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**RECENT DEVELOPMENTS  
IN ECONOMETRIC METHODS:**  
An application to the demand for money in  
Kenya

CHRISTOPHER S. ADAM



AFRICAN ECONOMIC RESEARCH CONSORTIUM

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for money in Kenya**

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**Recent developments in econometric  
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for money in Kenya**

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University of Oxford*



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## Contents

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Introduction	1
I The methodology of econometrics	4
Valid conditioning and exogeneity	5
Data admissibility and dynamic specification	8
Dynamic specification and cointegration	9
Cointegration and error correction	12
Testing for cointegration	15
The simplification process	15
Encompassing	16
Conclusions	17
II The demand for narrow money in Kenya	18
Introduction	18
Modelling the demand for narrow money	20
The time-series characteristics of the data	21
Testing for cointegration	25
Developing a parsimonious error correction model	30
Diagnostic testing and stability analysis	34
Conclusions	42
Annex I	43
Diagnostic Tests	43
The LM autocorrelation test: AR(m)	43
The LM ARCH test: ARCH(m)	43
Jarque-Bera normality test	44
Heteroscedasticity tests	44
RESET test	45
Omitted variables tests	45
Encompassing	45
Annex II	47
The recursive least squares estimator	47
Updating formula - the RLS estimator	47
References	49

## List of tables

---

1	Modelling $y_t$ by OLS	12
2	a Dickey Fuller test statistics	23
	b Sargan-Bhargava test statistics	23
3	Unit root tests on quarterly data	25
4	5% critical values for cointegration tests	26
5	Cointegrating regression and test statistics	28
6	Tests on the significance of each lag	29

## List of figures

---

1 a	Stationary series	10
b	Non-stationary series	10
2 a	Real gross national income (log)	19
b	Real narrow money stock (log)	19
3 a	Quarterly interest rates	20
b	Ksh/US\$ exchange rate	20
4	Granger-Engle error correction	29
5	Equation 2.21: actual vs fitted	36
6	Forecast stability 1985-1989	36
7	RLS estimates ecm 1	39
8	RLS estimates dp	41
9	Recursive residuals	41
10	One-step Chow tests	42

## Introduction

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The dramatic increase in cheap computer technology over the past decade has revolutionized the practice of econometrics and in doing so has prompted a major re-evaluation of its underlying methodology. Unlike previously where a relatively minor aspect of the econometric process, that of deriving methods for estimating the parameters of models, dominated the practice and particularly the teaching of econometrics, such estimation can now be achieved with comparative ease. Coefficient values which once took hours or even days to calibrate can now be estimated in milliseconds, heralding a new revolution in econometrics which is rapidly relegating "most of estimation theory to footnotes about numerical approximation and is re-focusing attention on all of the issues surrounding methodology, inference, model formulation and equation selection."<sup>2</sup> The current debate on econometric methodology dates from the mid-1970s, but in recent years it has acquired an intellectual maturity from which is emerging a coherent and practical methodology. To a large degree the debate has been conducted within the domain of time-series econometrics, although similar developments have also occurred in the realms of cross-sectional and panel-data econometrics: the fundamental methodological issues are however identical across all the branches of the discipline.

Unlike much of the broader debate on the philosophy of science, the debate on econometric methodology is far from being simply a matter of epistemological discourse, and it is the immediate application of such techniques to the increasingly more demanding economic policy issues environment which attests to its relevance. This is particularly so in Africa, where the need for robust economic analyses in informing the public policy debate is perhaps greater now than at any time in recent history.

The purpose of this paper is therefore twofold. The first objective is to survey the main developments in econometric thinking and to outline the key

<sup>1</sup> This paper draws on lecture notes prepared for the AERC Technical Workshop in Econometric Techniques held in Nairobi in September 1991, but also reflects a number of issues and concerns developed in that workshop. I am grateful to the participants for discussion and comment on these issues.

<sup>2</sup> Hendry (1980) p. 399.

concepts of the what has been called the LSE/Oxford methodology.<sup>3</sup> The second, and more important aspect of the paper considers the practical application of this methodological approach to the particular environment encountered in Africa. An oft-heard criticism of the use of "sophisticated" econometric techniques in Africa queries the use of such techniques when the data is full of errors, there are missing observations, and the samples are invariably short. While such problems are, unfortunately, endemic to much of African macroeconomic data, the response must be that if we seek to quantify economic relationships in the data then it is important that we do so by adopting an approach and employing techniques that allow us to make the most valid inferences possible from the data at our disposal. Too frequently, poor econometric techniques result in inappropriate inferences being drawn from poor data. The second half of the paper will illustrate the applicability of this methodology in deriving statistically valid inferences from "questionable" data by developing an empirical model to characterise the transactions demand for money in Kenya.

