to force the unemployed back to the land, it is clear that we can make the economy better off. Keeping the minimum wage we will continue to get consumer good output of $X^*$ and with full employment insured by restricting migration agricultural output will be $X^*$. We will be producing on the production possibilities frontier at $Z$ and the economy is unambiguously better off than at $H." With visions
of achieving a welfare level of $U_3$ he is about to rush off to get approval of an enforced back-to-the-land policy. However, the Finance Minister interjects that there is enough trouble in the rural areas already. If, being denied the opportunity to export its labour, the rural sector as a whole were to be made worse off it would be political dynamite. He then turns to the Economist for advice. Our expert replies that unquestionably, Z would be better in an aggregate welfare sense than H. In this particular case it is also better than J or K although in general back-to-the-land may or may not be superior to wage subsidy. He points out that at Z the economy is still not achieving its potential optimum $L_0$ and that the value of marginal product in agriculture will remain lower than the minimum wage which with no unemployment is equal to the expected wage. It is clear, however, that at Z the economy will have as much of the consumer good and more of the agricultural good than at H, hence, there exists some pattern of income transfers that would make no one worse off and many better off than at H. The Finance Minister comments that he has already heard about lump-sum transfers and has gotten into hot water as a result. The Economist replies that Figure 4 may make the issue more clear. The line $N'T'$ represents production possibilities for the agricultural sector when some or all of its labour endowment is engaged in urban manufacturing. If its entire endowment of labor is devoted to agricultural production, it will produce a quantity $OM'$. However, by exporting its labour it can "produce" the manufactured good (wages are paid in the form of this good). Hence this kind of production possibilities frontier depends on market forces (wage levels and unemployment) as well as on purely technological factors. The slope of the frontier is the marginal rate of transformation between agricultural and consumer goods for the sector and can be written as

$$\frac{\delta X_A}{\delta N_R} = \frac{\delta X_A}{\delta N_R} = \frac{\delta X_A}{\delta N_R} = \frac{\delta X_A}{\delta N_R} = \frac{\delta X_A}{\delta N_R} \quad \text{where} \quad -\delta N_A = -\delta N_R, \quad X_R \quad \text{is the total amount of} \quad \text{the manufactured good received in the form of wages by the sector, and} \quad N_R \quad \text{is the total number of rural migrants in the urban labour force.}$$
The numerator gives the amount of agricultural production given up by transferring one worker into "labour export." The denominator is the amount of additional manufactured goods earned as a result of the transfer of an additional worker into labor export. Clearly, this latter term will depend on wages and unemployment. Since the expected wage is assumed to equal the actual wages received by the sector, the total "production" of consumer goods by the rural sector is equal to the expected wage times the number of individuals from that sector who are exporting labour.

Symbolically,

\[ (13) \quad \frac{R}{M} = u, \quad \frac{R}{N} = \frac{N}{u} \cdot \frac{N}{u} \]

Hence the denominator of (12) is

\[ \frac{\delta X^R}{\delta X^A} = \frac{\frac{N}{u} \cdot \frac{N}{u}}{\left( \frac{N}{u} \right)^2} \]

Thus we can rewrite (10) as

\[ (12') \quad \frac{dX^R}{dX^A} = \frac{f^R}{f^M} \left( \frac{N}{u} \right)^2 \left( \frac{N}{u} \right) \left( 1 - \frac{N}{u} \right) \]

In addition to these production possibilities, the rural sector also has the opportunity to trade some of its output with the urban sector in order to increase its welfare. Corresponding to each point on its production possibilities frontier there is a determinate price of the agricultural good. In an important question to ask, however, concerns the way in which rural welfare is affected by the choice of particular output - consumption constellation. It can be shown that the rural sector's welfare is maximized when equation (14) is satisfied (see Appendix for derivation).

\[ (14) \quad \frac{A_R}{A_u} = \frac{1 - R}{1 - u} \]

where, \( n \) is the elasticity of the permanent urban labour forces offer curve of the consumer good in exchange for the agricultural good, and, \( R \) is the rural sector's proportion of the total urban labour force.
Equation (14) can be interpreted in the following manner.

