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**MONEY AND MARKETS**

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## MONEY AND MARKETS <sup>1</sup>

### Abstract

This paper studies the efficiency properties with respect to transactions cost of a trading post set-up vis-a-vis a marketless trading arrangement under the well-known framework of Ostroy and Starr (1974). It is revealed from this exercise that the social institute of money works better when coupled with the social institution of markets and thereby providing an explanation of why, historically speaking, money and markets go hand in hand.

### 1. Introduction

The last two decades have witnessed the gradual emergence of a variety of models to explain the superiority of monetary over non-monetary trade from the point of view of transactions cost. In order to capture the essence of the problem, however, these models have generally dealt with institutionally vacuous economies, at least insofar as the traders are seen to confront each other in the absence of organized markets (Ostroy and Starr (1974), Jones (1976), Kiyotaki and Wright (1989) etc.). On the other hand, there is another stream of literature (see Foley (1970), Wallace (1972) etc.)

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<sup>1</sup>The author wishes to acknowledge many useful discussions with Dipankar Dasgupta

which concentrates exclusively on models which recognize the existence of markets or trading posts (of different forms, eg. wholesale or retail) to deal with exchanges of different pairs of goods.

The present paper is an attempt to marry the above two strands of theory in an order to explain the *simultaneous* existence of monetary trade and market specialization. More precisely, we try to examine the effect (on transactions cost) of combining the social institution of markets with monetary exchanges. In a recent work with a different approach but similar flavour, Starr and Stinchcombe (forthcoming) have shown how a set-up with trading posts using a medium of exchange can dominate a trading post set-up involving barter trades alone.

To study this issue we consider a pioneering contribution to the literature, viz. that of Ostroy and Starr (1974,1990). As is well-known, Ostroy and Starr deal with the question of *feasibility*. More specifically, they look for a condition to be satisfied by a good which, if used as a medium of exchange by the individual traders, can attain their desired allocations within a reasonable period of time. By introducing trading posts in the Ostroy-Starr set-up one can conclude that the time taken to attain the agents' desired allocations *to each given degree of approximation* is bounded above irrespective of the size of the economy; whereas no such bound may

exists in a marketless economy. Note, however, that in certain situations even in the presence of trading posts the time taken to attain the desired allocation *exactly* may not remain bounded as the economy becomes large.

The paper is arranged as follows. The next section compares the trading post set-up with the marketless trading arrangement. A concluding section sums up the findings.

## 2. Transactions Cost : Trading Posts vs Marketless Set-up :

We consider an economy consisting of  $K$  traders (indexed by  $k$ ) and  $n$  commodities (indexed by  $c$ ), the  $n$ -th commodity, denoted by  $M$ , being earmarked to serve as the medium of exchange. The set of all traders is denoted by  $K$  also and the set of commodities other than the  $n$ -th good by  $C$ . The initial endowments of the  $K$  traders are depicted by the  $K \times n$  matrix  $\tilde{W} = \|w_{kc}\|$ , where  $w_{kc}$  represents the level of good  $c$  in the initial endowment of agent  $k$ . The utility function of agent  $k$  is  $U_k : \mathbb{R}_+^n \rightarrow \mathbb{R}$ ,  $k = 1, 2, \dots, K$ . Prices

$$\tilde{p}' = (p_1, \dots, p_n)$$

are exogenously given at the equilibrium levels. Given the prices and the initial endowments, the Walrasian excess demands for the economy are given by the  $K \times n$  matrix  $\tilde{Z} =$

$\|z_{kc}\|$ , where  $z_{kc}$  is the level of agent  $k$ 's excess demand for (or supply of) good  $c$ .

Similarly, the final demands are given by the  $K \times n$  matrix  $\tilde{X} = \tilde{W} + \tilde{Z} = \|x_{kc}\|$ , where  $x_{kc}$  is the level of good  $c$  finally consumed by agent  $k$ .