A number of important caveats must however first be recorded. The first is that this paper cannot hope to be fully comprehensive in its review. Not only is it impossible to cover the breadth of the debate in full in just one paper, but also research in the area of econometric methodology is proceeding rapidly and in some respects new developments may challenge even the received wisdom presented in this paper. A second important issue is that throughout the paper the discussion will be in terms of the methodology of estimating single-equation econometric models rather than econometric systems. The rationale for this is straightforward: while practices and estimation techniques do differ, the methodology of econometrics and valid inference is identical no matter what the dimensions of any particular model. Moreover, the block-by-block approach adopted in the construction of most systems requires that the process of equation specification and evaluation is sequential, and a system can be evaluated in terms of the validity of its component single-equation behavioural relationships. Third, a more relevant omission in the paper is the lack of a review of the burgeoning area of the methodology of cross-sectional, and in particular, panel data econometrics. In part this reflects the fact that much of the methodological debate has occurred in the domain of time series econometrics, but principally it is simply because

3 This nomenclature follows that of Gilbert's (1986) article on "Professor Hendry's Methodology", and reflects in particular the pioneering work of many economists at the London School of Economics in the 1960s and 1970s under the guidance of Denis Sargan and, latterly, the formalization and extension of this by one of the key members of the LSE group, David Hendry, who is Professor of Econometrics at the University of Oxford. These labels are, however, somewhat narrow and, perhaps, nationalistic, since they deny the major work done in this area by econometricians elsewhere in Europe, but particularly also in the US (see for example C.W.J. Granger, P.C.B. Phillips, and countless others). Citations and references in the text will, it is hoped, rectify this imbalance somewhat.

the debate is too comprehensive to hope to do justice to here. Finally in Part II where the emphasis will be on the exposition of the methodology certain issues concerning choice of variables for the model, data revisions and measurement error etc will be not be dealt with in detail.

## Part I: The Methodology of Econometrics

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Modern econometrics dates from the pioneering work of Ragnar Frisch in the 1930s, and although the early theorists placed heavy emphasis on the methodological aspects of inference and valid specification, these were often overshadowed by the daunting task of computation. As a consequence of the high costs of estimation, the prevalent econometric methodology proceeded on the assumption that the correct functional form, specification and dynamic structure of the model being estimated, not to mention the composition of the set of explanatory variables, was known *a priori* and with certainty. The task of econometrics was then simply the quantification of the parameters of the model, and, if necessary, the *ex post* transformation of equation residuals (for example using the Cochrane-Orcutt procedure for first-order autocorrelation) to ensure that the equation error accorded with the requirements of the classical regression model. Unfortunately the poor performance of such models, not least in relation to even the most simple time-series autoregressive models,<sup>4</sup> added to concerns about the fragility of econometric estimates cast new light on the methodology of econometrics. Leamer's (1978) pioneering critique of the methodology of econometrics and the problems of data mining, combined with the growing empirical success of time-series analysis in the United States (see, for example, Box and Jenkins (1970) and Sims (1980)) provided a renewed challenge for econometric theorists. The issues considered in this paper reflect one particular response to this challenge. Stemming from the work of Denis Sargan and others in the 1960s, these ideas have been developed and formalized into a coherent methodology in recent years through the work of David Hendry and his associates at the LSE and Oxford. The methodology is, probably, the dominant approach to econometrics in the UK and Europe and is gaining increasing support in the US.<sup>5</sup>

4 The ability of an econometric model to out-perform its own time series representation must be a minimum condition of satisfactory model specification if we are to retain the notion that econometrics is concerned with the study of the inter-relationship between economic variables. Fundamentally if we believe that there is at least some causal relationship between variables then we should always require that the model  $A(L)y_t = B(L)X_t + u_t$  will perform better (in some sense) than  $A(L)y_t = u_t$ , where  $L$  is the lag polynomial and  $X$  is a vector of explanatory variables.

5 See the papers by Hendry (1979, 1983, 1985), and Hendry and Ericsson (1991) for further detail on the approach.

The philosophy underpinning the LSE theory of econometrics commences from the combination of a "falsificationist" approach to scientific discovery and the established fact that in the era of cheap computing, statistical estimation is computationally easy. Thus there exists no such thing as a "true" model (least of all *a priori*) and as a consequence econometrics is a process of design in pursuit of "congruent models which characterize the salient features of the (unknown) process generating the (observable) data". Hendry and Richard (1982) suggest that a congruent model is one which is: i) interpretable in terms of the structural parameters of interest; ii) data coherent (ie the error terms are white-noise); and iii) where there is parameter stability. The process of econometric discovery thus consists of the development of efficient processes of design, testing and selection which yield models which are consistent with the data and which inform our understanding of economic structures and/or provide the basis to test the implications of economic theory. The process of model design is an iterative one which is halted only when the researcher has identified a congruent model (one that is both *theory consistent*, *data admissible*, and is able to *encompass* all rival models). This is referred to as the *general-to-specific* approach to modelling. The following discussion reviews the main features of this approach while in Part II we provide an illustration of this process.