\[ P_f \left(1 - \frac{1}{n}\right) \] is the amount of consumer goods sacrificed by the rural sector as a result of removing one worker from producing the agricultural good which would have been exchanged for the consumer good at the market price \(1/E\). This quantity is less than the value of the marginal product of labor in agriculture (\(P_{Ag}\)) since the reduction in output has a favorable terms-of-trade effect. If the demand for the agriculture good is inelastic (\(n < 1\)) we reach the startling conclusion that the sacrifice becomes negative! This is of course, the familiar proposition that aggregate farm income may be increased through reducing output. The direct gain in consumer goods achieved by the rural sector through exporting an additional unit of labor is \(c_u(1-R)\). The migrant earns the expected wage \(c_u\), but his migration, by increasing unemployment, reduces the expected wage (and earnings) of all migrants already in the urban labor force.

As long as \(P_f \left(1 - \frac{1}{n}\right) < c_u(1-R)\) the welfare of the rural sector will be increased by allowing migration even though unemployment ensues and the economy as a whole sacrifices output. Since \(P_{Ag}\) and \(c_u\) are always positive and \(R < 1\), additional migration will always benefit the rural sector when \(n < 1\). In general, the lower is \(P_{Ag}\), \(n\), or \(R\) and the higher is \(c_u\) the more will the rural sector benefit from the opportunity to migrate.

Consider now the choice facing the individual prospective migrant. This individual, acting as a price taker, will migrate as long as \(P_f \left(1 - \frac{1}{n}\right) < c_u^E\).

We see, therefore, that his decision will not in general lead to welfare maximization for the whole sector. His expected gain in income from migration exceeds that of the sector to the extent that he neglects the effect of additional unemployment on the earnings of others from the same sector. On the other hand, he overstates the loss of income from reduced agricultural output by ignoring the terms of trade effect. If \(\frac{1}{n} > R\), there will be too little migration from the standpoint of the sector as a whole. Conversely, if \(\frac{1}{n} < R\), there will be too much migration when
decisions are made solely from the point of view of individual utility maximization.

From the foregoing, one can conclude that although a back-to-the-land policy will clearly improve aggregate welfare of the economy, it will likely require substantial compensation to the rural sector if it is not to be made worse off as a result of removing the opportunity for free migration. Clearly the permanent urban labour force will be made better off by becoming fully employed at the high minimum wage while also being able to buy food at a lower price. Those migrants who are allowed and continue to export labour will similarly earn more but this gain will be offset by reduced total labor exports and lower agricultural prices. Whether or not this will be true depends, of course, on the specific parameters for this economy. If \( n \) is sufficiently high, the rural sector could be made better off by restricting migration in the absence of compensation, but this is highly unlikely. Perhaps this is most clearly illustrated in Figure 4. \( \Pi^* \) corresponds to the initial unemployment equilibrium \( H \) (Figure 1). At that point the rural sector as a whole "produces" \( X_A^0 \) and \( X_M^0 \) of the two goods. It also has the opportunity to trade at the price \( P_A^0 \). By trading some of its agricultural output to the permanent urban sector for additional manufactured goods, it consumes \( \frac{X_A^0}{X_M^0} \) and achieves a welfare level of \( u_R^0 \). An enforced back-to-the-land policy results in the sector's producing \( X_A^1 \), \( X_M^1 \). If it could still trade at price \( P_A^0 \) it would clearly be better off but this is impossible. At \( P_A^1 \) the price of the agricultural good will fall to \( P_A^1 \) and with trade the best consumption hurdle attainable by the sector is \( \frac{X_A^0}{X_M^0} \), which corresponds to a lower level of welfare \( u_R^1 \). (Note that if \( P_A^1 \) did not cut \( M^1 \) there would be no incentive to migrate at \( H \).)

At this point enthusiasm for an enforced back-to-the-land policy is rapidly waning. The Ministers of Economic Planning and Finance throw up their hands. Maybe letting the economy return to its old equilibrium at \( H \) (Figure 1) wouldn't be so bad after all, they say in unease.

The Economist immediately replies, "there is still another possibility - a policy package that will be optimal."
D. The Optimal Policy Package; The Parable Concluded

Look back at Figure 3, he says. Even if the back-to-the-land policy can be implemented so that the economy produces at Z and the rural sector is compensated so as to be no worse off than at H, we could still do better. At Z the value of the marginal product in agriculture \( P f_A \) is less than the marginal product in the manufactured goods industry \( f_W = W \).