Thus, the economy under study is characterized by  $(\tilde{W}, \tilde{Z}, p)$ , where  $p$  is the transpose of  $\tilde{p}'$ . The following properties are assumed :

- (U.1) The value of total excess demand equals zero, which can be represented as  $\tilde{Z}.p = 0$ , where the price of the  $n$ -th good  $p_M$  is assumed to be positive.
- (U.2) The total quantity of a particular good purchased is exactly equal to the aggregate amount of that good sold. In other words,  $\sum_k \tilde{Z}_k = 0$ , where  $\tilde{Z}_k$  represents the  $k$ -th row vector of  $\tilde{Z}$ .
- (U.3)  $\tilde{Z} \geq -\tilde{W}$ , the inequality is assumed to hold entrywise. It implies that the final consumption of each good by every agent is non-negative.

The trading arrangement taken up by Ostroy and Starr is based on the assumption that in order to explore the trading possibilities, agents meet each other in pairs following an arbitrary sequence of meetings. This ad hoc sequence in no way depends on the goods an agent desires to demand and

supply. Further, when one pair meets, other pairs are allowed to meet simultaneously.

For the system described so far suppose the following condition introduced by Ostroy and Starr (1974) is satisfied

$$\sum_{c \neq M} p_c [z_{kc}]^+ \leq p_M w_{kM}, \forall k \quad (1)$$

where,

$c$  is the index for commodities,  $k$  the index for agents,  $M$  the commodity earmarked to denote the medium of exchange,  $p_c$  is the equilibrium price of good  $c$ ,  $z_{kc}$  is the excess demand (supply) for(of) good  $c$  for agent  $k$ ,  $[z_{kc}]^+ = \max[0, z_{kc}]$ ,  $w_{kM}$  is the initial endowment of good  $M$  for agent  $k$ .

Therefore, the left hand side of inequality (1) shows the total value of the agent  $k$ 's desired purchase of goods other than  $M$ . On the other hand, the right hand side of (1) expresses the value of the initial endowment of good  $M$  for agent  $k$ . Hence, condition (1) guarantees the fact that an agent possesses enough money in his initial endowment to cover his desired purchases of all the remaining goods.

It is shown in the literature that if there exists a commodity  $M$  satisfying condition (1) above in any given competitive economy with a finite population, then using that good as a medium of exchange an agent can fulfill his/her excess demands within *one round* i.e., the time required for each agent to meet every other agent once and only once. The

result holds irrespective of the sequence in which the agents meet in a marketless arrangement.<sup>2</sup>

Suppose now we ask whether a medium of exchange satisfying such a condition can function *better* in a trading post set-up. Thus, we consider an economy with trading posts or separate markets for dealing with each good against  $M$ . Each agent visits only those markets which are relevant for him.

Depending on the levels of excess demand vectors and a specific sequence of meeting between the agents or visiting markets a marketless arrangement may sometime take less time than a trading post set-up. However, in general for economies with large population the advantages of the social institution of markets is clearly revealed. To see this, an appropriate framework to consider may be that of Debreu and Scarf (1963). More precisely, the economy consists of specific classes of agents where agents belonging to a particular class have identical characteristics. In other words, the set-up under consideration is a replication of a smaller economy. We represent an economy of this nature satisfying condition (1) by  $E$  and its  $\nu$  times replication by  $E_\nu$ .

<sup>2</sup>A more plausible feasibility criterion compared to (1) is that a good, say  $M$ , is demanded in positive quantity by every agent i.e.,

$$z_{kM} + w_{kM} > 0, \text{ for all } k$$

Such a good can be used as a medium to attain equilibrium in finite time. (See, Rajeev(1990) and Dasgupta and Rajeev (forthcoming).