### Valid conditioning and exogeneity

We start from the most general notion that observable economic data is generated by an extremely large and complex process (the economy) which is not fully understood. This is known as the *data generating process*, or DGP, and the task of the econometrician is to characterize the main, or relevant, features of this DGP through the construction of theory-consistent and statistically valid models. Since in most economies data sets generally consist of a relatively small sample of observations on a large number of variables, statistical manageability and economic clarity require that analysis focuses on a sub-set of the variables. We can think of the DGP very generally in terms of the joint probability distribution of a vector of variables  $V$ , which depends on the entire history of the vector  $V$  and a set of parameters,  $\Theta$ . Let this DGP be  $D(V_t/V_{t-1}, \Theta)$ . Suppose, however, that we are particularly interested only in some specific parameters, denoted  $\lambda_1$  (chosen because of their theory relevance to the problem being analyzed by the researcher).<sup>6</sup>

<sup>6</sup> For example, we may wish to think of these as say, the marginal propensity to consume, or the price of foreign exchange etc.

The model to be estimated, containing only the parameters of interest,  $\lambda_1$ , which are a subset of  $\Theta$ , is obtained by a process of factorization of the joint distribution.<sup>7</sup> Let  $w$  be the set of variables which are not of relevance to the problem, and let  $X = (y, z)$  be the variables that are. The DGP can then be factorized into its conditional and marginal processes to yield

$$(1.1) \quad D(V_t|V_{t-1}, \Theta) = D(X_t | w_t, V_{t-1}, \phi_1) D(w_t|V_{t-1}, \phi_2)$$

where  $\Theta = \{\phi_1, \phi_2\}$ . Once the researcher has reduced the scope of the problem by marginalizing the "nuisance variables",  $w$  in this manner further factorization is performed to arrive at a preferred specification.

$$(1.2) \quad D(y_t, z_t|X_{t-1}, \phi) = D(y_t|z_t, X_{t-1}, \lambda_1) D(z_t|X_{t-1}, \lambda_2).$$

Equation 1.2, where  $\phi = (\lambda_1, \lambda_2)$ , expresses the modelled variables of interest,  $y$ , in terms of only the explanatory variables  $z$ , their history, and the parameters of interest. All econometric models are essentially conditional models of the form (1.2), which are arrived at through a process of factorization of the DGP, either explicitly or, more often, implicitly.

Such a process may seem to be somewhat random, being dependent on the problem in hand, and the individual researcher's decomposition of the vector  $V_t$ , and it may seem to lay itself open to the charge of data mining - to wit that with a finite and non-replicable data set an infinite number of regressions (re-parameterizations) the researcher will eventually generate the "right" answer. This criticism does not necessarily hold since both economic theory and econometric technique can provide the means to ensure that the factorization of the DGP follows a logical and testable path, although economic theory may lead us in our (frequently untested) initial marginalization of the  $w_t$ . The validity of the conditioning and marginalization processes are captured in the notion of exogeneity.

Exogeneity concerns itself with whether, as a result of conditioning, there is any loss of information when explaining  $y$  by  $z$  *without* explaining  $z$  at the same time. If a loss of information occurs, in other words if  $y$  and  $z$  are jointly determined, then it is invalid to condition on  $z$ , and thus  $z$  is not exogenous. Rather the process generating  $z$  needs to be modelled explicitly and jointly with  $y$  in order to fully explain the behaviour of  $y$ , and the variables of both processes need to be estimated using a simultaneous estimator. Obviously, however, because of the imperfect transition from theoretical

<sup>7</sup> For any pair of random variables  $m$  and  $n$ , their joint probability distribution  $D(m, n)$  can be expressed as  $D(m, n) = D(m|n)D(n)$  or  $D(n|m)D(m)$ .

concepts to statistical reality in most practical situations there will always be *some* loss of information in conditioning, but the practical issue is whether this loss of information is grievous enough to "damage" the conditional model. The notion of valid conditioning therefore needs to be interpreted in a probabilistic rather than a strict sense. Depending on the type of conditional analysis that is being undertaken, there are three different types of exogeneity.

If the precise specification of the marginal distribution  $D(z|X_{1-1}, \lambda_2)$  is irrelevant for the specification of the parameters of the conditional model,  $\lambda_1$ , then  $z$  is said to be *weakly exogenous* for those parameters. Weak exogeneity is a necessary condition for the valid and efficient estimation of the values of unknown parameters of the conditional model. If  $z$  is weakly exogenous, and is not "Granger-caused" by  $y$ , then  $z$  is said to be *strongly exogenous*. Granger causality exists when the history of  $y_t$  does not determine  $z_t$ .<sup>8</sup> While strong exogeneity is not required for direct inference, it is a necessary condition if a researcher is seeking to forecast future values of  $y$  using future values of  $z$ . If there is Granger causality then future values of  $y$  (ie those being forecast) will have feedback effects on  $z$ . Finally, if the parameters of the conditional model  $\lambda_1$  are *invariant* to any change in the marginal distribution density of  $z$  and if  $z$  is weakly exogenous for  $\lambda_1$ , then  $z$  is said to be *super-exogenous*.<sup>9</sup> Super-exogeneity is the econometric analogue of the Lucas Critique, namely that the changes in governments or in the policy environment will be incorporated into the expectations formation process of agents which will therefore alter their underlying (conditional) behaviour of agents. Consequently, empirical studies which take behavioural responses to policy as invariant will break down and policy simulation will be misleading, especially in the context when simulations are made conditional on agents responses.<sup>10</sup>