For general welfare maximization for the economy as a whole we know that \( P f_A = f_W \) only then will the marginal rate of transformation in production equal the marginal rate of substitution in consumption. Clearly, this is point L. In order to reach this point some wage subsidy will have to be paid since you insist on maintaining the minimum wage.

It will have to equal \( U_L - X^L \) when \( f_A = P f_A \). However, when this point is reached, since there is full employment, \( U^L = U_L \) but, \( P f_A < U^L \). Hence individuals will still find it in their interest to migrate and you will repeat the process of moving to J (Figure 3). Clearly, measures must be taken to restrict further migration. We see therefore, that we can reach the best possible position only through planning and implementing with appropriate shadow prices and migration control. The rural sector will be better off if the economy moves from Z to L (Fig. 3) (in the absence of compensation) providing at both Z and L, \( f_A^L (1 - \frac{P}{W}) U^L (1 - \frac{P}{W}) \) which is likely although clearly not necessary. Even if we can ascertain that moving from Z to L represents a welfare improvement, we cannot in general assert that L represents an improvement over H from the standpoint of the rural sector.

The entire analysis of the preceding section can be applied to compare H and L. It is plausible, but not necessary, that the move from H to L will require compensation to the rural sector if it is not to be made worse off \( U_L \). But we can say with certainty that if compensation is required, it will be smaller at L than at Z and furthermore it should be easier to finance since if the rural sector remains at the same welfare level as at H, the welfare of the urban sector at L will be higher at L than at Z.

In summary, Gentlemen, says the Economist, the optimal policy for you to pursue is the following. Give a wage subsidy as long as the marginal product of labour in the consumer goods industry exceeds the...
value of its marginal product in agriculture. When they are equated, the subsidy should remain at the level necessary to maintain the relationship and direct measures to prohibit further rural-urban migration must be instituted (a prohibitive discriminatory tax on additional migrants would be equivalent to direct controls).

Even so there are corresponding fiscal requirements that cannot be taken lightly. If, to make the policy package politically feasible, compensation of the rural sector is required as well as wage subsidy (or operating deficit of government-owned plants), any government may in fact find it difficult to find non-distorting taxes capable of raising sufficient revenue. Perhaps a head-tax on all urban residents would be feasible although this too raises the issue of the terms in which minimum wages are set (unions in Tropical Africa have been quite aware of net real wages and have, in many cases, successfully fought to maintain the real after-tax wage). A tax on rural land is ruled out if there must be net subsidy to the rural sector which leaves an urban land tax as the remaining potential tax (we have assumed the absence of pure profits in industry).

This time the Economist, after being thanked, heads for the airport confident that his advice will not boomerang.

IV. Suggested Extensions and Modifications

The model which we have used in this paper is extremely simple. As always, simplicity is gained only through compromises with reality. Nevertheless, we believe that this model captures the essence of the migration cum unemployment process that is a prevalent phenomenon in the "third world." As such, it gives insight into a pressing issue of public policy. In this section we will suggest some directions in which the analysis can and should be extended in order to increase its realism and usefulness.

The most obvious limitation of the analysis is its static character. We intend to remedy this by incorporating migration in response to expected income differentials into a growth model in a subsequent paper. If savings propensities differ between the sectors, if investment is sector
specific, and if there are differential rates of technical progress 
(arising through "learning by doing") between the sectors, then some of 
the conclusions based solely on arguments of static efficiency may have 
to be modified. Migration is a disequilibrium phenomenon and even casual 
observation in Africa suggests that the flows are, if anything, still 
increasing. A general model of dynamic disequilibrium would be desirable 
although such models are notoriously intractible. The assumption we have 
made that all migrants actually earn the expected wage, difficult to 
accept but necessary for analytical purposes, actually derives from a 
more realistic construct set in context of a growing economy. Despite this, our preliminary investigations in this direction give us 
is confidence that the model presented here is a useful analytical tool.