We consider an arrangement where all markets are functioning simultaneously and an agent chooses a trading post at *random* from the set of trading posts he desires to visit (which depends on the goods he wishes to demand and supply). In a parallel fashion we assume for the marketless trading arrangement that agents meet each other at random. We can now compare the average time involved for all possible sequences of meetings, i.e.  $E\hat{T}_{ML}$ , with the corresponding  $E\hat{T}_{TP}$ .

In the case of the marketless trading arrangement, an agent meets another agent at random without any reference to the goods they want to exchange. It is assumed to take one unit of time to confront any agent and exchange (if possible).

We may choose rational numbers from  $[z_{ic} - \varepsilon, z_{ic} + \varepsilon]$  and  $[z_{jc} - \varepsilon, z_{jc} + \varepsilon]$  for all  $c$  and denote them respectively by  $\chi_i^c$  and  $\Omega_j^c$  respectively. We then consider the problem of achieving these rational values instead of the original  $z_{ic}$ 's and  $z_{jc}$ 's. The total time needed to arrive at the  $\varepsilon$ -neighbourhood of a competitive allocation would be represented by  $[\hat{T}_{TP}](\varepsilon, \nu)$ . Parallel notations, viz.  $[\hat{T}_{ML}](\varepsilon, \nu)$  would be used to denote the same for the alternative marketless framework. For a set-up of this nature we have the following results.

**Proposition 1** For an economy  $E_\nu$  described above, there exists an integer  $\bar{N}(\varepsilon)$  independent of  $\nu$  s.t

$$E(\hat{T}_{TP}(\varepsilon, \nu)) < \bar{N}(\varepsilon) \quad , \forall \nu$$

and hence

$$\text{Prob}\{\hat{T}_{TP}(\varepsilon, \nu) < \infty, \text{ for any } \nu\} = 1.$$

**Proof :** See Appendix.

Note that if the original excess demands were themselves rational numbers, then the above result would go through without the need for the  $\varepsilon$  approximation.

On the other hand, a conclusion as general as the above does not hold for the marketless trading arrangement. In fact, it is possible to construct a large number of economies (with  $z_{kc} \in \mathcal{Q}$ , the set of rational numbers,  $\forall k, \forall c$ ) where expected time taken in a marketless trading scheme diverges as  $\nu$  increases. More precisely,

**Proposition 2** There exists at least one economy of the type  $E_\nu$  for which

$$E(\hat{T}_{ML}(\nu)) \rightarrow \infty \text{ as } \nu \rightarrow \infty$$

**Proof :** See Appendix.

Once again this result shows clearly the advantages of a trading post set-up for large economies. Also, from the Remark (in the Appendix) it is clear that one can construct large number of competitive economies where Proposition 2 holds.

It is important to emphasize the fact that in the case of the above result (Proposition 1) the rational approximation played a crucial role. In fact it would be impossible to establish the uniform bound if the agents' excess demand vectors were comprised of irrational components and they tried to achieve these demand points exactly. As Example 1 below shows clearly, no such uniform bound would exist even if the economy with trading posts consists of two goods.

**Example 1:** Consider an economy  $\tilde{E}$  having two goods (say,  $c$  and  $M$ ) and 3 demanders and 3 suppliers with excess demands and supplies for good  $c$  against  $M$ . Let  $z_i^d$  and  $z_j^s$  ( $i, j = 1, 2, 3$ ) denote respectively the excess demands and excess supplies of the  $i$ -th and  $j$ -th agents for good  $c$  with

$$(z_1^d, z_2^d, z_3^d) = (1, \sqrt{2}, \sqrt{3})$$

$$(z_1^s, z_2^s, z_3^s) = (1, \sqrt{2}, \sqrt{3})$$

Initial endowments of the (excess) demanders are represented by  $w_1^d = (0, 1)$ ,  $w_2^d = (0, \sqrt{2})$ ,  $w_3^d = (0, \sqrt{3})$ , where the first components refer to good  $c$  and the second components refer to good  $M$ . Similarly initial endowments of the (excess) suppliers are given by,  $w_1^s = (1, 0)$ ,  $w_2^s = (\sqrt{2}, 0)$  and  $w_3^s = (\sqrt{3}, 0)$ . It can be easily checked that condition (1) of Ostroy and Starr is trivially satisfied here (where the equilibrium price vector  $p' = (1, 1)$ ).