While weak exogeneity is a necessary condition for any valid conditioning, and thus any robust econometric modelling, strong and super-exogeneity are necessary only in the case of forecasting and for policy simulation respectively. Testing for the presence or absence of exogeneity is not straightforward, but in all cases non-constant parameters, increasing forecast errors, and structural breaks in the model are symptomatic of the violation of exogeneity.<sup>11</sup> We shall consider the details of such testing procedures in the example in Part II.

The conditioning and marginalization process therefore reduces the size of the econometric problem, and if the validity of the conditional model is established the researcher may now deal with a question of a manageable size,

8 Formally, Granger non-causality exists if and only if  $D(z_t | X_{t-1}, \lambda_1) = D(z_t | Z_{t-1}, \lambda_1)$ .

9 Note that here we distinguish between parameter invariance to changes in the value of  $z$  (as required for weak exogeneity) and parameter invariance to the process generating the  $z$  series (super exogeneity).

10 See Engle and Hendry (1986), and latterly Hendry and Ericsson (1991) for an analysis of the empirical import of super-exogeneity in the context of the monetarist theory of inflation.

11 See Engle and Hendry (1983).

focusing only on the series of interest and the explanatory variables, and with greater confidence that inference drawn will be valid.

### Data admissibility and dynamic specification

The process through which these conditional models are developed combines aspects of economic theory and time-series analysis to capture the salient features of the data. Recall that earlier we noted the key point that it is assumed that the exact structure of econometric models is not known and that consequently econometrics is a process of design around a core conditional model derived from theory-based considerations of the parameters of interest. In moving from this maintained general model to a specific dynamic specification the methodology is more agnostic: economic theory may richly endow us with static or long-run theoretical constructs, but it only rarely informs on the nature of the processes of short-run dynamic adjustment to these equilibria. These dynamics may be, and indeed often are, inordinately complex and therefore specification of the short-run dynamics of the model can be determined by considerations of data admissibility only. This feature of the methodology, which has resulted from attempts to incorporate the methodological ideas of the time series tradition (see Harvey (1990)) stands in contrast to countless other treatments of dynamic specification seen in the economic literature which *impose* very specific dynamic adjustment mechanisms (such as the partial adjustment form of model) with often unacceptable results.

A log linear unrestricted autoregressive distributed lag model (ADL)<sup>12</sup> of the form

$$(1.3) \quad A(L)y_t = B(L)z_t + \varepsilon_t,$$

where  $L$  is the lag operator,<sup>13</sup> and  $z$  the vector of explanatory variables, and where the largest lag possible are allowed for on all the  $y$  and  $z$  variables, generally provides the most "agnostic" initial specification to ensure that any possible complexity in the short-run dynamics will be captured by the model. Though intentionally over-parameterized, (1.3) establishes the most general form of the model, and allows an examination of the detailed structure and correlation between the variables of the model. It also provides an initial indication of the equation standard error, and, by providing an extensive

<sup>12</sup> So called because the process in  $y_t$  is autoregressive, and the  $x_t$  process is a distributed lag.

<sup>13</sup> The lag operator is such that  $Lx_t = x_{t-1}$ . In other words  $L^2 = x_{t-2}$ . The polynomial  $A(L)$  is denoted  $A(L) = \sum \alpha_i L^i$ .

analysis of the dynamics of the model, frequently gives the researcher an indication of how the salient features of the model and its dynamics can be characterized.

Moving from this over-parameterized model to a tightly defined and economically interpretable model presents a number of challenges. However research in the 1980s has led to important developments in the analysis of dynamic structures and the development of optimal dynamic specification, particularly amongst non-stationary series. To appreciate these developments we must first introduce the basic ideas used in time-series analysis.

#### *Dynamic specification and cointegration*

The contrasts between stationary and non-stationary series can be illustrated in terms of Figure 1. Both series are cases of a simple AR(1) model of the form