Within the static context it would be desirable to introduce more 
complete and realistic demand functions. Distribution effects could not 
be ignored if income elasticities differing from unity were introduced 
and/or if tastes differed systematically between groups. However, our 
results derive only from price of a good being inversely related to changes 
in the relative quantity of its production. (Even this condition is 
stronger than necessary, see p.436). This need not be universally true 
but is consistent with a large mass of empirical observation within 
limited ranges of income redistribution.

Another desirable modification would be to increase the number of 
sectors to be considered from a welfare standpoint. In the rural sector 
it would be useful to consider landowning and non-landowning classes 
separately since, as extended family relationships weaken and land becomes 
more scarce, the assumption of intra sectoral population homogeneity and 
compensation becomes less tenable. Similarly, distinction between capi-
talist and worker classes in the urban area could be useful. Introduction 
of an urban "semi-modern" production sector which hires labour at wages 
below the official minimum would also add realism. 

Our specification of the expected urban wage is much too simple. 
We are presently in the early stages of empirically testing the migration
hypothesis in Kenya using a more complicated function for expectation formation. So far casual observation by us and statements by experienced observers suggest that the hypothesis is broadly correct. p. (183) So long as expected income is positively related with minimum wages and inversely related with unemployment, this model gives qualitatively correct results. It is quite possible that equilibrium could only occur with positive rural-urban wage differentials in the absence of unemployment as a result of preferences for rural life; (some sociological studies of the magnetic attraction of cities might suggest the opposite). If such differentials reflect preferences which the social welfare function respects, then the optimality criterion of equal values of marginal product between sectors has to be appropriately modified. Somewhat similarly, if migration gives rise to social costs through increased infrastructure requirements, higher crime rates, etc., the optimality criterion will also have to be modified to take this into account.

Finally, we have considered compensation only through lump-sum taxes and subsidies. If instead we realistically add the use of commodity taxes as a fiscal tool for redistribution the analysis will have to be modified along lines suggested by Diamond and Mirlees. Their essential point is that if lump-sum taxes and subsidies are infeasible, optimality requires productive efficiency without equating marginal rates of substitution and transformation.

V. Conclusions

Rural-urban migration in the face of urban unemployment and positive earnings in agriculture has been shown to be a rational response by individuals when urban wages are kept at a high level through minimum wage legislation or collective action. The crucial assumption, the implications of which we have explored, is that individuals migrate in response to expected income differentials.

We have shown that the standard remedy for such unemployment, namely, the expansion of job opportunities in the manufacturing sector through labour subsidies or direct government hiring in accordance with a
profitability criterion using a shadow price for labour will lead either to overexpansion of the urban sector or will fail to eliminate unemployment. Furthermore, the fiscal requirements for subsidy are likely to prove extremely onerous. Such a policy may or may not lead to an improvement in welfare even with perfect lump-sum compensation. At least, it will lead to a productively efficient but non-Pareto-optimal position.

A forced back-to-the-land movement of all unemployed individuals will lead to an unambiguous aggregate welfare improvement. However, in the absence of compensation it is highly likely that the rural sector (defined to include all migrants who retain ties to the sector) will be made worse off. If their welfare is politically important, such a policy would be extremely difficult to pursue. This policy will lead to an efficient but non-Pareto-optimal point.

Pareto-optimality can be achieved only through a policy package combining employment expansion in accordance with shadow pricing and migration restriction. Again, the rural sector is likely to require compensation if it is not to be made worse off, but the required compensation will be less than under a back-to-the-land policy alone.

Governments, no doubt, are caught in a dilemma. Neither eliminating minimum wage legislation nor placing physical controls on migration are likely to be politically palatable although the latter policy is somewhat of an anathema. The alternative, however, is to continue to suffer substantial and growing levels of open unemployment in urban areas.