From  $z_i^d$ 's and  $z_j^s$ 's it is clear that  $z_2^d, z_3^d, z_2^s, z_3^s \in \mathcal{Q}^1$ , the set of irrational numbers. Also  $\text{rank}(1, \sqrt{2}, \sqrt{3}) = 3$ , in the field of integers.

Given the above competitive economy, consider the following sequence of meetings. The first demander meets the first supplier, the second demander meets the second supplier and so on as depicted through the following diagrammatic scheme:

Demander	→	Supplier
1	→	1
2	→	2
3	→	3

Let the time taken in attaining equilibrium (starting from the initial endowments) when this economy replicates  $\nu$  times be denoted by  $\hat{T}_{TP}(\nu)$ . Clearly,  $\hat{T}_{TP}(1) = 1$  for the aforementioned sequence of meetings.

Let us now consider a  $\nu = 3$  times replication of this economy. That is, there are 9 demanders as well as suppliers. Let

$$(z_1^d, z_2^d, \dots, z_9^d) = (1, \sqrt{2}, \sqrt{3}, 1, \dots, \sqrt{3}).$$

Similarly

$$(z_1^s, z_2^s, \dots, z_9^s) = (1, \sqrt{2}, \sqrt{3}, 1, \dots, \sqrt{3})$$

Consider the following sequence of meetings shown by the arrows in the figure below. Arrangement I is related to the

first 3 demanders and suppliers. Similarly, Arrangement II is related to the next 3 demanders and suppliers (viz. 4, 5 and 6) and so on.

Arrangement I			Arrangement II		
Demander	→	Supplier	Demander	→	Supplier
1	→	1	4	→	4
2	→	2	5	→	5
3	→	3	6	→	6

#### Arrangement III

Demander	→	Supplier
7	→	7
8	→	8
9	→	9

Let  $V_{id}^t(V_{js}^t)$  denote demander (supplier)  $i$ 's ( $j$ 's) unsatisfied demand (supply) for (of) good  $c$  at the beginning of time period  $t$ . Thus, from trading arrangement I,

$$V_{1d}^2 = V_{2d}^2 = V_{3d}^2 = V_{1s}^2 = V_{2s}^2 = V_{3s}^2 = 0.$$

From trading arrangement II

$$V_{4d}^2 = V_{5d}^2 = V_{4s}^2 = 0$$

$$V_{6d}^2 = \sqrt{3} - 1, V_{5s}^2 = \sqrt{2} - 1, V_{6s}^2 = \sqrt{3} - \sqrt{2}$$

Thus, 3 agents get satisfied and 3 agents remain to be satisfied. The reason behind this is quite simple. When two agents meet, at least one of them would get satisfied. But not

more than 3 traders would get satisfied because  $|1 - \sqrt{2}| \neq |\sqrt{2} - \sqrt{3}| \neq |\sqrt{3} - 1| \neq 0$ .

In trading arrangement III for the same reason, only 3 agents would be satisfied and

$$V_{8d}^2 = \sqrt{2} - 1, V_{9d}^2 = \sqrt{3} - \sqrt{2}, V_{9s}^2 = \sqrt{3} - 1$$

In period 2, consider the following sequence of meetings.

Demander		Supplier
6	→	9
8	→	5
9	→	6

This would result in all the agents reaching equilibrium in two periods. Thus,

$$\hat{T}(3) = 2$$

As a next step consider the  $\nu = 3^2$  times replication of this economy and a similar sequence of meetings. One can easily check that  $\hat{T}(3^2) = 3$  so on.