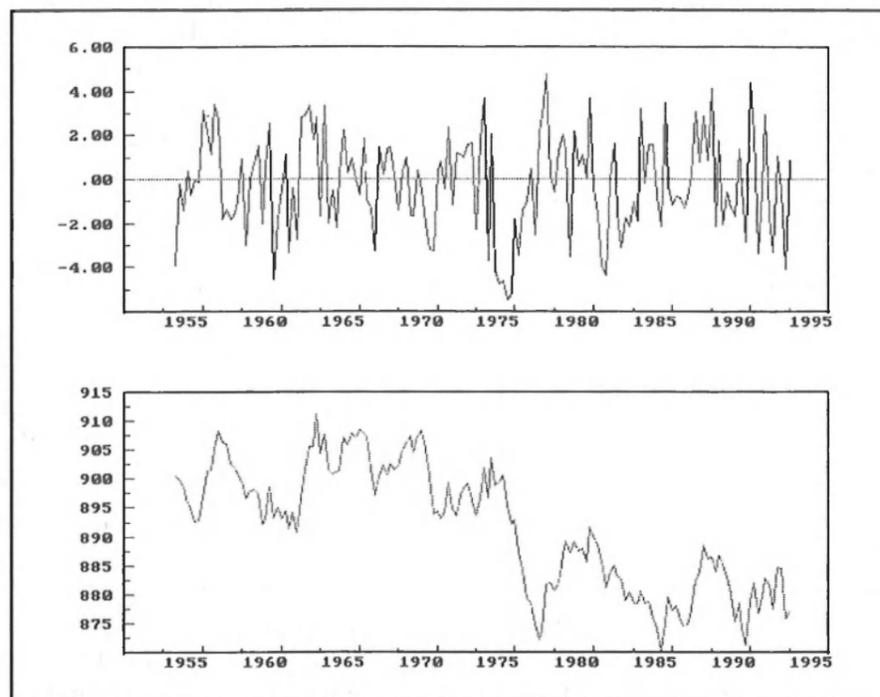
$$(1.4) \quad y_t = \alpha y_{t-1} + u_t; \quad y_0 = 0.$$

A stationary series is one where  $|\alpha| < 1$ . Stationary series have a finite variance, transitory innovations from the mean, and a tendency for the series to return to its mean value. This can be clearly seen from the Figure. Consequently, the value of the mean of a stationary series is independent of time, and thus, intuitively, no matter at what point in its history the series was examined we would always recover the same information about its structure (in a probabilistic sense). In contrast, the non-stationary series is one where  $|\alpha| \geq 1$ . Non-stationary series have a variance which is asymptotic infinite; the series rarely crosses the mean (in finite samples), and innovations to the series are permanent.<sup>14,15</sup>

<sup>14</sup> A special case of the class of non-stationary series is where  $\alpha = 1$ . This is the random walk which can be expressed as  $y_t = y_{t-1} + \epsilon_t$  or  $\Delta y_t = \epsilon_t$ .

<sup>15</sup> In contrast to the stationary series, the information recovered on the series from the analysis of a sub-period of the non-stationary series would depend heavily on the period chosen.

**Figure 1(a)** Stationary series  $Y_t-I(0)$   
**Figure 1(b)** Non-stationary series  $Y_t-I(1)$



It has been common practice to refer to the structure of series in terms of their *order of integration*. A series is said to be integrated of order  $d$  if it becomes stationary after differencing  $d$  times. Such a series is denoted as  $x_t \sim I(d)$ . Using this terminology, a stationary series is an  $I(0)$  series. Most non-stationary series (specifically the random walk) are  $I(1)$ , but it is entirely possible for non-stationary series to be of order 2 or even higher - in other words the first difference or growth rate of the series is itself non-stationary. In general, however, the properties of  $I(1)$  series apply to all non-stationary series. In Part II we shall discuss the procedure for testing the order of integration.

The importance of concentrating on such issues, which again draw heavily on pure time-series analysis, reflects a number of concerns. First, while virtually the entire body of statistical estimation theory is based on asymptotic convergence theorems (such as the Weak Law of Large Numbers) which assume that data series are stationary, these tools are increasingly being brought to bear on non-stationary data which is not, even asymptotically,

consistent with these notions of convergence. Second, far from being a special case, non-stationarity is, as we shall see, extremely common in macroeconomic time-series such as income, consumption, money, prices and trade data. Moreover, as data samples get longer and the extent of the non-stationarity has become more pronounced, the failure to appropriately deal with this non-stationarity has progressively more serious implications. This alone suggests why concern about non-stationarity has been a phenomenon of the 1980s and 1990s. From the end of the Second World War until the economic crises of the early 1970s the macroeconomic environment of most countries, both developing and industrialised, was relatively stationary. World inflation was low and stable, while income growth, investment and even commodity prices did not fluctuate greatly. In such a world the costs of treating non-stationary series as if they were stationary were not great. It was only after the mid-1970s when it became clear that the economic world was not stationary and the major economic aggregates began to fluctuate much more widely that the costs of inappropriate time-series specification became apparent. It was specifically for this reason that many hitherto well-established econometric relationships began to break down in the late 1970s. Not only did they systematically fail to predict outcomes, but it became increasingly obvious that this predictive failure was not so marked amongst pure time-series models. While considerable time and effort was spent attempting to rescue existing models through re-selection of explanatory variables and the addition of "auxiliary" hypotheses, a more productive response emerged which drew heavily on the comparative success of pure time series modelling. This saw the introduction of a more comprehensive treatment of the time-series characteristics into econometric modelling and the development of the notion of cointegration (see Hendry (1986) and Granger (1986)).