The long run implications of this phenomenon (i.e., the loss of potential output and, perhaps more importantly, the proliferation of social and political unrest) are only now beginning to be recognized in newly independent developing nations.
We want to derive the conditions for welfare maximization by the rural sector. First form the Lagrangean expression

\[ q = U(x_A^*, x_M^*) + \lambda_1 \left( f_A(N_A^*, \bar{L}) - x_A \right) + \lambda_2 \left( \frac{N_R - N_A^*}{u} \frac{U}{N_M^*} - x_M \right) \]

+ \lambda_3 \left( p_A (x_A - x_A^*) + (x_M - x_M^*) \right). \]

The \( x^* \) terms are quantities consumed while the \( x \) terms are quantities produced. The first term is a utility function applicable to the sector since we assume identical tastes and complete egalitarian redistribution within the sector. The second term is the agricultural production function, while the third term shows that "production" of the consumer good by the rural sector is equal to the expected wage times the number of labour units expected. Finally, the last term reflects the budget or trading constraint when \( (x-x^*) \) is the quantity sold.

Maximizing (1) we get the following first order conditions:

\[ \frac{\delta q}{\delta N_R} = \frac{\delta U}{\delta N_A^*} + \lambda_2 \left( \frac{N_R - N_A^*}{u} \frac{U}{N_M^*} - \lambda_3 \right) = 0 \]

\[ \frac{\delta q}{\delta N_A} = \frac{\delta U}{\delta x_A} - \lambda_1 + \lambda_3 \left( \frac{f_A(N_A^*, \bar{L})}{x_A} \left( \frac{p_A}{x_A} \right) \right) = 0 \]

\[ \frac{\delta q}{\delta x_A} = \lambda_1 \left( \frac{\delta f_A}{\delta x_A} \right) + \lambda_2 \left( \frac{N_R - N_A^*}{u} \frac{U}{N_M^*} \left( \frac{U}{N_M^*} \right)^2 \right) = 0 \]

\[ \text{recalling that } dN_a = -dN_u \, ; \]

\[ \frac{\delta q}{\delta x_A^*} = f_A(N_A^*, \bar{L}) - x_A = 0 \]

\[ \frac{\delta q}{\delta x_M} = (N_R - N_A^*) \left( \frac{U}{N_M^*} \right) - x_M = 0 \]

\[ \frac{\delta q}{\delta x_M^*} = p_A (x_A - x_A^*) + (x_M - x_M^*) = 0 \]
Substituting from (2), (4) and (5) into (6) we get

\[ \frac{\Delta U}{\Delta X} \left( P_A + \frac{\delta P_A}{\delta X_A} \frac{\delta X}{\delta X_A} \right) \frac{\delta P_A}{\delta X_A} \frac{\delta X}{\delta X_A} = \frac{\Delta W}{\Delta X} \left( \frac{W}{N_u} - \left( N_R - N_A \right) \frac{W}{N_u} \right). \]

Note that \( \frac{\delta P_A}{\delta X_A} = -\frac{\delta P}{\delta X_A} \) and let us define \( \eta = \frac{\delta (X - X_A)}{\delta P_A}, \) which is the price elasticity of demand (uncompensated) for the agricultural good evaluated at the quantity marketed by the rural sector. \( \eta \) can also be interpreted as the elasticity of the permanent urban sector's offer curve. Further let us define \( (N_R - N_A) = W_R, \) the total amount of urban labor originating from the rural sector (i.e., total labor export by the rural sector) and let \( \frac{W_R}{W_u} = \text{We} \) can rewrite (10) as

\[ (11) \quad P_A \left( 1 - \frac{\eta}{\text{We}} \right) f^*_A = W^*_u \left( 1 - \text{E} \right) \]

which is the formula utilized in the text.
FOOTNOTES

1. For some empirical evidence on the great magnitude of these earnings differentials in less developed economies, see Berg (2) also Chai (10) and International Labor Office (16).

2. We do not make the special assumption of an agricultural labour surplus for the following reasons. Most available empirical evidence to date tends to cast doubt on the labour surplus argument in the context of those economies of Southeast Asia and Latin American countries where such a surplus would be most likely (22) (23) (5). Moreover, few if any economists would seriously argue that general labour surplus exists in tropical Africa, the area to which this paper is most directly related.

3. For a dynamic model of labour migration in which urban unemployment rates and expected incomes play a pivotal role in the migration process, see M. Todaro (24). However, unlike the present model which attempts to view the migration process in the context of aggregate and intersectoral welfare considerations, this model was strictly concerned with the formulation of a positive theory of urban unemployment in developing nations. As such, it did not specifically consider the welfare of the rural sector. Nor was it concerned with some of the broader issues of economic policy considered in the present paper.

