

**THE CAPITAL ASSET PRICING MODEL
WITH NON-HOMOGENEOUS EXPECTATIONS:
THEORY AND EVIDENCE ON SYSTEMATIC RISKS
TO THE BETA**

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ABSTRACT

This paper introduces non-homogeneity of expectations (NHE) among investors on the parameters of the probability distribution of assets rates of return and derives an equilibrium return-risk relationship which is non-linear. This relationship shows a new and additional form of risk called theta risks I and II which are the systematic biases to the beta risk arising from NHE among investors on the mean and variance (covariance) respectively of the rates of return. The beta is no longer a complete measure of risk. Under the traditional homogeneous expectations (HE) assumption, or if the theta risks vanish, the CAPM of Sharpe and Lintner is a special case. An errors-in-variables model is used to provide an indirect test and the results are explained within the framework of the model. It appears that the empirical anomalies on the CAPM are due to attempts to fit a linear model on a fundamentally non-linear return-risk relationship.

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**The Capital Asset Pricing Model
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by

Clodualdo R. Francisco*

I. INTRODUCTION AND SUMMARY

The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1969) is an equilibrium model of asset price determination based on the mean-variance portfolio selection of Markowitz (1952). This model, which is the basis for most of the recent works in capital market theory and finance, postulates that under certain assumptions, there is a linear relationship between the return of an asset and its non-diversifiable risk.^{1/}

Tests of the CAPM indicate that the postulated linear return-risk relationship is not consistent with the data and that

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^{1/} For some works which provide a landscape of the literature on the theoretical and empirical development on the CAPM, see Fama (1968), Sharpe (1970), Jensen (1972), Friend (1977), Roll (1977), and Brealey and Myers (1981).

the evidence does not support a riskless market rate of return consistent with the actual measures of riskless rates of return.^{2/}

Aside from the empirical attempts to explain the apparent deficiencies of the CAPM on measurement and other statistical grounds, theoretical extensions of the model, by relaxing some of its assumptions, were also used to explain the evidence.^{3/}

One of the crucial assumptions of the CAPM is that of homogeneous expectations (HE) among investors on the mean and variance of the probability distributions of rates of return to assets. This assumption is observed to be quite restrictive and a number of writers have attempted to relax this assumption.

^{2/}

Friend and Blume (1970), Black, Jensen and Scholes (1972), Blume and Friend (1973), Fama and MacBeth (1973), and Petit and Westerfield (1974). For more recent tests taking into consideration Roll's (1977) critique, see Friend, Westerfield and Granito (1978), Cheng and Grauer (1980), and Best and Grauer (1985).

^{3/}

Among the earliest writers who extended the CAPM by relaxing some of its assumptions were Black (1972) and Mayers (1972). Black dropped the assumption of the existence of unlimited borrowing and lending opportunities at a market risk-free rate and arrived at the well-known zero-beta portfolio. Mayer relaxed the assumption that all assets are marketable by introducing non-marketable assets. Both model appear to provide explanation to the results documented by Black, Jensen and Scholes (1972). For a complete statement of all the assumptions of the CAPM, see for instance, Black (1972).

Lintner (1969) and Sharpe (1970) considered heterogeneity of expectations and arrived at similar conclusions that the existence of diverse opinion does not change the linear structure of equilibrium prices and portfolios. Gonedes (1976) derived equilibrium conditions under non-homogeneous expectations (NHE) similar to that of Black (1972) and Sharpe (1970). Rabinovitch and Owen (1978) derived equilibrium prices and portfolios under NHE and apply their model to different structures of inside information for trading purposes. Mayshar (1983) extends the CAPM by incorporating transaction costs and NHE. He assumes a particular type of probability distribution of asset return and like Lintner, assumes a specific type of investors' preferences. He concludes that "...given divergence of opinion, equilibrium prices can be considered as determined simultaneously by the average and marginal investors" (p. 127). Rabinovitch and Owen did not derive an explicit return-risk relationship for the CAPM with NHE. Like Mayshar, they did not provide tests for their models.

The purpose of this paper is to; extend the CAPM by introducing NHE following Rabinovitch and Owen (1978); derive an explicit equilibrium non-linear return-risk relationship for a given asset; provide an indirect test of the CAPM with NHE; and attempt an alternative explanation to the evidence documented by Black, Jensen and Scholes (1972) and other related test results on the CAPM.

The rest of the paper is summarized as follows:

Section II derives the equilibrium vector of prices and portfolios under NHE and shows that the simple CAPM is a special case. An explicit non-linear return-risk relationship for each asset is derived which shows the existence of two new and additional types of risks. Due to lack of an appropriate name, they are termed as "theta risks I and II". These risks are observed to be a form of "systematic risks" to the beta arising from NHE among investors on the vector of means and covariance matrix, respectively, of returns to assets. Approximations for the theta risks yield alternative return-risk relationships which are comparable to the simple CAPM. The corresponding equilibrium vectors of portfolio are also derived and related to that of the simple CAPM.

Section III provides a two-stage indirect test for the CAPM with NHE using an errors-in-variables model. The primary consideration for this test procedure is the fact that the theta risks are basically unobservable. The first stage tests the simple CAPM where the test procedure attempts to overcome some limitations of previous tests in the literature, such as the use of indirect estimates for the betas using the market model, equal weight per security in the market portfolio, and stationary assumption on the probability distribution of securities rates of return. The second stage postulates that the theta risks for a given security among investors, investing in a particular

subgroup of securities, are relatively smaller than the theta risks among all investors in the entire market investing on the same security such that tests of the simple CAPM using sectoral groups of securities should yield better results. This is expected since the theta risks introduce non-linearity into the return-risk relationship and if the theta risk vanish, then the resulting return-risk relationship becomes linear and is given by the simple CAPM.

Section IV presents and explains the test results. A pattern of significant test results is observed and these results are explained within the framework of the CAPM with NHE. An alternative explanation of the empirical finding of Black, Jensen and Scholes (1972) and other related works is provided. Section V provides some concluding remarks.

II THE CAPM WITH NHE : EXPLICIT NON-LINEAR RETURN-RISK RELATIONSHIPS

A. NHE Assumption, Individual Portfolio Selection Problem and the Market Clearing Condition

Let V be an $n \times 1$ vector, each element of which represents the random one-period total return of asset j , $j = 1, \dots, n$; Ω be an $n \times n$ variance-covariance matrix of total returns which is assumed to be positive definite and where each element $\omega_{jk} = \text{var}(V_j)$, $j = k$ and $\omega_{jk} = \text{cov}(V_j, V_k)$, $j \neq k$, $k = 1, \dots, n$. Given that there are m investors in the market, the homogeneous expectations (HE) assumption of the simple CAPM implies that all investors perceive V and Ω in the same manner. However, if an individual i is instead assumed to perceive V and Ω differently, i.e., there exist V_i and Ω_i such that they are not necessarily equal to V_k and Ω_k respectively, $k = 1, \dots, m$, $k \neq i$, then this means that the investors have non-homogeneous expectations.

Given NHE, the portfolio selection problem of investor i is to

$$\text{maximize } U_i(E_i, \phi_i) \quad (1)$$

$$\text{subject to: } W_i = P'X_i - d_i \quad (2)$$

where

$$E_i = V_i'X_i - \theta d_i \quad (3)$$

$$\phi_i = X_i'\Omega_i X_i \quad (4)$$

and W_i is the total marketable assets of investor i at the beginning of the period, X_i is an $n \times 1$ vector, each element of which represents the fraction of the total market value of asset

j held by investor i , d_i is the net debt of investor i at the beginning of the period and P is an $n \times 1$ vector, each element of which represents the total market value of asset j at the beginning of the period, and $\theta = 1 + R_F$, R_F being the one-period risk-free rate of return. A prime denotes transposition. Also define ρ_j and Q such that

$$\rho_j = (V_j/P_j) = ((P_j + P_j r_j)/P_j) = 1 + r_j \quad (5)$$

where r_j is the one-period random rate of return of asset j , Q is an $n \times n$ variance-covariance matrix of rates of return which is assumed to be positive definite and where each element σ_{jk} is such that $\sigma_{jk} = \text{var}(r_j)$, $j = k$ and $\sigma_{jk} = \text{cov}(r_j, r_k)$, $j \neq k$.

Let $S = (S_1, S_2, \dots, S_n)'$ be the $n \times 1$ vector of the number of outstanding shares of assets in the market, i.e., S_j is the total number of shares of asset j . Further, let $s_i = (s_{i1}, s_{i2}, \dots, s_{in})'$ be the $n \times 1$ vector representing the portfolio of risky assets of investor i in terms of the total number of shares of asset j held by investor i . The market is cleared if all outstanding shares of all assets are held by the m investors so that

$$\sum_{i=1}^m s_{ji} = S_j, \quad j = 1, \dots, n \quad \text{or} \quad \sum_{i=1}^m s_i = S. \quad (6)$$

By definition, X_{ji} is the fraction of the total market value of asset j held by investor i , that is,

$$X_{ji} = s_{ji} P_j / S_j P_j. \quad (7)$$

where P_j is the prevailing price per share of asset j , so that

$$\sum_i X_{ji} = \sum_i s_{ji} P_j / S_j P_j = \frac{P_j}{P_j S_j} \sum_i s_{ji} = \frac{P_j S_j}{P_j S_j} = 1. \quad (8)$$

Therefore, the market-clearing condition given by (6) can be written as

$$\sum_i X_i = G \quad (9)$$

where G is an $n \times 1$ vector with all elements equal to 1.