The essence of the problem lies with the presence of *spurious regression* which arises where the regression of non-stationary series, which are known to be un-related, indicates that the series are correlated. Consider the following example. Suppose two series  $x$ , and  $y$  are *known* to be un-correlated and generated by the following known processes:

$$(1.5) \quad y_t = y_{t-1} + \mu_t, \quad \mu_t \sim \text{IID} (0, \sigma_\mu^2)$$

$$(1.6) \quad x_t = x_{t-1} + v_t, \quad v_t \sim \text{IID} (0, \sigma_v^2)$$

where we know that  $E(\mu_t v_s) = 0 \forall t, s$ ;  $E(\mu_t \mu_{t-k}) = E(v_t v_{t-k}) = 0 \forall k$ . These restrictions ensure that  $x$  and  $y$  are un-correlated random walks. Consequently, we would expect that the coefficients  $\beta_0$  and  $\beta_1$  in the regression model

$$(1.7) \quad y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

would tend to zero and  $R^2$  would also tend to zero. We can test this implication directly by using Monte Carlo procedures to generate the series (1.5) and (1.6) subject to the restrictions that they are independent, and with a sample size of 100, and use the resulting  $y$  and  $x$  data to run the regression (1.7)<sup>16</sup>. The regression results are as follows, in Table 1.

**Table 1** Modelling  $y_t$  by OLS

Variable	Coefficient	Std Error	t-value
$x_t$	-.4778	.12964	-3.68
Constant	9.4917	.46649	20.34
$R^2 = .122$ $\sigma = 3.143$ $DW = .128$ $F(1, 98) = 13.59 [0.0004]$			

Despite the fact that we constructed the data series to be unrelated the OLS regression produces strongly significant  $t$  and  $F$  statistics for the model. This result occurs because both series are  $I(1)$  and are both dominated by their persistence or trend component. It is the correlation of this effect which biases the OLS regression and hence the correlation in the series cannot be interpreted in the way it could be between stationary series.

We therefore seek a method which reject spurious regression results but which will not at the same time reject correlation between non-stationary series where the correlation is in fact structural rather than spurious. Recently the cointegration of series has been used as a guide to appropriate dynamic specification in the presence of exactly this problem.

#### *Cointegration and error correction*

Cointegration is a special case within the analysis of the order of integration. Formally, if  $y_t \sim I(d)$  and  $x_t \sim I(b)$  and the linear combination of the two,

<sup>16</sup> The Monte Carlo experiment was performed using PC-NAIVE and the estimation using PC-GIVE. See Hendry (1989).

namely,  $z_t = y_t - kx_t \sim I(d-b)$  then  $x_t$  and  $y_t$  are cointegrated. In general we deal with cases where  $y_t$  and  $x_t$  are  $I(1)$ . Thus if their linear combination (or alternatively, the residuals from the regression of  $y$  on  $x$ ) is  $I(0)$ , then we have cointegration.

Where does this lead us? The first point is that cointegration analysis provides a powerful discriminating test for spurious correlation: conducting cointegration analysis between apparently correlated  $I(1)$  series and finding cointegration validates the regression. Failing to find cointegration is an indication that spurious correlation may be present, and thus the invalidity of inference drawn from such correlation.

Second, an important result from the cointegration analysis concerns the specification of processes of dynamic adjustment. Frequently empirical models of adjustment have tended to address the issue of dynamic adjustment by focusing on the partial adjustment mechanism (see Chow (1960)).<sup>17</sup> In terms of a simple bivariate model this can be represented as

$$(1.8) \quad y_t = \alpha_0 + \alpha_1 z_t + \alpha_2 y_{t-1} + \varepsilon_t.$$

While this model of adjustment may indeed be a true characterization of the actual adjustment process, evidence has shown that such partial-adjustment models face a host of problems when the series  $y$  and  $x$  are non-stationary (as is likely in many models of macroeconomic adjustment). First they experience high levels of (possibly spurious) multicollinearity amongst the regressors  $z_t$  and  $y_{t-1}$ ; they tend to exhibit high levels of first-order autocorrelation; and moreover, as  $\alpha_2$  tends towards unity (which occurs as the series becomes non-stationary), the implicit rate of adjustment towards the equilibrium become inordinately slow.<sup>18</sup> Finally, as a result of the above, the partial adjustment specification is generally dominated by the non-stationary time-series characteristics of  $y_t$ , so that it fails to capture the structural characteristics between  $y$  and  $z$ .

By contrast the Engle-Granger Representation theorem (Engle and Granger (1987)), states that if two series are cointegrated then they will be most efficiently represented by an error correction specification, and furthermore, if the series are cointegrated, this dynamic specification will encompass any other dynamic specification, including the partial adjustment model. The error correction parameterization of the ADL takes the form

$$(1.9) \quad \Delta y_t = \alpha_0 + \alpha_1 \Delta z_t - \alpha_2 (y - kz)_{t-1} + \varepsilon_t,$$

<sup>17</sup> See Adelkunle (1968), Mweha (1990), and Agheveli (1978) for examples of this adjustment mechanism in the demand for money.