4. In tropical Africa especially, this notion that migrants retain their ties to the rural sector is quite common and manifested by the phenomenon of extended family ties and remittances to rural relatives of large proportions of urban earnings. However, the reverse flow, i.e., rural-urban monetary transfers, is also quite common in cases where the migrant is temporarily unemployed and, therefore, must be supported by rural relatives.

5. Again, the qualitative conclusions of the model do not depend on the precise nature of the selection process. We have assumed random selection, however, not merely for analytic convenience but also because it directly corresponds to an appropriate dynamic construct developed in Todaro (24). There it is shown that over time expected and actual earnings will converge to a positive number even though the rate of job creation is less than the rate of migration so that unemployment is increasing.

6. A sufficient, but not necessary, condition for this assumption is a unitary income elasticity of demand for both goods with all individuals in the economy having the same preference map. Again, the assumption is made for analytical convenience. The qualitative conclusions of our analysis will remain unaffected under several plausible assumptions about distribution of income and tastes.

7. \( \phi(a) = 0 \) is purely arbitrary. If, instead, we assume \( \phi(a) = 0 \) where \( a \) can take on any value, migration will cease when the urban-rural expected wage differential is equal to \( a \). None of the subsequent analysis is affected qualitatively by specifying \( a = 0 \). (8) would merely be written as \( W_A + a = W_u \).

8. It is theoretically possible, but not likely, that the institutional minimum wage could be equal to the free market full employment equilibrium wage in which case the actual and expected urban wage would be identical. In such a case conventional analysis suffices.

9. Even if we were more realistic and allowed for income and redistribution effects on demand, the conclusion of excess demand would generally hold if income elasticities for agricultural goods were low.
10. Several issues arise regarding the finance and sale of aid-provided food which would require substantial modification of the model through adding government and foreign-trade sectors. We will not pursue this further here.

11. Hagen (12) states, "a subsidy per unit of labour equal to the wage differential between agriculture and industry will increase real income further than a tariff and if combined with free trade will permit attaining an optimum optimum. Bardhan (17) p. 379) similarly adds. "The best remedy for the misallocation caused by a wage differential is... an appropriate subsidy to the use of labour in the manufacturing industry." It is important to recall that this argument is dependent on variable proportions production functions. If production coefficients are fixed, wage subsidy will have no effect in the short run. The classic statement of this case is Echaus (7) and Bardhan (17) explores its implications for subsidy in a dynamic context. Both of these papers, however, posit surplus labour in agriculture, an assumption we do not wish to make in an African context.

12. Note that aggregate output of consumer goods remains constant, hence price is determined by output of the agricultural good per (3).

13. In considering the welfare of the rural sector as a whole we are making the tacit assumption that there is redistribution of goods between individuals in this sector. Although this is a very strong assumption, yet there is considerable evidence from Tropical Africa that employed urban migrants repatriate substantial portions of their earnings to their kinsmen remaining in the rural areas and conversely that income both in cash and kind is received by unemployed migrants from kinsmen remaining on the farm. To the extent that the extended family system does redistribute goods between members, this assumption may be tenable as a first approximation.

14. Note that if the urban unemployment was experienced only by migrants, this term would equal zero since the total amount of earnings through labour export would be constant. It can be positive only because the permanent urban labour force shares in unemployment, thereby reducing its share of the constant wage bill in the manufactured good industry.

15. For example, compensation might be effected through support of agricultural prices. If this is accomplished through keeping the producers' price higher than the sales price other taxes will have to be increased in order to finance the subsidy and urban real wages will rise through cheaper food prices. Alternatively if producers' price is supported and sales price is maintained at the same level, surpluses will accrue and these will have to be financed from government revenue.

16. At present, Tanzania is attempting to cope with its unemployment problem through just such a back-to-the-land policy. See (14) for an analysis of the economics of the Tanzanian program.

14a. As drawn in Fig. 3, L must represent a higher welfare level than H for the rural sector since P rises (e.g. 3) and the sector produces more of both goods. In fact 'if L lies along BN northwest of the ray going through H there will be an unambiguous sectoral welfare improvement. However if L lies southeast of the ray on BN, the rural sector could be worse off than at H since P_r falls.
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