Proceeding similarly it can be shown that in general there exists at least one sequence of meetings s.t.  $\hat{T}(3^\ell) = \ell + 1$  and hence the time taken for attaining equilibrium rises unboundedly as the economy replicates indefinitely.

**Note:** 1. The above example shows that if we allow the excess demand vectors to take irrational values then one can construct economies where even in the presence of markets

equilibrium can take infinite time. However this certainly does not rule out the possibility that there can exist economies with irrational demand and supply vectors where time needed to attain equilibrium is finite. A trivial example is where demand and supply vectors contain identical irrational elements.

2. When the demand and supply figures are rational numbers, the time taken to attain ones desired bundle cannot go to infinity due to the following reason. Let  $g$  be the greatest common divisor of the non-zero  $z_{kc}$  values, for all  $k$  and for all  $c$ . That is  $z_{kc} = g \cdot I_{kc}$ , where  $I_{kc}$  is a natural number. Hence any pairwise meeting would reduce the agents' excess demands and supplies at least by  $g$ . Since  $z_{kc}$ 's are finite, after a finite number of stages every agent would reach equilibrium.

A greatest common divisor need not exist when some  $z_{kc}$ 's are irrational numbers. Therefore, a *fixed* reduction in excess demands and supplies after each pairwise meeting cannot be ensured for all such competitive economies. This gives rise to the possibility that there can exist economies where the unsatisfied excess demand is reduced by a decreasing sequence requiring in its turn infinite time to satisfy ones demand.

#### 4. Conclusion:

This paper formally establishes the intuitively clear fact

that from the point of view of time as a transactions cost, a trading post set-up dominates a marketless trading arrangement. Thus, money and markets complement each other.

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## Appendix

### Proof of Proposition 1

We choose rational numbers from  $[z_{ic} - \varepsilon, z_{ic} + \varepsilon]$  and  $[z_{jc} - \varepsilon, z_{jc} + \varepsilon]$  for all  $c$  and denote them respectively by  $\chi_i^c$  and  $\Omega_j^c$  respectively. We then consider the problem of achieving these rational values instead of the original  $z_{ic}$ 's and  $z_{jc}$ 's. The associated time cost would then be denoted by  $[T_c]_\varepsilon$ . An exactly analogous problem would be considered for each of the  $(n - 1)$  trading posts that exists for dealing with each good against  $M$  (the medium of exchange). Let  $K_c^S$  and  $K_c^D$  represent respectively the sets of suppliers and demanders of good  $c$ .

Let  $g$  be the greatest common divisor of the elements of the set

$$A = \{\chi_i^c, \Omega_j^c : c \in C, i \in K_c^D, j \in K_c^S\}$$

That is,  $g$  is the largest real number such that  $z_{kc} = g \cdot I_{kc} \forall k, \forall c$  where  $I_{kc}$  is a natural number.

At each period, an agent chooses a trading post at random from the set of trading posts he wishes of visit. Let  $\eta_k^t$  denote the number of trading posts which  $k$  wishes to visit at the beginning of time period  $t$ . Let

$$B_k^t = \{c : |V_{kc}^t| > 0, c \neq M\}$$

then

$$\eta_k^t = \#B_k^t$$

Since  $|V_{kc}^t|$  is a decreasing function of  $t$ , therefore,  $\eta_k^t$  decreases in time.

According to our assumption, at period  $t$ , agent  $k$  chooses a trading post with probability  $= 1/\eta_k^t$

Let us denote the probability for a particular demander  $i (\in K_c^{Dt}$ , the set of unsatisfied demanders of good  $c$  at the beginning of time period  $t$ ) being present in trading post  $(c, M)$  at period  $t$  by  $\rho_{ic}^t$ . Hence,

$$\rho_{ic}^t = \frac{1}{\eta_i^t} \geq \frac{1}{n-1} \quad (1)$$

Similarly, for a supplier  $j (\in K_c^{St})$

$$\rho_{jc}^t \geq \frac{1}{n-1} \quad (2)$$

We denote by  $\delta_c^t$  and  $\psi_c^t$  respectively the actual number of demanders and suppliers *present* in trading post  $(c, M)$ , at time period  $t$ . The following, relations can hold between  $\delta_c^t$  and  $\psi_c^t$

$$(i) \min (\delta_c^t, \psi_c^t) = \delta_c^t$$

This implies each buyer would meet a seller with probability 1 and hence unsatisfied demand of each buyer present would reduce at least by  $g$ .