<sup>18</sup> The mean lag is  $\alpha_1/(1-\alpha_2)$ , so that as  $\alpha_2$  tends to unity  $\alpha_1/(1-\alpha_2)$  tends to infinity.

where  $k$  is the long run proportionality between  $x$  and  $y$ .<sup>19</sup>

The error correction model has existed as a dynamic specification for a significant period of time (Sargan's (1984) paper on real wage determination being the first use), and has been found to be validated for the specification of consumption, investment and money demand across a range of economies (see Davidson *et al.* (1978), (Hendry (1985), Domowitz and Elbadawi (1987)). The specification thus relates the short run change in the dependent variable  $\Delta y$  to the short run change in the explanatory variable  $\Delta z$  (*the impact effect*), but ties the change to the long run proportionality between  $y$  and  $z$  (*the long run effect*), through a feedback mechanism. In doing so it powerfully allows us to exploit information on the equilibrium relationship between non-stationary series (if such equilibrium relationship exists) within a stationary, and therefore statistically consistent, model. Moreover, the presence of cointegration permits the combination of long-run and short-run information in the same model, and thereby overcomes some of the drawbacks associated with the loss of information that occurs from other attempts to address non-stationarity through differencing (as in the growth rate model).

Note, however, the corollary of the Engle-Granger theorem, namely that if cointegration is not accepted then there is no long-run relationship between the non-stationary series and thus there will be no information in the  $\alpha_2$  coefficient. The imposition of a long-run error-correction term will be rejected by the data. The correct specification in this instance will be some other form of specification in which no long run relationship appears.

While the error correction model is grounded in an intuitive adjustment model,<sup>20</sup> it also has a number of statistically attractive properties when dealing

19 In many textbook presentations, it is assumed for convenience that  $K=I$ , and it consequently is omitted. There is no necessity that this restriction holds.

20 A theoretical link between the long-run equilibrium conditions and short-run dynamics can be established if we assume that agents seek to minimize the costs of adjustment from their actual to their desired positions, when costs are defined in terms of a quadratic loss function of the form:

$$L = \sum \alpha^t [\beta_1 (x_{t+1} - x^*)^2 + (x_{t+1} - x_{t+1})^2 - 2\beta_2 (x_{t+1} - x_{t+1})(x^* - x_{t+1})].$$

Where  $x$  is the control variable,  $x^*$  the target variable,  $\alpha$  the rate of time discount, and  $\beta_1, \beta_2 > 0$  (see Nickell (1985) and Domowitz and Hakkio (1987)). Minimizing this loss function with respect to  $x_{t+1}$  results in a second order difference equation of the form

$$\alpha x_{t+1} - (1 + \alpha + \beta_1)x_{t+1} + x_{t+1} = \alpha \beta_2 x_{t+1} - (\alpha \beta_2 + \beta_2 + \beta_1)x_{t+1} + \beta_2 x^*_{t+1}$$

This can be re-written bringing all the  $x$  and  $x^*$  terms together as  $y$

$$\alpha y_{t+1} - (1 + \alpha + \beta_1)y_{t+1} + y_{t+1} = -\beta_1(1 - \beta_2)x_{t+1}$$

where  $y_{t+1} = x_{t+1} - \beta_2 x^*_{t+1}$ . Factorising and solving, this difference equation solves out to

$$\Delta x_t = \beta_2 \Delta x^*_t + (1 - \mu) \{ \beta_2 x_{t+1} + (1 - \beta_2)(1 - \alpha \mu) \sum (\alpha \mu)^i x_{t+1} - x_{t+1} \}$$

where  $\mu$  is the stable root of the difference equation. This relates changes in the actual control variable,  $x$ , to the change in the target variable  $x^*$  and to the error in the previous period (ie the deviation of  $x_{t+1}$  from the growth trajectory for  $x^*$ ), and is commonly termed an error-correction function. Note that if  $\beta_2$  is set to zero in the original loss function (ie the third term of the loss function is ignored), then the error correction function reverts to a simple partial adjustment model.

with the modelling of non-stationary data. First, by transforming the series the worst excesses of the multicollinearity problems of the partial adjustment model are avoided: while  $y_{t-1}$  and  $z_t$  may be highly correlated<sup>21</sup>,  $\Delta y_t$  and  $(y_t - y_{t-1})$  will be less so, and the covariance matrix will thus be more orthogonal. Second, the error correction model does not impose the same restrictive lag structure as the partial-adjustment model, and finally the error correction model approach ensures that all the variables in the model have the same order of integration, namely  $I(0)$ .<sup>22</sup>

### *Testing for cointegration*

Tests for cointegration are based on the same procedures developed for analyzing the order of integration of individual series (the Dickey-Fuller (DF) and Sargan Bhargava Durbin Watson (SBDW) tests) which will be presented in Part II, except in this instance we apply such test to one specific series namely the residuals of the "cointegrating regression" of  $y$  on  $x$ . Recall that if  $y$  and  $x$  are  $I(1)$  then the residual from the regression of those series would also be  $I(1)$ , *unless* they are cointegrated. Thus if the residuals are in fact  $I(0)$  then we accept cointegration. Despite its intuitive appeal, there are problems with this cointegration analysis, especially for researchers dealing with small samples. This arises because both the DF and the SBDW tests have significant biases in certain circumstances and that these biases differ between the "cointegrating regression" and the solution to the dynamic model (see Banerjee *et al.* (1986)). It therefore becomes necessary to check the validity of the ECM before it is included in the dynamic specification of the models. The procedures for doing so are discussed in Part II of this paper.