B. The Equilibrium Vector of Prices and Portfolio Under NHE and HE

Solving the individual Portfolio selection problem, the portfolio vector of investor i is

$$X_i = h_i \Omega_i^{-1} (V_i - \theta P), \quad i = 1, \dots, m, \quad (10)$$

where

$$h_i = - \frac{\partial U_i}{\partial E_i} / 2 \frac{\partial U_i}{\partial \phi_i}, \quad (11)$$

a measure of risk aversion. Slightly rearranging (10), introducing the market-clearing condition (9) and summing up over all investors result in

$$\theta \sum_i h_i \Omega_i^{-1} P = \sum_i h_i \Omega_i^{-1} V_i - \sum_i X_i$$

or

$$\theta P = \left(\sum_i h_i \Omega_i^{-1} \right)^{-1} \left(\sum_i h_i \Omega_i^{-1} V_i - G \right). \quad (12)$$

This gives the equilibrium vector of prices. The equilibrium vector of portfolio is arrived at by substituting (12) in (10), so that

$$X_i = h_i \Omega_i^{-1} \left(V_i - \left(\sum_i h_i \Omega_i^{-1} \right)^{-1} \left(\sum_i h_i \Omega_i^{-1} V_i - G \right) \right). \quad (13)$$

The traditional or simple CAPM is a special case of (12) and (13). More specifically, if there is HE, i.e., $V_i = V$ and $\Omega_i = \Omega$ for all i , then the equilibrium price vector is

$$\theta P = V - \frac{1}{h} \Omega G \quad (14)$$

where

$$h \equiv \sum_i h_i$$

For each asset j ,

$$\begin{aligned} V_j &= \theta P_j + \frac{1}{h} \sum_k \omega_{jk} \\ &= \theta P_j + \frac{1}{h} \text{cov}(V_j, V_M), \end{aligned} \quad (15)$$

where $V_M = \sum_j V_j$. But $\text{cov}(V_j, V_M) = P_j P_M \text{cov}(\rho_j, \rho_M)$, where $P_M = \sum_j P_j$, so that dividing (15) by P_j results in

$$\rho_j = \theta + \frac{1}{h} P_M \text{cov}(\rho_j, \rho_M) \quad (16)$$

Taking expectation and subtracting 1 from both sides give

$$E(r_j) = R_F + \frac{P_M}{h} \text{cov}(\rho_j, \rho_M). \quad (17)$$

Appendix 1 shows that

$$h = \frac{P_M \sigma_{r_M}^2}{E(r_M) - R_F} \quad (18)$$

where $E(r_M)$ is the expected market rate of return and $\sigma_{r_M}^2$ is the variance of r_M . Also, $\text{cov}(\rho_j, \rho_M) = \text{cov}(r_j, r_M)$ so that (17) becomes

$$E(r_j) = R_F + (E(r_M) - R_F) \beta_j \quad (19)$$

where

$$\beta_j = \frac{\text{cov}(r_j, r_M)}{\sigma_{r_M}^2}. \quad (20)$$

Equation (19) is the simple CAPM due to Sharpe, (1964) and Lintner (1965).

Again, if there is HE, the equilibrium vector of portfolio of investor i given by (13) becomes

$$X_i = h_i \Omega^{-1} \left(V - \left(V - \frac{1}{h} \Omega G \right) \right)$$

or

$$X_i = \frac{h_i}{h} G . \quad (21)$$

An investor i holds all assets in his portfolio in the same proportion and it is a function of his degree of aversion to risk relative to the sum of the degree of risk aversion of all ^{4/} investors.

The equilibrium solutions given by (12) and (13) are general results. To simplify, following Rebinovitch and Owen (1978), define V_i and Ω_i such that $V_i = V + e_i$ and $\Omega_i = \Omega + B_i$, where e_i is an $n \times 1$ vector and B_i is an $n \times n$ matrix representing component by component the bias of V_i and Ω_i from V and Ω respectively. More conveniently, let $\Omega_i = (I - A_i) \Omega$ where I is an identity matrix and $B_i = -A_i \Omega$. Further, assume that the matrix $(I - A_i)^{-1}$ exists for all i and can be approximated such

^{4/}

This is an important limitation of the CAPM which has been the object of criticism and investigations. See for example Levy (1978) and Green (1986).

that $(I - A_1)^{-1} \approx I + A_1$ and $A_1 A_j \approx 0$ for all $1 \neq j$,
 $A_1 e_j = 0$, $i, j = 1, \dots, m$. These assumptions imply that the
differences in perceptions are "small".^{5/}

Substituting for v_1 and w_1 in (12), using the assumption of
small differences in perceptions and simplifying yield an
approximate price vector under NHE, i.e.,

$$\theta P = V - \frac{1}{h} \Omega G + \sum_i \delta_i \left(e_i + \frac{1}{h} A_i \Omega C \right) \quad (22)$$

where $\delta_i = h_1/h$. If e_i and A_i are zero for all i , that is,
there is HE, or if there is NHE but the weighted sums of e_i and
 A_i over all i are zero with δ_i as weights, the solution reverts
back to the simple CAPM.

Similarly, the equilibrium vector of portfolio for investor
 i can be derived as

$$x_i = \delta_i \left(G + h \Omega^{-1} \bar{e}_i + \Omega^{-1} \bar{A}_i \Omega C \right) \quad (23)$$

where

$\bar{e}_i = e_i - \sum_j \delta_j e_j$ and $\bar{A}_i = A_i - \sum_j \delta_j A_j$. If there is HE, i.e., $e_i =$
and $A_i = 0$ for all i , the equilibrium vector of portfolio
reverts back to (19) which is that of the simple CAPM. Even if
there is NHE, as long as e and A are equal to zero, (19) still
holds.

^{5/}

For added details on this assumption, see Rabinovitch and
Owen (1978), p. 579.

C. Non-Linear Return-Risk Relationship and the Theta Risks

Rabinovitch and Owen (1978) did not provide an explicit return-risk relationship. An explicit return-risk relationship is given here. It will be seen that this relationship has two new and additional components of risks, the "systematic risks" to the beta risk, which are referred to as "theta risks I and II".

Equation (22) can be re-written so that

$$\theta P = V - \frac{1}{h} \Omega G + \frac{1}{h} \sum_i h_i e_i + \frac{1}{h^2} \sum_i h_i A_i \Omega G. \quad (24)$$

Appendix 1 shows that

$$h = \frac{\omega_{V_M}^2}{(E(V_M) - \theta P_M)} = \frac{P_M^2 \sigma_{\rho_M}^2}{P_M (E(\rho_M) - \theta)} \quad (25)$$

Substituting the value of h in (24) gives

$$\theta P = V - \frac{P_M (E(\rho_M) - \theta)}{P_M^2 \sigma_{\rho_M}^2} (\Omega G - \sum_i h_i e_i) + \frac{P_M^2 (E(\rho_M) - \theta)^2}{P_M^4 \sigma_{\rho_M}^4} (\sum_i h_i A_i \Omega G). \quad (26)$$

For each asset j ,

$$V_j = \theta P_j + (E(\rho_M) - \theta) \left(\frac{\sum_i \text{cov}(V_j, V_i)}{P_M \sigma_{\rho_M}^2} - \frac{\sum_i h_i e_{ji}}{P_M \sigma_{\rho_M}^2} \right) - (E(\rho_M) - \theta)^2 \left(\frac{\sum_i h_i A_{ji} \Omega G}{P_M^2 \sigma_{\rho_M}^4} \right). \quad (27)$$

The term $\sum_i \text{cov}(V_j, V_i) = \text{cov}(V_j, V_M) = P_j P_M \text{cov}(\rho_j, \rho_M)$.

call that $V_i = V + e_i$ so that $V_{ji} = V_j + e_{ji}$, $i = 1, \dots, m$;

$= 1, \dots, n$. For asset j ,

$$e_{ji} = V_{ji} - V_j \quad (28)$$

Dividing both sides of (28) by P_j yields

$$\begin{aligned} \frac{e_{ji}}{P_j} &= \frac{V_{ji}}{P_j} - \frac{V_j}{P_j} \\ &= \rho_{ji} - \rho_j \\ &= (1 + E(r_{ji})) - (1 + E(r_j)) \\ &= E(r_{ji}) - E(r_j). \end{aligned} \quad (29)$$

$E(r_{ji})$ is the expected forecast of the rate of return of asset j by investor i . Define $\mu_{ji} \equiv E(r_{ji})$ and $\mu_j \equiv E(r_j)$. Then

$$e_{ji} = P_j (\mu_{ji} - \mu_j) \quad (30)$$

and therefore the term $\sum_i h_i e_{ji}$ in (27) becomes

$$\sum_i h_i P_j (\mu_{ji} - \mu_j) = P_j \sum_i h_i (\mu_{ji} - \mu_j).$$

Substituting values in (27) gives

$$\begin{aligned} V_j &= \theta P_j + (E(\rho_M) - \theta) \left(\frac{P_j P_M \text{cov}(\rho_j, \rho_M)}{P_M \sigma_{\rho_M}^2} - \frac{P_j \sum_i h_i (\mu_{ji} - \mu_j)}{P_M \sigma_{\rho_M}^2} \right) \\ &\quad - (E(\rho_M) - \theta)^2 \left(\frac{\sum_i h_i A_{ji} \Omega G}{P_M^2 \sigma_{\rho_M}^4} \right). \end{aligned} \quad (31)$$

Dividing through by P_j and knowing that $\rho_j = 1 + \mu_j$, $\theta = 1 + R_F$, $\text{cov}(\rho_j, \rho_M) = \text{cov}(r_j, r_M)$ and $\sigma_{\rho_M}^2 = \sigma_{r_M}^2$, (31) becomes

$$\begin{aligned}
E(r_j) = R_F + (E(r_M) - R_F) & \left(\frac{\text{cov}(r_j, r_M)}{\sigma_{r_M}^2} - \frac{\sum_i h_i (\mu_{ji} - \mu_j)}{P_M \sigma_{r_M}^2} \right) \\
& - (E(r_M) - R_F)^2 \left(\frac{\sum_i h_i A_{ji} \Omega G}{P_j P_M^2 \sigma_{r_M}^4} \right) \quad (32)
\end{aligned}$$

Equation (32) provides an explicit non-linear return-risk relationship for the CAPM with NHE.

Now, define θ_{Ij} and θ_{IIj} so that

$$\theta_{Ij} \equiv \frac{\sum_i h_i (\mu_{ji} - \mu_j)}{P_M \sigma_{r_M}^2} \quad (33)$$

and

$$\theta_{IIj} \equiv \frac{\sum_i h_i A_{ji} \Omega G}{P_j P_M^2 \sigma_{r_M}^4} \quad (34)$$

Let $R_F = \alpha_0$ and $(E(r_M) - R_F) = \alpha_1$, the market risk premium.