$$(ii) \min (\delta_c^t, \psi_c^t) = \psi_c^t$$

This implies unsatisfied supply of each seller present would be reduced at least by  $g$ .

$$(iii) \delta_c^t = \psi_c^t$$

This implies excess unsatisfied supply as well as demand of the respective agents would be reduced at least by  $g$ .

At each period  $t$ , the expected reduction of  $i$ 's ( $i \in K_c^{Dt}$ ) unsatisfied demand and/or  $j$ 's ( $j \in K_c^{St}$ ) unsatisfied supply is at least

$$E(\rho_{ic}^t g) \geq \frac{g}{n-1}, \quad i \in K_c^{Dt}$$

and

$$E(\rho_{jc}^t g) \geq \frac{g}{n-1}, \quad j \in K_c^{St}$$

The alternatives depend on (i), (ii) and (iii) above.

Let at time period  $t$ ,  $\bar{R}^t$  be the reduction that occurs to the expression

$$\tilde{V}_c^t = \max_{i \in K_c^{Dt}} V_{ic}^t + \max_{j \in K_c^{St}} |V_{jc}^t|$$

The  $(c, M)$  trading post would be completely cleared (i.e.  $\tilde{V}_c^t = 0$ ) after at most  $\tau$  periods if

$$\sum_{t=1}^{\tau} \bar{R}^t \leq \max_t \tilde{V}_c^t = |z_{lc}| + |z_{hc}|$$

where

$$z_{lc} \geq z_{ic}, \forall i \in K_c^D, i \neq l \in K_c^D$$

and

$$|z_{hc}| \geq |z_{jc}|, \forall j \in K_c^S, j \neq h \in K_c^S$$

We have,

$$\begin{aligned}
 E\left(\sum_{t=1}^{\tau} \bar{R}^t\right) &= EE\left(\sum_{t=1}^{\tau} \bar{R}^t \mid \tau\right) \\
 &= EE(\bar{R}^1 + \dots + \bar{R}^{\tau}) \\
 &= E\{E\bar{R}^1 + E\bar{R}^2 + \dots + E\bar{R}^{\tau}\} \\
 &\geq E\left(\frac{g}{n-1}\tau\right) \\
 \therefore E(\tau) &\leq \frac{(n-1)[z_{lc} + |z_{hc}|]}{g}
 \end{aligned}$$

i.e.

$$E(\hat{T}_C) \leq \frac{(n-1)[z_{lc} + |z_{hc}|]}{g},$$

which is independent of  $\nu$ .

A similar exercise can be performed for other trading posts. Adding over all  $c$  we get,

$$E(\hat{T}_{TP}(\nu)) \leq \bar{N}(\varepsilon); \text{ independent of } \nu.$$

Coming now to the second part of the result, consider the sequence of random variables  $\hat{T}_{TP}(\nu)$ . We have shown that

$$E(\hat{T}_{TP}(\nu)) \leq \bar{N}(\varepsilon) \quad \forall \nu$$

$$\therefore \text{Prob}\{\hat{T}_{TP}(\nu) < \infty \text{ for any } \nu\} = 1. \quad \blacksquare$$

**Proof of Proposition 2 :**

Consider an economy with 2 types of agents, viz. type 1 and type 2 and two goods, to be called  $c$  and  $M$ . The total number of type 1 agents is equal to the total number of

type 2 agents. Each type 1 agent has one unit of good  $c$  and demands in return 1 unit of good  $M$ . A type 2 trader on the other hand has one unit of good  $M$  and demands in return 1 unit of good  $c$ .

It is clear that when a type 1 agent meets a type 2 agent, they can trade and attain their desired commodity bundles. Suppose each type consists of  $X$  number of agents. Then, the probability that an arbitrary type 1 agent, say  $k$ , meets a complementary trading partner (i.e. a type 2 agent) in the first period is

$$p_1 = \frac{X}{X + X - 1} = \frac{1}{2 - \frac{1}{X}} \leq \frac{2}{3} \text{ if } X \geq 2.$$

The denominator of this probability (i.e.  $X + X - 1$ ) refers to the total number of agents excluding  $k$  and the numerator (i.e.  $X$ ) represents the number of complementary trading partners.

The probability of not meeting any type 2 agent by  $k$  in the first period is

$$1 - p_1 = 1 - \frac{X}{2X - 1} \geq \frac{1}{3}.$$

Let  $X_t^U$  denote the number of type 1 agents (or, equivalently type 2 agents) remaining unsatisfied at the beginning of time period  $t$ .

$$\therefore X_1^U = X$$

Clearly,  $X_t^U, t > 1$ , is a random variable and hence we would be concerned here with  $EX_t^U$ . Since the probability of remaining unsatisfied during the first period is  $(1 - \frac{X}{2X-1})$  and there are a total of  $X$  type 1 traders in the economy

$$E(X_2^U) = X \cdot (1 - \frac{X}{2X-1}) \geq \frac{1}{3}X.$$

Let us define an indicator function

$$\begin{aligned} I_{X_2^U=x} &= 1, \text{ if } X_2^U = x \\ &= 0, \text{ otherwise} \end{aligned}$$

Then we can write

$$\begin{aligned} E(X_2^U) &= E(X_2^U I_{X_2^U=1} + X_2^U I_{X_2^U \geq 2}) \\ &= E(X_2^U I_{X_2^U=1}) + E(X_2^U I_{X_2^U \geq 2}) \end{aligned}$$

Let us now calculate

$$\begin{aligned} E(X_2^U I_{X_2^U \geq 2}) &= \sum_{\gamma=2}^X \gamma Pr\{X_2^U = \gamma\} \\ &= \sum_{\gamma=2}^X \gamma \binom{X}{\gamma} (1-p_1)^\gamma p_1^{X-\gamma} \\ &= X(1-p_1) \sum_{\gamma=1}^{X-1} \binom{X-1}{\gamma-1} (1-p_1)^{\gamma-1} p_1^{X-\gamma} \\ &= X(1-p_1)(1-p_1^{X-1}) \geq \frac{1}{3}X(1-p_1), \text{ if } X \geq 2 \end{aligned}$$

$$E(X_2^U I_{X_2^U \geq 2}) \geq (\frac{1}{3})^2 X \quad (3)$$

Let us consider period  $t = 3$ . Suppose at the beginning of the third period,  $X_2^U$  is the number of unsatisfied agents. The probability that a type 1 agent gets satisfied during period 2 is

$$p_2 = \frac{X_2^U}{X_2^U + X_2^U - 1}$$

$$\therefore 1 - p_2 = 1 - \frac{X_2^U}{2X_2^U - 1} \geq \frac{1}{3}, \text{ if } X_2^U \geq 2$$