Substituting values in (32) gives

$$E(r_j) = \alpha_0 + \alpha_1 (\beta_j - \theta_{Ij}) - \alpha_1^2 \theta_{IIj} \quad (35)$$

providing a simplification for (32).

One may interpret θ_{Ij} as the "systematic bias" of the systematic risk β_j that arises from the non-homogeneity in the forecast of the rate of return μ_j , and θ_{IIj} as the "systematic bias" that arises from non-homogeneity in the forecast of the covariances α_{jk} , $j, k = 1, \dots, n$. For lack of an appropriate name, θ_{Ij} and θ_{IIj} are referred to as "theta risks I and II" respectively.

D. Approximations for the Theta Risks and the Corresponding Alternative Return-Risk Relationships

Appendix 2 provides approximations for the theta risks I and II. More specifically,

$$\theta_{Ij} = \bar{\theta}_{Ij} = \frac{\epsilon_j}{\alpha_1} \quad (36)$$

and

$$\theta_{IIj} = \bar{\theta}_{IIj} = \frac{\bar{A}_{j..}}{\alpha_1} \quad (37)$$

where

$$\epsilon_j = \frac{\sum_{i=1}^m (\mu_{ji} - \mu_j)}{m}$$

and

$$\bar{A}_{j..} = \frac{\sum_{i=1}^m \left(\sum_{k=1}^n A_{jki} / n \right)}{m} \quad (38)$$

In brief, ϵ_j and \bar{A}_j represent some crude approximations for the average differences in perceptions by the m investors in the market from the first two moments respectively of the parameters of the probability distribution of the rate of return of asset j .^{6/}

Substituting (36) and (37) in (35) gives

$$E(r_j) = \alpha_0 + \alpha_1 \left(\beta_j - \frac{\epsilon_j}{\alpha_1} \right) - \alpha_1^2 \left(\frac{\bar{A}_j}{\alpha_1} \right) \quad (40)$$

If there is HE, then (40) becomes

$$E(r_j) = \alpha_0 + \alpha_1 \beta_j$$

which is the simple CAPM. Under varying assumptions on NHE, three alternative return-risk relationships can be derived from the approximations given by (40).

Suppose NHE only exists on V but not on Ω , i.e., $V_k \neq V_i$ for some k and i but $\Omega_k = \Omega$ for all i . Then, (40) becomes

$$E(r_j) = \alpha_0 + \alpha_1 \left(\beta_j - \frac{\epsilon_j}{\alpha_1} \right)$$

or

$$E(r_j) = (\alpha_0 - \epsilon_j) + \alpha_1 \beta_j \quad (41)$$

^{6/}

Alternatively, ϵ_j and \bar{A}_j could arise from the differences in perceptions on the probability distributions of assets rates of return by the "marginal-opinion investors" as exemplified by "insiders" who possess or believe that they possess information not available to the rest of the investors. See Rabinovitch and Owen (1978, pp. 581-585), Mayshar (1983, pp. 114-115), and Givoly and Palmon (1985, p. 85).

The resulting return-risk relationship is linear. The effect of theta risk I is only to shift the intercept by the amount ϵ_j . Even if NHE exists for both V and Ω but $\bar{A}_{j..} = 0$, (41) still holds. However, if (36) does not hold, α_1 does not cancel out and the effect of theta risk I is to introduce a bias on β_j .

Suppose NHE only exists on Ω but not on V , i.e., $V_i = V$ for all i but $\Omega_i \neq \Omega_k$ for some k and i . Then, (40) becomes

$$E(r_j) = \alpha_0 + \alpha_1 \beta_j - \alpha_1^2 \left(\frac{\bar{A}_{j..}}{\alpha_1} \right)$$

or

$$E(r_j) = \alpha_0 + \alpha_1 (\beta_j - \bar{A}_{j..}). \quad (42)$$

The resulting return-risk relationship is linear and the presence of theta risk II introduces a bias on the beta risk. Even if NHE exist for both V and Ω but $\epsilon_j \neq 0$, (42) still holds. However, if (37) does not hold, α_1 does not cancel out and the relationship is no longer linear.

Finally, suppose NHE exists for both V and Ω and that $\epsilon_j \neq 0$ and $\bar{A}_{j..} = 0$. The (40) becomes

$$E(r_j) = \alpha_0 + \alpha_1 \left(\beta_j - \frac{\epsilon_j}{\alpha_1} \right) - \alpha_1^2 \left(\frac{\bar{A}_{j..}}{\alpha_1} \right)$$

or

$$E(r_j) = (\alpha_0 - \epsilon_j) + \alpha_1 (\beta_j - \bar{A}_{j..}). \quad (43)$$

Under NHE in both V and Ω , the resulting return-risk relationship is linear but the intercept is shifted by ϵ_j and β_j is biased by the magnitude \bar{A}_j . However, if (36) and (37) do not hold, α_1 does not cancel out in (43) and the return-risk relationship is non-linear.

E. The Corresponding Equilibrium Vector of Portfolio

In (23), define $C \equiv \Omega^{-1}$ and where c_{jk} are the elements of C , $j, k = 1, \dots, n$. Using (30), the term $h\Omega^{-1}\bar{e}_i$ in (23), for each j becomes

$$h \sum_k c_{jk} (e_{ji} - \sum_i \delta_i e_{ji})$$

or

$$h \sum_k c_{jk} P_j ((\mu_{ji} - \mu_j) - \frac{1}{h} \sum_i h_i (\mu_{ji} - \mu_j)). \quad (44)$$

Using (38), (44) becomes approximately equal to zero, i.e.,

$$h \sum_k c_{jk} P_j (e_j - \frac{1}{h} \epsilon_j \sum_i h_i) \approx 0. \quad (45)$$

Now, consider the third term inside the parenthesis of (23), i.e., $C\bar{A}_i \Omega G$. Substituting the values of \bar{A}_i results in

$$C \bar{A}_i \Omega G - C \sum_i \delta_i \bar{A}_i \Omega G \quad (46)$$

Without considering C in (46), for each asset j , this becomes

$$\begin{aligned} \sum_k A_{jki} P_k P_M \text{cov}(r_k, r_M) - \frac{1}{h} (h_1 (\sum_k A_{jki} P_k P_M \text{cov}(r_k, r_M))) + \dots \\ + \frac{1}{h_m} (\sum_k A_{jki} P_k P_M \text{cov}(r_k, r_M)). \end{aligned} \quad (47)$$

By (A2.6) and (A2.7) it is assumed that $\bar{A}_{j.1}$ and $\bar{\sigma}_{.M}$ exist such that

$$\frac{\sum_k A_{jki}}{n} = \bar{A}_{j.1} \quad \text{and} \quad \frac{\sum_k \sigma_{kM}}{n} = \bar{\sigma}_{.M}$$

Substituting these values in (47) results in

$$P_M^2 \bar{A}_{j.1} \bar{\sigma}_{.M} - \frac{1}{h} P_M^2 \bar{A}_{j.1} \bar{\sigma}_{.M} \sum_i h_i = 0. \quad (48)$$

Thus, based on the assumptions which yield the crude approximations for theta risks I and II, the last two terms in the parenthesis of (23) are zero. This means that under the alternative approximate return-risk relationships given by (41), (42) and (43), the equilibrium vector of portfolio of investor i is given by (21) which is that of the simple CAPM.

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it might be worthwhile to suggest that the theta risks might be able to provide insights on studies on capital market segmentation as well as a possible alternative explanation of the occurrences of unexplained large fluctuations in stock prices.

III. INDIRECT TEST OF THE CAPM WITH NHE

A. Design and Test Procedure

Designing a precise empirical procedure to test (32) is not easy since estimating the theta risks may not be possible. This is due to two reasons. First, h_i which is a measure of the i th investor's risk aversion may not be measurable; second, μ_{ji} and A_{ji} which reflect the perceptions of investor i on μ_j and σ_{jk} respectively are not observable.

However, an approximate and two-part indirect set of tests might be provided for (32) by using the approximation given by (43) of which (41), (42) and (19) are special cases. More specifically, the following model can be estimated.

$$r_j^* = \alpha_0 + \alpha_1 \beta_j^* + e_j \quad (49)$$

where

$$r_j^* = r_j + \epsilon_j \quad (50)$$

$$\beta_j^* = \beta_j - \bar{A}_{j..} \quad (51)$$

and e_j is the regression error term with the usual properties assumed. The variables r_j^* and β_j^* are not measurable due to ϵ_j and $\bar{A}_{j..}$ which are not observable.

Substituting values in (49) and simplifying give

$$(r_j + \epsilon_j) = \alpha_0 + \alpha_1 (\beta_j - \bar{A}_{j..}) + e_j$$

or

$$r_j = \alpha_0 + \alpha_1 \beta_j + e_j^* \quad (52)$$

where

$$e_j^* = (e_j - \epsilon_j - \alpha_1 \bar{A}_{j..}). \quad (53)$$

Since r_j and β_j are the ones measurable, (52) can instead be estimated and it can be taken as a model with errors in variables, the errors being ϵ_j and $\bar{A}_{j..}$.

Except for ϵ_j^* , (52) is similar to (19). If ϵ_j and $\bar{A}_{j..}$ are in fact equal to zero or negligible in magnitude, i.e., the theta risks vanish, then the correct model is given by a testable model for (19) and test of (52) should yield significant linear return-risk relationship, and that the estimate of the intercept α_0 should be equal to R_F . On the other hand, if ϵ_j and $\bar{A}_{j..}$ are non-zero, then the errors will yield insignificant results and the hypothesized equality between α_0 and R_F may not be expected. This is Part I of the two-part indirect test of the CAPM with NHE given by (32).

Looking at (35) which is a simplification of (32), it is seen that if θ_{Ij} and θ_{IIj} approach zero, then (35) reverts back to the simple CAPM given by (19). The lesser the degree of NHE among investors on the parameters of the probability distribution of the rate of return of an asset, the smaller the values of the theta risks.

Given a security within a sectoral group, for example, mining, the differences in perceptions on the parameters of the probability distribution of this security among investors investing on this group could be relatively smaller compared to the differences in perceptions of the parameters of the

distribution of this particular security among all investors in the market. In effect, within a sectoral group, the theta risks could be relatively smaller compared to the theta risks for the entire market.