At the beginning of period  $t = 2$ ,  $X_2^U$  agents have remained unsatisfied and they have a probability  $(1 - p_2)$  of remaining unsatisfied during the third period

$$\begin{aligned} \therefore E(X_3^U) &= EE(X_3^U | X_2^U) \\ &= E\left(X_2^U \left(1 - \frac{X_2^U}{2X_2^U - 1}\right)\right) \\ &= E\left[X_2^U \left(1 - \frac{X_2^U}{2X_2^U - 1}\right) I_{X_2^U=1}\right] \\ &\quad + E\left[X_2^U \left(1 - \frac{X_2^U}{2X_2^U - 1}\right) I_{X_2^U \geq 2}\right] \\ &= 0 + E\left[X_2^U \left(1 - \frac{X_2^U}{2X_2^U - 1}\right) I_{X_2^U \geq 2}\right] \\ &\geq \frac{1}{3} [X_2^U I_{X_2^U \geq 2}] \\ &\geq \frac{1}{3} \cdot \left(\frac{1}{3}\right)^2 X = \frac{1}{3^3} X \quad (\text{using (3)}) \end{aligned}$$

As before in an exactly parallel manner let us calculate

$$E[X_3^U I_{X_3^U \geq 2}]$$

$$\begin{aligned}
&= EE[X_3^U I_{X_3^U \geq 2} | X_2^U] \\
&= E\left[\sum_{\gamma=2}^{X_2^U} \gamma Pr\{X_3^U = \gamma\}\right] \\
&= E[X_2^U (1 - p_2)(1 - p_2^{X_2^U - 1})] \\
&= E\left[X_2^U \left(1 - \frac{X_2^U}{2X_2^U - 1}\right)(1 - p_2^{X_2^U - 1})\right] \\
&\quad [I_{X_2^U = 1} + I_{X_2^U \geq 2}] \\
&= 0 + E\left[X_2^U \left(1 - \frac{X_2^U}{2X_2^U - 1}\right)(1 - p_2^{X_2^U - 1}) I_{X_2^U \geq 2}\right] \\
&\geq \frac{1}{3} E[X_2^U (1 - p_2) I_{X_2^U \geq 2}] \\
&\geq \left(\frac{1}{3}\right)^2 E(X_2^U I_{X_2^U \geq 2}) \geq \left(\frac{1}{3}\right)^2 \left(\frac{1}{3}\right)^2 X \\
&\quad \therefore (X_3^U I_{X_3^U > 2}) \geq \left(\frac{1}{3}\right)^4 X \quad (4)
\end{aligned}$$

Using (4), we get as before

$$\begin{aligned}
E(X_4^U) &\geq \frac{1}{3} E(X_3^U I_{X_3^U > 2}) \\
&\geq \frac{1}{3} \cdot \left(\frac{1}{3}\right)^4 X = \frac{1}{3^5} X
\end{aligned}$$

In general,

$$E(X_t^U) = \frac{1}{3^{2(t-2)+1}} = \frac{1}{3^{2t-3}}$$

If we replicate the economy  $\nu$  times

$$E(X_t^U(\nu)) \geq \frac{X}{3^{2t-3}} \cdot \nu$$

Let us define  $T_\nu$  as

$$\bar{T}_\nu = \min\left\{t \mid \frac{\nu X}{3^{2t-3}} \leq 2\right\}$$

Then,

$$E\hat{T}_{ML}(\nu) \geq \bar{T}_\nu$$

Now given any  $t$ , we can find a  $\nu$  s.t.

$$\frac{\nu X}{3^{2t-1}} \geq 2.$$

$\therefore$  as  $\nu \rightarrow \infty$ ,  $\bar{T}_\nu \rightarrow \infty$ , and hence  $E\hat{T}_{ML}(\nu) \rightarrow \infty$ . ■

**Remark** From this example it is clear that the result holds even when we consider a similar economy with 4 types of agents (each type having  $X$  agents) and 4 commodities. A type 3 agent demands one unit of good 3 against one unit of good 4. A type 4 agent demands one unit of good 4 against one unit of good 3 and so on. Each type (type 3 and 4) has one unit of good  $M$  so that condition (1) is satisfied. Thus, a type 3 agent can trade only when he meets a type 4 agent and vice-versa.

In a parallel fashion we can consider 6 types of agents with 6 goods in the economy and so on. Thereby, one can construct a large member of competitive economies where Proposition 2 holds.

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