Since ϵ_j and $\bar{A}_{j..}$ are directly proportional to theta risks I and II respectively, relatively smaller theta risks imply that these "error" terms are also relatively small and the results of the tests for (52) using sectoral betas should show improvement over the test results using betas for the entire market. If the results of the tests in fact show such improvement, then this is an indirect confirmation of the existence of the theta risks. Test using sectoral groups of securities comprise Part II of the two-part indirect test of the CAPM with NHE.

B. Data

Most tests of the simple CAPM use indirect estimates of the betas using Sharpe's (1963) diagonal or market model. This model relates the rate of return of a security to a market index and the regression coefficient estimate provides the estimate for the beta risk of that security. Also, these tests assume an equal weight for each security in computing for the market portfolio rate of return r_m .^{7/}

^{7/}

Friend and Blume (1970), Black, Jensen and Scholes (1972) and Pama and MacBeth (1973). Brown and Barry (1984, p. 814) following Roll's (1977) conjecture conclude that "...we have found evidence that observed anomalies in excess returns are associated with misspecification in the market model used to estimate systematic risk."

Available Philippine data could overcome these two test limitations since the total number of securities is relatively small. Monthly data published by the Manila Stock Exchange are available for a total of 72 securities for the entire period from January 1976 to December 1979. If the time period is further extended to the past, the number of securities which can be included is reduced since some securities are newly-listed. Other securities are also delisted through time, further reducing the total number of securities. The total of 72 is therefore the maximum number of securities which can be included for the 48-month period. These securities are categorized into three sectoral groups: 18 Bank, Commercial and Industrial (BCI); 19 Mining; and 35 Small Board (mostly oil) issues. The average yield of the 49-day Treasury Bill is used to estimate R .

F

The off-diagonal terms of the 72×72 variance-covariance matrix computed cannot be accounted for entirely by random deviations from zero so that Sharpe's diagonal model is inadequate. Also, using the outstanding value of each security as basis for computing the weight of each security in the market portfolio shows that the equal weight per security assumption is inadmissible.

Based on (20), precise estimates of the betas were computed. The results of the calculations for the period 1976-1979 for all the 72 securities are shown in Table 1. Similar calculations

TABLE I

Calculation Results for the 72 Securities
for the Period 1976-1979

PERIOD 1976-1979		MARKET r_M :	0.1229	VARIANCE OF THE MARKET $\sigma_{r_M}^2$		0.7259
SEQ	SEC NR	WEIGHT	MEAN RETURN	VARIANCE	COV(\bar{R}_J, r_M)	BETA-J
1	44	0.0049	0.4621	7.8975	1.1763	1.6204
2	64	0.0019	0.1354	3.4681	0.6192	0.8529
3	71	0.0001	-0.0445	0.3284	-0.0720	-0.0992
4	1	0.0256	0.2329	0.8787	0.2256	0.3108
5	2	0.0233	0.2662	1.0295	-0.1158	-0.1595
6	8	0.0575	-0.0996	2.7013	1.0872	1.4977
7	7	0.0294	-0.0621	1.6608	0.2747	0.3784
8	8	0.0829	0.0898	2.8762	0.9866	1.3591
9	28	0.1423	0.0305	0.8703	0.5768	0.7945
10	27	0.0082	0.4921	1.8909	0.0990	0.1363
11	33	0.0013	0.0704	2.9305	0.3443	0.4743
12	34	0.0436	-0.1805	1.7363	0.7769	1.0702
13	35	0.0792	0.0622	1.6045	0.8517	1.1732
14	22	0.0324	0.4878	10.1840	0.5836	0.8040
15	43	0.0013	-0.3996	1.9412	0.5639	0.7768
16	40	0.0072	-0.3504	2.1983	0.8507	1.1719
17	45	0.0021	-0.1276	8.5314	0.9118	1.2560
18	49	0.0007	-0.2080	6.3441	1.0878	1.4985
19	31	0.0025	0.1098	1.0520	0.2788	0.3841
20	54	0.0022	-0.4872	4.1243	0.9445	1.3011
21	72	0.0094	0.1460	2.7127	0.6168	0.8497
22	73	0.0048	0.1108	10.9461	1.4074	1.9388
23	85	0.0007	-0.1770	4.8468	0.7246	0.9982
24	62	0.0043	0.1513	2.1449	0.4799	0.6612
25	69	0.0007	0.2700	3.5703	0.4508	0.6210
26	81	0.0015	-0.2284	6.5677	1.4572	2.0074
27	79	0.0019	0.0079	4.2631	0.8143	1.1218
28	5	0.0460	0.2189	0.7723	0.3803	0.5239
29	9	0.0756	0.4543	5.6961	0.7522	1.0362
30	11	0.0206	0.3274	1.2653	0.5579	0.7685
31	12	0.0032	-0.2150	1.0701	0.0277	0.0382
32	13	0.0144	0.1378	1.3201	0.2610	0.3595
33	14	0.0197	-0.1043	3.5443	0.8729	1.2024
34	15	0.0353	-0.1204	3.1173	1.0166	1.4005
35	16	0.0151	-0.1047	1.0810	0.3130	0.4311
36	18	0.0018	0.1232	0.2053	0.0887	0.1222
37	21	0.0169	0.1752	4.9767	0.7003	0.9647
38	29	0.0109	0.0695	0.6986	0.4214	0.5805

Table 1 (cont'd)

SEQ	SEC NR	WEIGHT	MEAN RETURN	VARIANCE	COV(\bar{R}_J .RM)	BETA-J
39	46	0.0003	-0.3151	4.7235	-0.3463	-0.4770
40	75	0.0001	-0.5397	6.4785	0.5283	0.7278
41	78	0.0247	0.6770	11.0981	1.7393	2.3960
42	85	0.0042	0.0900	3.2487	0.9474	1.3051
43	84	0.0029	0.0642	5.9394	0.9882	1.3613
44	3	0.0283	0.1915	0.3347	0.0471	0.0648
45	4	0.0041	0.1610	0.3234	0.1966	0.2708
46	28	0.0003	-0.1621	4.6335	0.1115	0.1538
47	32	0.0009	0.0773	0.8091	0.2850	0.3928
48	47	0.0023	-0.1325	3.5621	1.0686	1.4721
49	17	0.0017	0.1281	0.1392	0.0697	0.0960
50	68	0.0009	0.0051	2.1815	0.7142	0.9838
51	36	0.0256	0.3606	3.8089	1.3691	1.8859
52	37	0.0172	0.2450	3.7678	1.3830	1.9051
53	38	0.0009	-0.4604	8.4052	-0.1189	-0.1638
54	51	0.0004	0.0675	1.0351	0.2241	0.3087
55	70	0.0005	0.0386	16.5145	-0.0822	-0.1132
56	95	0.0005	-0.0000	4.5677	0.9327	1.2848
57	91	0.0014	-0.1450	9.0980	1.2788	1.7616
58	92	0.0005	-0.2708	4.5856	0.9974	1.3740
59	93	0.0005	-0.2339	9.7199	1.4656	2.0189
60	94	0.0003	-0.1338	2.6321	0.5592	0.7703
61	90	0.0016	-0.2140	4.9195	0.9517	1.3110
62	89	0.0023	0.1244	4.4605	0.8115	1.1179
63	39	0.0010	-0.3785	1.4236	0.2440	0.3361
64	59	0.0042	-0.1396	5.0084	1.3384	1.8437
65	58	0.0070	-0.0775	1.5486	0.6658	0.9171
66	41	0.0104	0.0071	3.4429	0.8854	1.2197
67	87	0.0012	-0.1689	6.3086	1.4521	2.0004
68	56	0.0118	0.3293	8.0484	1.1023	1.5184
69	52	0.0009	-0.2268	2.5053	0.6495	0.8947
70	61	0.0005	-0.1067	0.9525	0.2297	0.3164
71	65	0.0050	0.3574	6.0202	0.9140	1.2591
72	76	0.0039	0.2532	5.8979	0.8428	1.1610

were done for the annual periods. The same calculations were done for each of the three sub-groups of securities. The results of the calculations for BCI for 1976-1979 are shown in Table 2. Table 3 shows the market portfolio rates of return and variances for the entire market and for the three sectoral groups of securities for the annual as well as the 1976-1979 periods. The riskless rates of return are also shown for each period.

C. Test Results

Initially, (52) was tested using the data over the 48-month period for the entire market using all the 72 securities. The result of the regression analysis using ordinary least squares is insignificant.

The observed volatility of the market indicates the possibility of significant differences in market behavior over time such that the parameters of the probability distributions of the rates of return of securities may not be stationary over the 48-month period. The betas could also change over time.^{8/} To be able to take into account these possibilities, tests were made using annual data. The results are shown in Table 4.

The results of the tests using annual data show some improvement over the results of the test using the data for the

^{8/} For evidence which shows that the beta could change over time, see Jacob (1971), Blume (1975), Fabozzi and Francis (1978) Olson and Rosenberg (1982) and Bos and Newbald (1984).

48-month period. On the whole, however, the results are generally poor. For the year 1976, where there is a significant positive return-beta relationship, only 11 percent of the variations are explained by the explanatory variable beta and the intercept is at -0.2937 , significantly less than R_F . In fact for 1979, a perverse result is obtained giving a negative estimate for the coefficient of the beta.

These results could be interpreted to mean that the magnitudes of the errors in (52) are large which could also mean that the theta risks are relatively large thereby giving insignificant results. Due to the presence of the theta risks, Part I of the indirect test of the CAPM with NHE attempted to fit a linear model on a basically non-linear relationship. Thus, the generally insignificant test results may not be unexpected.^{9/}

^{9/} To preclude the possibility that the generally poor test results is due to some other factors, investigations were made on: 1) the possible non-stationarity of the probability distributions of securities rates of return; 2) the possible autocorrelation of monthly rates of return following the test procedure of Brown (1979); 3) tests by groups of securities using ranked betas to reduce the possible effects of measurement errors similar to the procedure used by Black, Jensen and Scholes (1972); 4) possible non-stationarity of the betas; and, 5) tests of the CAPM using the variance as risk surrogate following Levy's (1978) constraints on the number of securities in the investor's portfolio. All these did not provide adequate explanation for the generally poor test results. The probability distributions of securities rates of return were found to exhibit departures from normality and some degree of leptokurtosis and asymmetry. For further details, see Francisco (1983). These findings on leptokurtosis are in consonance with those of Mandelbrot (1963) and Fama (1965, 1976). For evidence on asymmetric distributions, see Fieletz and Smith (1972) and Leitch and Paulson (1975). The existence of leptokurtosis and asymmetry could be interpreted as possible indications of the presence of NHE among investors.

TABLE 2

Bank-Commercial-Industrial Securities

PERIOD 1976-1979		MARKET r_M : 0.2578		VARIANCE OF THE MARKET $\sigma_{r_M}^2$		0.6229
SEQ	SEC NR	WEIGHT	MEAN RETURN	VARIANCE	COV(RJ.RM)	BETA-J
1	1	0.0744	0.2329	0.8787	0.1207	0.1938
2	2	0.0677	0.2662	1.0295	-0.0584	-0.0937
3	7	0.0855	-0.0621	1.6608	0.6267	1.0060
4	27	0.0238	0.4921	1.8909	0.1010	0.1621
5	22	0.0941	0.4878	10.1840	1.0562	1.6956
6	31	0.0073	0.1098	1.0520	0.2024	0.3249
7	5	0.1338	0.2189	0.7723	0.3865	0.6204
8	9	0.2197	0.4543	5.6981	1.5802	2.5368
9	11	0.0600	0.3274	1.2653	0.4209	0.6757
10	12	0.0092	-0.2150	1.0701	0.1260	0.2023
11	13	0.0419	0.1378	1.3201	0.3744	0.6010
12	16	0.0440	-0.1047	1.0810	0.1508	0.2421
13	18	0.0053	0.1232	0.2053	0.1215	0.1951
14	29	0.0316	0.0695	0.6966	0.1930	0.3098
15	3	0.0822	0.1915	0.3347	0.0583	0.0937
16	4	0.0120	0.1610	0.3234	0.1036	0.1663
17	32	0.0025	0.0773	0.8091	0.1427	0.2291
18	17	0.0050	0.1281	0.1392	0.0389	0.0624

TABLE 3

**Riskless Rate and Portfolio Rates of Return and Variances^{a/}
for the Market and for Sectoral Groups of Securities
(Manila Stock Exchange, Philippines)**

Period	^{b/} Riskless	Market		BCI		Mining		Small Board	
	Rate	^{c/} 72 Securities		18 Securities		19 Securities		35 Securities	
	R_F	r_M	σ_M^2	r_M	σ_M^2	r_M	σ_M^2	r_M	σ_M^2
1976	0.1022	-0.0033	1.3641	0.1573	0.3636	-0.0980	1.6386	0.1518	8.8508
1977	0.1074	0.0434	0.3703	0.2717	0.0601	-0.1075	0.8945	0.1108	1.0750
1978	0.1040	0.4910	0.7244	0.6442	1.4656	0.3339	1.3747	0.7080	1.2202
1979	0.1194	-0.0565	0.4585	-0.0648	0.2398	-0.0704	1.0803	0.0269	0.7647
1976-79	0.1083	0.1229	0.7259	0.2578	0.6229	0.0141	1.1916	0.2685	3.2510

^{a/} The portfolio rates of return and variances are computed using value weights.

^{b/} The average yield of the 49-day Treasury Bill, Philippines.

^{c/} Notationally, σ_M^2 and $\sigma_{r_M}^2$ are equivalent.

TABLE 4

Results of Tests of the CAPM (OLS)
72 Securities
(Manila Stock Exchange)

$$(r_j = \alpha_0 + \alpha_1 \beta_j + e_j^*)$$

<u>Period Covered</u>	<u>Intercept</u>	<u>Beta Coefficient</u>	<u>R²</u>	<u>D.W. Statistic</u>
Jan.-Dec. 1976	-0.2937* (0.0780) ^b	0.1550* (0.0522)	0.1104	2.25
1977	-0.0418 (0.0555)	0.0575 (0.0396)	0.0289	2.07
1978	0.2600 (0.0809)	0.0628 (0.0717)	0.0107	2.18
1979	0.0226 (0.0693)	-0.2272* (0.0670)	0.1393	1.56
Jan. 1976 - Dec. 1979	-0.0045 (0.0515)	0.0246 (0.0464)	0.0039	1.92

a/

See equation (52) in the text. The results here comprise Part I of the 2-part indirect test of the CAPM with non-homogeneous expectations.

b/

Values in parenthesis are the standard errors.

*Significant at least at the .005 level.

Table 5 shows the results of Part II of the indirect test for the CAPM with NHE using sectoral groups of securities for the annual and the 1976-1979 periods for BCI, Mining and Small Board.

For BCI, the coefficient of the sectoral beta is significant for 1977 and the 1976-1979 periods and $\alpha_0 = R_F$ at .05 level. This means that the simple CAPM explains the data. For Mining, the coefficient of the sectoral beta is significant only for 1977 but $\alpha_0 < R_F$. This means that the simple CAPM cannot explain the data. Finally, the coefficient of the sectoral beta for Small Board is significant for 1977 and the 1976-1979 periods but $\alpha_0 < \frac{R_F}{10}$. Again, the simple CAPM is unable to explain the data.

Of the total 15 regression estimates made, only five showed significant relationship at the .05 level. Except for BCI in 1977, the explanatory power of the sectoral beta, in terms of the coefficient of determination for the significant portion of the test results, is generally low, with at most 25% of the variations in the dependent variable explained.

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These different results by sectoral groups might also be interpreted in the light of the segmentation hypothesis. For a study on capital market segmentation using the CAPM framework, see Errunza and Losq (1985).

TABLE 5

Results of Indirect Tests of the CAPM with NHE (OLS)

$$(r_j = \alpha_0 + \alpha_1 \beta_j + e_j^*)^a$$

Period Covered	Group	Intercept	Beta Coefficient	R ²	D.W. Statistic
Jan.-Dec. 1976 :	1 ^b	0.0666 (0.0629) ^c	0.0246 (0.0713)	0.0070	2.70
	2 ^b	-0.4286 (0.1712)	0.1962 (0.1498)	0.0869	1.43
	3 ^b	-0.2480 (0.1597)	0.2409 (0.2044)	0.0393	2.24
1977 :	1	0.1358* (0.0417)	0.0742* (0.0114)	0.7145	2.27
	2	-0.4068* (0.1577)	0.3102* (0.1504)	0.1912	1.58
	3	-0.2214* (0.0839)	0.1739* (0.0752)	0.1358	1.80
1978 :	1	0.3225 (0.1330)	0.1790 (0.1468)	0.0805	2.02
	2	0.1706 (0.2037)	0.0800 (0.1762)	0.0113	1.71
	3	0.2312 (0.0915)	0.1686 (0.0944)	0.0859	1.93
1979 :	1	0.1129 (0.1569)	-0.1627 (0.1591)	0.0579	2.42
	2	-0.0744 (0.2439)	-0.0464 (0.2584)	0.0018	2.12
	3	-0.1051 (0.1140)	-0.1448 (0.1357)	0.0324	1.87
Jan. 1976 - Dec. 1979 :	1	0.1056* (0.0521)	0.1319* (0.0630)	0.2052	2.42
	2	-0.2805 (0.1484)	0.2229 (0.1442)	0.1172	1.56
	3	-0.1786* (0.0638)	0.3113* (0.0917)	0.2530	2.08

a/

See equation (52) in the text. The results here comprise Part II of the 2-part indirect test of the CAPM with non-homogeneous expectations.

b/

Numbers 1, 2 and 3 refer to tests of the CAPM using 18 securities of Bank-Commercial-Industrial(BCI), 19 securities of Mining, and 35 of Small Board (mostly oil) respectively.

c/

Values in parenthesis are the standard errors.

*Significant at least at .05 level.

Figure 1 shows the graph of the significant results. ^{11/}

These results using sectoral groups of securities show significant improvement over the test results for the entire market. This improvement could be interpreted that the errors ϵ_j and \bar{A}_j embedded in e_j^* of (52) are relatively smaller which could also mean that the theta risks within a sectoral group of securities could be smaller than those for the entire market.

Two observations can be made on the foregoing results. First, the results for BCI are better than the results for Mining and Small Board. Second, there is a pattern of significant return-risk relationship. More specifically, the significant relationship is noted only for 1977 and for the 1976-1979 period for all the three groups of securities except for Small Board. The next section provides explanation for these two sets of observations.

^{11/}

The graph for Small Board for 1976-1979 is included although it is not significant at the .05 level. The t ratios of the beta coefficient and the intercept are 1.55 and -1.89 respectively. It is postulated below that the nature of the data for 1977 which yields significant results for all the three sectoral groups is the primary reason for the significant result for the 1976-1979 period since tests for this period include the 1977 data.

IV. EXPLAINING THE TEST RESULTS

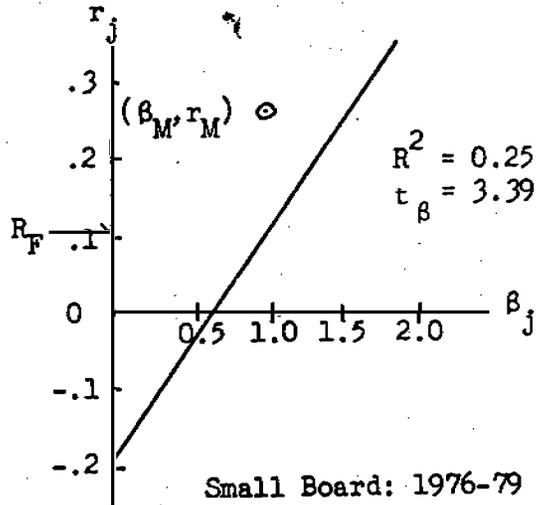
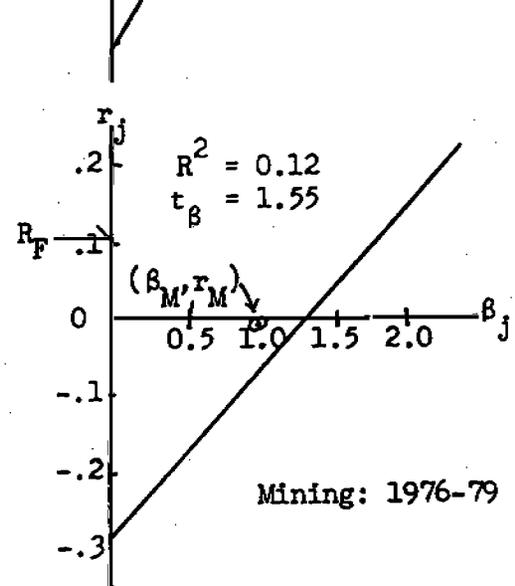
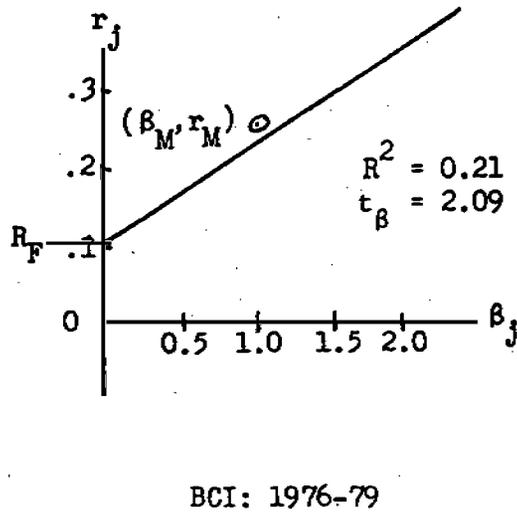
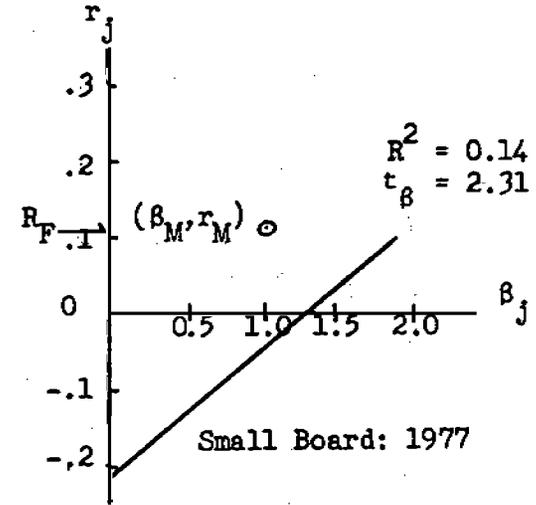
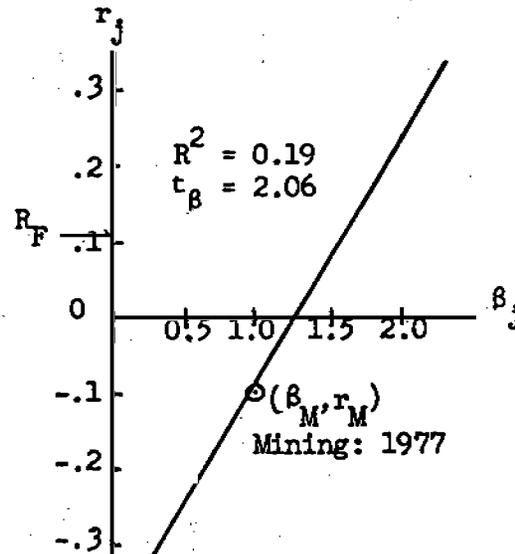
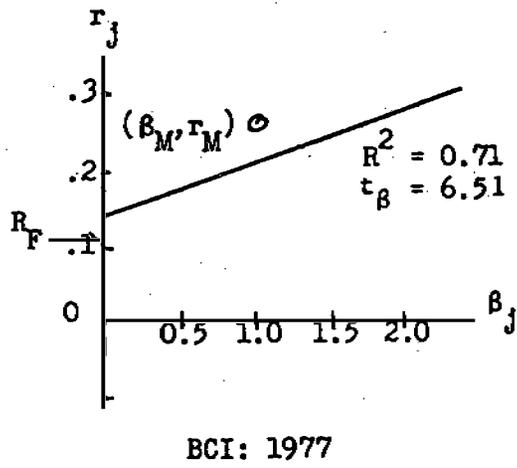
A. Sectoral Theta Risks

Given a security within a sectoral group, say Mining, the differences in perceptions on the parameters of the probability distribution of this security among investors investing on this subgroup could be relatively small compared to the differences in perceptions on the same parameters of the probability distribution of this particular security among all investors in the entire market. In effect, within a sectoral group, the theta risks could be relatively small compared to the theta risks for the entire market. Tests of (52) using sectoral groups of securities are therefore expected to yield better results. This is the underlying hypothesis of the two-part indirect test of the CAPM with NHE. The overall improvement of the tests results in Part II over those in Part I appears to confirm this.

Moreover, the relatively less risk-averse investors are expected to invest more on the more risky assets such as Mining and Small Board (mostly oil). On the other hand, the relatively more risk-averse investors are expected to invest more on the less risky assets such as BCI. For those investing on the more risky assets, the differences in opinions among them are expected

FIGURE I

Plots of Significant Test Results for CAPM with Non-Homogeneity of Expectations
Using Sectoral Groups of Securities



to be more diverse. Correspondingly, the theta risks are expected to be larger.^{12/} For those investing on the less risky assets, the differences in opinions among them are expected to be less diverse. The theta risks are expected to be smaller. Also, it is known that information on the BCI, which consists mostly of the "blue chips", are generally more reliable and readily available than information about Mining and Small Board. Thus, a more significant test result is expected for BCI than for Mining and Small Board.^{13/} This provides an explanation for the better test results observed for BCI compared to that of Mining and Small Board.

12/

In other words, the departure from the simple CAPM is larger if the overall level of risk is larger. This is in line with the conclusion of Friend and Blume (1970 p. 574) that "...our analysis raises some questions about the usefulness of the theory in its present form to explain market behavior. The Sharpe, Treynor and Jensen one-parameter measure of portfolio performance based on this theory seem to yield seriously biased estimate of performance, with the magnitudes of the bias related to portfolio risk.

13/

Equality of risk premia among BCI, Mining and Small Board for 1977 and 1976-1979 is rejected at least at .01 level of significance using two-tailed t test, with BCI having the lowest risk premium. While the estimates may have been biased by the presence of the theta risks which is an added form of risks, it may not be unreasonable to expect that the theta risks directly affect the risk premium. The risk premium could therefore change over time as investors' expectation change. See for example Corhay, Hawawini and Michael (1987) for seasonal variations of risk premium.

B. Theta Risks and the General Market Conditions

The general market conditions could affect the overall perception of investors on the prospects of their stock investments. Correspondingly, the values of the theta risks could change.

For an assumed value of the market risk premium α_1 , changes in the values of the theta risks correspondingly affect the values of ϵ_j and $\bar{A}_{j..}$. In (33), for any security j , the denominator is fixed so that θ_{Ij} depends only on the value of h_i and $(\mu_{ji} - \mu_j)$. Suppose that h_i , $i = 1, \dots, m$ is fixed at least for the period under investigation.^{14/} Then the magnitude of theta risk I is entirely determined by μ_{ji} which is the one-period expected rate of return of security j as perceived by individual i . Since this rate of return is a function of the ending price of the security, the greater the differences in expectations among investors on the ending price of the security, the greater the magnitude of $(\mu_{ji} - \mu_j)$, $i = 1, \dots, m$ and $\sum_i (\mu_{ji} - \mu_j)$ may not be zero.

Specifically, when the market is bullish or active, there are relatively more investors in the market who may tend to be

^{14/}

This assumption may not be unreasonable especially if one considers tests over a one-year period, noting that h is a measure of risk aversion.

optimistic and hence may accept relatively higher risk per unit of portfolio return.^{15/} This implies that these investors choose a security or subgroup of securities which have relatively higher risks as measured by their betas. Since $(\mu_{ji} - \mu_j)$ is the difference in perception of individual i from the true rate of return of security j , an optimistic investor may tend to overestimate, μ_j such that $(\mu_{ji} - \mu_j) > 0$. The preponderance of investors with $(\mu_{ji} - \mu_j) > 0$ means that $\sum_i (\mu_{ji} - \mu_j) > 0$. Since $h_i > 0$ for all i , $\theta_{Ij} > 0$ by (33) and (36) $\epsilon_j > 0$ given that $\alpha_i > 0$.

On the other hand, during periods when the market is bearish or dull, i.e., the market is relatively inactive as indicated by a relatively subdued trading, the less risk-averse individual may minimize investment or entirely pull out of the market, leaving the relatively more risk-averse investors. These investors tend to be more cautious and conservative in their forecasts of the ending price and hence their estimate of μ_j . Thus, μ_{ji} could be close to or lesser than μ_j such that $\sum_i (\mu_{ji} - \mu_j) \leq 0$.

^{15/}

There are some indications which show that investors may shift investments from one group of stocks to another or may enter and pull out of the market. For the period under study, percentage changes in the total monthly values of shares traded ranged from 372 to 984% for the entire market. By sector, the corresponding percentage changes are 282 to 625% for BCI, 199 to 1,209% for Mining and 387 to 7,074% for Small Board.

Consequently, $\theta_{Ij} < 0$ and by (36) $\epsilon_j \leq 0$.

In (34), if h_i , $i = 1, \dots, m$ is assumed fixed, at least over the period under investigation, then the magnitude of θ_{IIj} depends only on A_{ji} and P_j since Ω , G , P_M^2 and $\sigma_{r_M}^4$ are the same or common to all securities. Since Ω is the variance-covariance matrix of return, the elements of Ω are functions of the rates of return of securities which are in turn determined by the ending prices of securities. Thus, the differences in perceptions on Ω reflected on A_{ji} are due to the differences in expectations on the ending price of security j embedded in P_j . In effect, theta risk II is large when differences in perceptions among investors are large; it is small when these differences are small. Similarly as in the case of theta risk I, theta risk II is expected to be relatively large when the market is bullish or active; it is expected to be relatively small when the market is bearish or dull. A non-zero value of the theta risk II implies that $\bar{A}_{j..} \neq 0$.

The year 1977 is observed to be a relatively dull market compared to the other periods. Figure 2 shows that the monthly total values of shares traded are relatively much lower than those of the other years for each of the three sectoral groups of securities. Also Figure 3 shows that the sectoral ranges of price indices given by the difference between the monthly high(H) and low (L) price indices are relatively smaller in 1977

FIGURE 2

Monthly Total Values of Shares Traded
(Million Pesos (P))

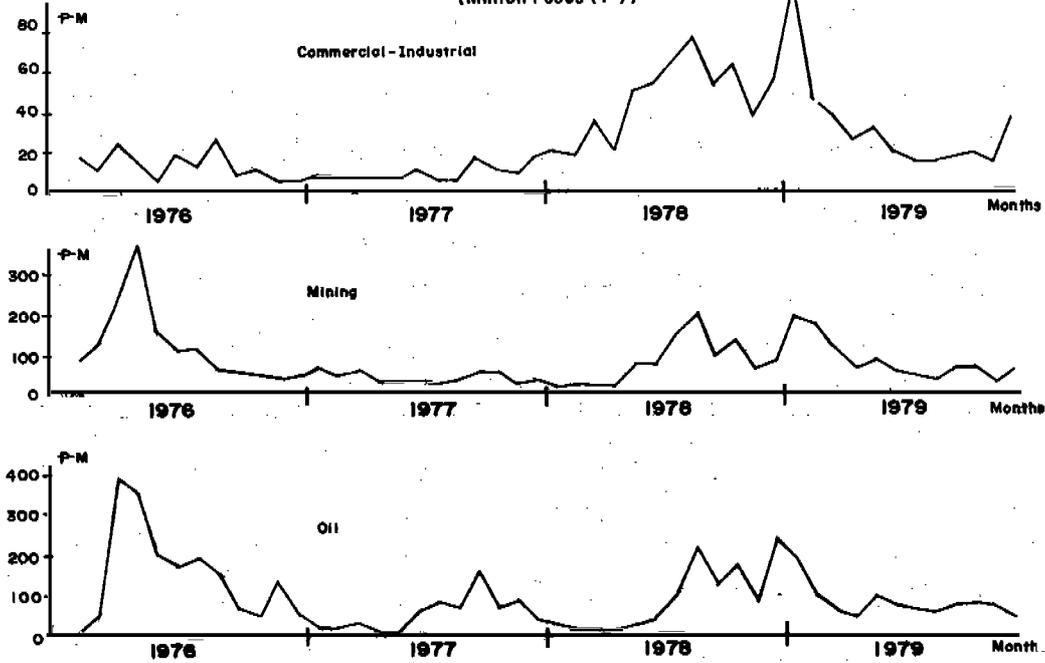
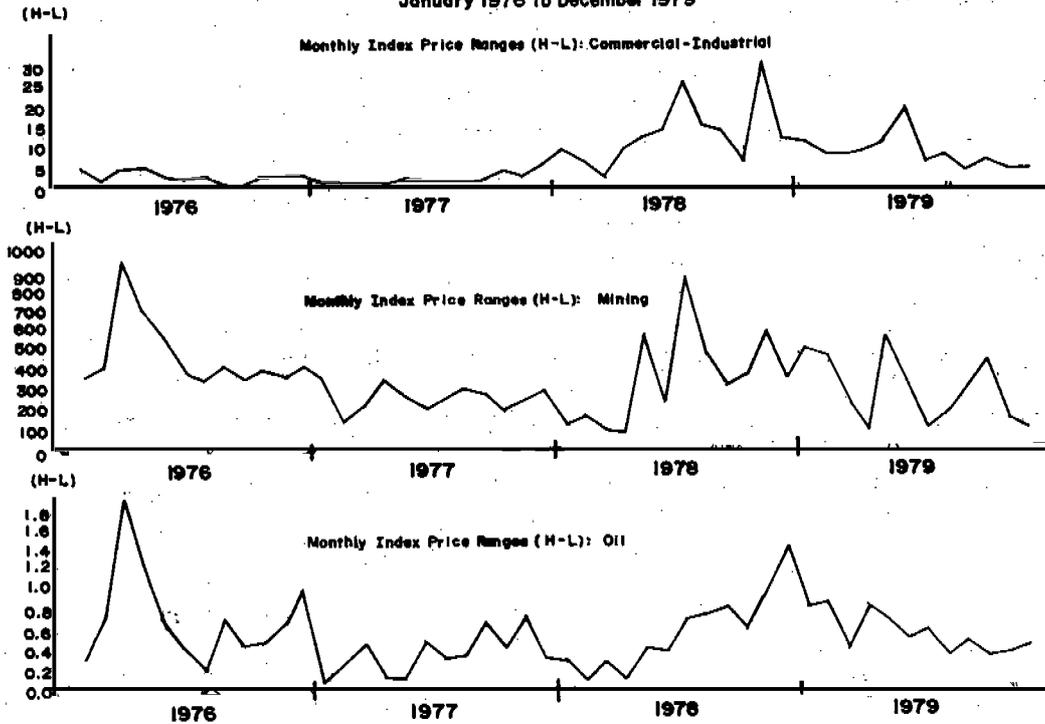


Table 3'

Monthly Ranges of Price Indices of Sectoral Groups of Securities
January 1976 to December 1979



compared to the other periods.^{16/} These could be interpreted to mean that the theta risks for 1977 are generally smaller than the

theta risks for the other periods.^{17/} If the theta risks are generally small, ϵ_j and $\bar{A}_{j..}$ are correspondingly small such that tests based on (52) could yield relatively more significant results.

This provides an explanation of the pattern of significant results for 1977 for each of the three sectoral groups of securities. The significant relationship observed for the 1976-1979 period could be attributed to the effects of the 1977 data which form part of the data used for the tests for the 1976-1979 period. For the other periods, the theta risks could be relatively large so that (52) is an entirely inadequate regression model. Hence, the generally insignificant results for these periods.

^{16/}

F-statistics comparing the variances of price ranges (high-low) between periods for each sectoral group of securities show that in general the ranges in prices in 1977 are significantly smaller than those for the other years.

^{17/}

One possible implication of this is that the theta risks could influence stock price movements. Thus, the theta risks might provide an alternative explanations for unexplained large stock price fluctuations. For more recent development in this area of study, see Poterba and Summer (1986).

The significant test results can be classified according to the three alternative approximate linear return-risk relationships given by (41), (42) and (43). These are summarized in Table 6 below. The graphs for these are previously shown in Figure 1.

TABLE 6

Model Classification for the Indirect and Significant Test
Results for the CAPM with Non-Homogeneous Expectations (NHE)

Year/Period	BCI	Mining	Small Board
1977	(41) or (42)	(41) or (43)	(41) or (43)
1976-1977	(41) or (42)	(41) or (43)	(41) or (43)

C. An Alternative Explanation of U.S. Results on the CAPM

Equation (41), (42) and (43) might provide an alternative explanation to the test results of the CAPM in the U.S. The works of Friend and Blume (1970), Black, Jensen and Scholes (1972), Blume and Friend (1973), Fama and MacBeth (1973) and others do not support the CAPM hypothesis. One result which has been subjected to extensive investigation is that high beta securities have intercept less than R_F and low beta securities have intercept greater than R_F (Black, Jensen and Scholes, 1972).

In the U.S., it may not be unreasonable to suppose that NHE in V exists^{18/} but with negligible NHE in Ω . This means that θ_{Ij} is non-zero but θ_{IIj} vanishes. A higher value of h_i implies that individual i may accept relatively higher risk per unit of portfolio return and therefore a security j chosen has relatively higher risk as measured by its beta. A relatively less risk-averse investor believes that $(\mu_{ji} - \mu_j) > 0$, otherwise he may not invest on security j . In effect $\theta_{Ij} > 0$ and by (36), $\epsilon_j > 0$. The intercept in (41) is $(\alpha_0 - \epsilon_j)$ and thus, there is a downward bias on the intercept for high beta securities.

On the other hand, a lower value of h_i implies that an individual i may accept relatively lesser risk per unit of portfolio return, i.e., a more risk-averse investor. Therefore, a security j chosen could have a relatively lower risk as measured by its beta. In this case, the perception of investor i for the rate of return of security j could be conservative or an under-estimate such $(\mu_{ji} - \mu_j) \leq 0$. In effect, $\theta_{Ij} \leq 0$ and by (36) $\epsilon_j \leq 0$. The intercept in (41) is

^{18/}

The presence of insiders confirms this. On insiders, see Jaffe (1974), Finnerty (1976) and Givoly and Palmon (1985).

($\alpha_0 - \epsilon_j$) giving an upward bias on low beta securities.^{19/} If NHE in Ω also exists and do not vanish, then (43) holds and there is a bias on both the intercept and beta. This could perhaps explain the generally poor explanatory power of the beta which made investigators conclude that the CAPM of Sharpe (1964) and Lintner (1965) is not consistent with the data.^{20/}

^{19/}

Since the h s for low beta securities are relatively smaller than the h s for high beta securities, it is not unreasonable to expect a preponderance of downward biases on the intercept, assuming that the number of investors in the low beta and the high beta securities are approximately equal. This is observed for the results shown in Figure 1. Otherwise, if there are relatively more risk-averse investors, then there could be an upward bias on the intercept as in the case of BCI for 1977.

^{20/}

The alternative explanation here for the empirical anomalies of the CAPM might be more appealing in the light of more recent doubts raised on conclusions drawn from zero-beta based tests for the CAPM (Best and Grauer, 1986, p. 98).

V. CONCLUDING REMARKS

When investors' diverse expectations on the probability distributions of assets rates of return are considered, the resulting equilibrium return-risk relationship is non-linear. This is due to the existence of a new and additional form of risk called theta risks I and II. These risks are the systematic biases to the beta arising from non-homogeneity of expectations among investors on the mean and variance (covariance) respectively of the rate of return of an asset.

Under NHE, the beta is not a complete measure of risk of an asset. If the theta risks vanish, the resulting return-risk relationship reverts to the simple CAPM of Sharpe (1964) and Lintner (1965).

Since NHE is a more appropriate characterization of the market than HE, the apparent anomalies in the existing empirical evidence on the CAPM could perhaps be explained by the attempt to fit a linear model on a fundamentally non-linear return-risk relationship.

There are methodological and data limitations on the indirect test of the CAPM with NHE undertaken here. The theta risks are basically unobservable and an errors-in-variables model is used. Also, tests using data from a larger and more mature capital market, such as that of the U.S., are necessary to further verify the theoretical predictions of the model. Also,

it might be worthwhile to suggest that the theta risks might be able to provide insights on studies on capital market segmentation as well as a possible alternative explanation of the occurrences of unexplained large fluctuations in stock prices.

APPENDIX I

To show that:

$$h \equiv \sum_{i=1}^m h_i = \frac{P_M \sigma_{r_M}^2}{E(r_M) - R_F}$$

Proof:

The problem given by (1) and (2) is a constrained maximization problem with the Lagrangian equation given by

$$L = U_i (E_i, \phi_i) + \lambda_i (W_i - P'X_i + d_i), \quad i = 1, \dots, m. \quad (A1.1)$$

Differentiating with respect to d_i and X_i , equating to zero and substituting values yield

$$\frac{\partial U_i}{\partial E_i} V + \frac{\partial U_i}{\partial \phi_i} 2 \Omega X_i - \theta \frac{\partial U_i}{\partial E_i} P = 0. \quad (A1.2)$$

Define

$$f_i = \partial U_i / \partial E_i \quad (A1.3)$$

$$g_i = \partial U_i / \partial \phi_i \quad (A1.4)$$

$$h_i = -f_i / 2g_i. \quad (A1.5)$$

By assumption, $f_i > 0$, $g_i < 0$ such that $h_i > 0$ for all i .

Dividing A(1.2) through by $\partial U_i / \partial E_i$ and substituting the defined variables yield

$$V + \frac{g_i}{f_i} 2 \Omega X_i - \theta P = 0. \quad (A1.6)$$

or

$$\frac{1}{h_i} \Omega X_i = V - \theta P. \quad (A1.7)$$

For each security j , (A1.7) means that

$$h_i = \frac{\sum_k \omega_{jk} X_{ki}}{V_j - \theta P_j}, \quad j = 1, \dots, n. \quad (\text{A1.8})$$

The condition for optimality of individual i , $i = 1, \dots, m$ is that his marginal rate of substitution between ϕ_i and E_i are equal for all securities.* That is,

$$\left(- \frac{\partial \phi_i}{\partial E_i} \right)_1 = \dots = \left(- \frac{\partial \phi_i}{\partial E_i} \right)_j = \dots = \left(- \frac{\partial \phi_i}{\partial E_i} \right)_n = h_i \quad (\text{A1.9})$$

Thus, summing up the numerator and denominator over all j in (A1.8) will not change the value of h_i , i.e.,

$$h_i = \frac{\sum_j \sum_k \omega_{jk} X_{ki}}{\sum_j (V_j - \theta P_j)}. \quad (\text{A1.10})$$

Since by definition, $h \equiv \sum_i h_i$,

$$h = \sum_i \left(\frac{\sum_j \sum_k \omega_{jk} X_{ki}}{V_M - \theta P_M} \right)$$

or

$$h = \frac{\sum_j \sum_k \omega_{jk} \sum_i X_{ki}}{V_M - \theta P_M} = \frac{\sum_j \sum_k \omega_{jk} X_k}{V_M - \theta P_M}. \quad (\text{A1.11})$$

By (9), $X_k = 1$ for all k , $k = 1, \dots, n$. By definition

$$\sum_j \sum_k \omega_{jk} = \sum_j \text{cov}(V_j, V_M) = \omega^2 V_M.$$

*

On this point, see for instance, Mayers (1972, p. 227).

Thus,

$$h = \frac{\omega_{V_M}^2}{V_M - \theta P_M} = \frac{P_M^2 \sigma_{\rho_M}^2}{P_M (\rho_M - \theta)}$$

or

$$h = \frac{P_M \sigma_{r_M}^2}{E(r_M) - R_F} .$$

Approximations for Theta Risks I and II

A. Theta Risk I

By (33),

$$\theta_{Ij} = \frac{\sum_i h_i (\mu_{ji} - \mu_j)}{P_M \sigma_{r_M}^2}, \quad i = 1, \dots, m; j = 1, \dots, n.$$

Estimating h_i and $(\mu_{ji} - \mu_j)$ may not be possible since h_i and μ_{ji} are not observable. To have an idea of the magnitude of θ_{Ij} , suppose

$$\frac{\sum_i (\mu_{ji} - \mu_j)}{m} = \epsilon_j \quad (A2.1)$$

where ϵ_j is defined as the average deviation of the forecasts from μ_j by all investors in the market.* Then a crude approximation for θ_{Ij} is

$$\bar{\theta}_{Ij} = \frac{\sum_i h_i \epsilon_j}{P_M \sigma_{r_M}^2} = \frac{\epsilon_j \sum_i h_i}{P_M \sigma_{r_M}^2} = \frac{\epsilon_j h}{P_M \sigma_{r_M}^2}. \quad (A2.2)$$

From Appendix 1,

$$h = \frac{P_M \sigma_{r_M}^2}{E(r_M) - R_F}$$

* In effect, (A2.1) enables us to substitute ϵ_j for $(\mu_{ji} - \mu_j)$ as a very crude approximation.

so that

$$\bar{\theta}_{Ij} = \frac{\epsilon_j P_M \sigma_{r_M}^2}{(E(r_M) - R_F) P_M \sigma_{r_M}^2} = \frac{\epsilon_j}{E(r_M) - R_F} = \frac{\epsilon_j}{\alpha_1}$$

B. Theta Risk II

By (34),

$$\theta_{IIj} = \frac{\sum_i h_i A_{ji} \Omega G}{P_j P_M^2 \sigma_{r_M}^4}, \quad i = 1, \dots, m; j = 1, \dots, n.$$

It is readily seen that

$$\begin{aligned} \Omega G &= (\text{cov}(V_1, V_M) \text{cov}(V_2, V_M) \dots \text{cov}(V_n, V_M))' \\ &= (P_1 P_M \text{cov}(r_1, r_M) P_2 P_M \text{cov}(r_2, r_M) \dots P_n P_M \text{cov}(r_n, r_M))'. \end{aligned} \quad (\text{A2.4})$$

Thus,

$$A_{ji} \Omega G = \sum_k^n A_{jki} P_k P_M \sigma_{kM} \quad (\text{A2.5})$$

Again, to have an idea of the approximate magnitude of θ_{IIj} ,

suppose

$$\frac{\sum_k A_{jki}}{n} = \bar{A}_{j.i} \quad (\text{A2.6})$$

and let

$$\frac{\sum_i \sigma_{kM}}{n} = \bar{\sigma}_{.M} \quad (\text{A2.7})$$

(A2.6) means that $\bar{A}_{j.i}$ is the average value of the j th row of $n \times n$ matrix A_i and (A2.7) means that $\bar{\sigma}_{.M}$ is the average covariance of rate of return of a security with the market rate of return.

Then

$$A_{ji} \Omega G = P_M \bar{A}_{j.i} \bar{\sigma}_{.M} \sum_k P_k \quad (A2.8)$$

But

$$\sum_k P_k = P_M$$

so that

$$\theta_{IIj} = \frac{\sum_i h_i P_M^2 \bar{A}_{j.i} \bar{\sigma}_{.M}}{P_j P_M^2 \sigma_{r_M}^4} = \frac{\bar{\sigma}_{.M} \sum_i h_i \bar{A}_{j.i}}{P_j \sigma_{r_M}^4} \quad (A2.9)$$

Further, suppose there exists a value $\bar{A}_{j..}$ such that

$$\begin{aligned} \frac{\sum_i A_{j.i}}{m} = \bar{A}_{j..} &= \frac{\sum_k A_{jkl} / n + \dots + \sum_k A_{jkm} / n}{m} \\ &= \frac{\sum_i \sum_k A_{jki}}{mn} \end{aligned} \quad (A2.10)$$

Then (A2.9) becomes

$$\theta_{IIj} = \frac{\bar{\sigma}_{.M} A_{j..} \sum_i h_i}{P_j \sigma_{r_M}^4} = \frac{\bar{\sigma}_{.M} \bar{A}_{j..} h}{P_j \sigma_{r_M}^4} \quad (A2.11)$$

stituting the value of h yields

$$\begin{aligned} \theta_{IIj} &= \frac{\bar{\sigma}_{.M} A_{j..} P_M \sigma_{r_M}^2}{P_j \sigma_{r_M}^4 (E(r_M) - R_F)} \\ &= \left(\frac{\bar{A}_{j..}}{E(r_M) - R_F} \right) \left(\frac{\bar{\sigma}_{.M}}{\sigma_{r_M}^2} \right) \left(\frac{P_M}{P_j} \right) \end{aligned} \quad (A2.12)$$

By definition, $\bar{\sigma}_{.M} = \sum_k \sigma_{kM} / n$. But $\sum_k \sigma_{kM} = \sigma_{r_M}^2$ so that $\bar{\sigma}_{.M} = \sigma_{r_M}^2 / n$.

Thus,

$$\frac{\bar{\sigma}_{.M}}{\sigma_{r_M}^2} = \frac{\sigma_{r_M}^2}{n \sigma_{r_M}^2} = \frac{1}{n} \quad (A2.13)$$

For the n securities, let \bar{P}_j be the average total market value of security j at the beginning of the period. Since $P_M = \sum_j P_j$,

$$\bar{P}_j = \sum_i P_j / n = P_M / n. \quad (A2.14)$$

Thus, a very crude approximation for θ_{IIj} is given by

$$\theta_{IIj} = \bar{\theta}_{IIj} = \left(\frac{\bar{A}_{j..}}{E(r_M) - R_F} \right) \left(\frac{1}{n} \right) n = \frac{\bar{A}_{j..}}{\alpha_1}. \quad (A2.15)$$

It is noted that for any security j and for given values of $A_{j..}$ and α_1 , (A2.15) means that

$$\text{if } P_j > \bar{P}_j \Rightarrow \theta_{IIj} < \bar{\theta}_{IIj} \quad (A2.16)$$

and

$$\text{if } P_j < \bar{P}_j \Rightarrow \theta_{IIj} > \bar{\theta}_{IIj}. \quad (A2.17)$$

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