### The simplification process

The third stage of the general-to-specific strategy concerns the simplification of the general over-parameterized model. This is the least mechanistic aspect of the general to specific modelling process, and relies more on experience and intuition (and luck) than on set procedures. The object of the simplification procedure is to reduce the over-parameterized ADL to a "parsimonious

<sup>21</sup> Which is clearly the case if the series are co-integrated.

<sup>22</sup> If  $y$  and  $z$  are  $I(1)$  then  $\Delta y_t$  and  $\Delta z_t$  are  $I(0)$ , and if they are cointegrated then, by definition,  $(y-z)_{t-1}$  is also  $I(0)$ .

simplification of the general representation". Thus through a series of linear transformations of the general model and the imposition of coefficient restrictions, we seek to produce a model which is consistent with theory and with the data. Even given the force of the Engle-Granger theorem indicating the applicability of the error-correction specification, deriving a data admissible "parsimonious" is still a time consuming business, and an iterative one. At each stage in the process the transformations and simplifications are tested for their effect on the explanatory power of the model using "goodness of fit" statistics, and for the effect of the transformations on the structure of the error term using a battery of diagnostic tests. While the purpose of the former tests is self-explanatory the latter reflects an important point about model design. Unlike earlier modelling traditions which sought to "correct" the equation error term, the Hendry approach emphasises the fact that the nature of the error term in a statistical model is not a function of the data, or of the structure of the economy, but rather is a statistical artifact of the model. Thus the presence of deviations from the classical assumption that the error term is independently and normally distributed merely reflects dynamic mis-specification in the model. In other words the presence of any systematic pattern of behaviour in the error term is simply an indication that that systematic part of the data is not being adequately captured by the model, and this indicates the need to re-specify the dynamic specification of the model. Diagnostic tests are thus criteria to guide model design.

### Encompassing

It is not uncommon to see models "confirmed" by only the fewest of diagnostic statistics such as the  $R^2$  and Durbin Watson test for first-order autocorrelation. The LSE methodology however acknowledges that ultimately econometric model building consists not of developing models which *confirm* theory, but rather which *survive* repeated attempts to falsify the model. There is no ultimate test through which models are unconditionally accepted, although there are tests through which models are unconditionally rejected. This brings us to the final but perhaps central aspect of the methodology namely encompassing. As Gilbert (1986) notes,

"... the very arbitrariness of the LSE simplification process makes encompassing crucial. One may derive a parsimonious representation any way one chooses; its congruency is established or otherwise by a battery of tests; if it is the only congruent model, one will tentatively accept it; otherwise one will attempt to encompass it. The plausibility of a model

derives not from the procedure by which it was discovered (many important scientific discoveries, for example penicillin, are the results of accidents), but by the procedure by which it is validated - the more rival models that are encompassed the more plausible the encompassing model."

The purpose of encompassing is to provide a formalized way of pitting one model against another to explain a set of data. Essentially if we have two competing models we are asking the question: by believing in Model 1 can we explain the data at least as well as Model 2 can? If we can, we say that Model 1 encompasses Model 2. In keeping with the thrust of the general-to-specific strategy, Mizon (1984) introduce the concept of *parsimonious encompassing* which captures the notion a simple model encompasses a more general one. It is clear that a model in which there are many regressors is intuitively less powerful than a model which can explain the same amount on the basis of a more simple model.<sup>23</sup> Some of the main forms of encompassing tests are provided in Annex I.

The final aspect of modelling is therefore to consider whether the maintained model can encompass any rival models - including the pure time-series representation of the data - not only over the existing sample, but also over out of sample data as it becomes available. It is only through this process of continual confrontation of a model with attempts to refute and encompass it can a model maintain methodological credibility.

## Conclusions

This first section has briefly reviewed the main elements of the general to specific approach to the development of robust time series econometric models. Though the process has been somewhat stylised it has attempted to describe the principal elements involved in ensuring that for any given data set, irrespective of its nature, statistical inferences drawn are valid. In the remainder of this paper we shall attempt to add flesh to this discussion by considering the specific example of the demand for narrow money in Kenya.

<sup>23</sup> In the limit, of course, a model that describes everything, explains nothing.

## **Part II: The Demand For Narrow Money in Kenya**

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### **Introduction**

The purpose of this section is to establish an empirically robust and theoretically consistent model for the demand for narrow money in Kenya for the period from 1973 to 1990. Though the principal purpose of the section is expositional it is useful to motivate the discussion of the results by considering the main features of the process we are seeking to model.<sup>24</sup>

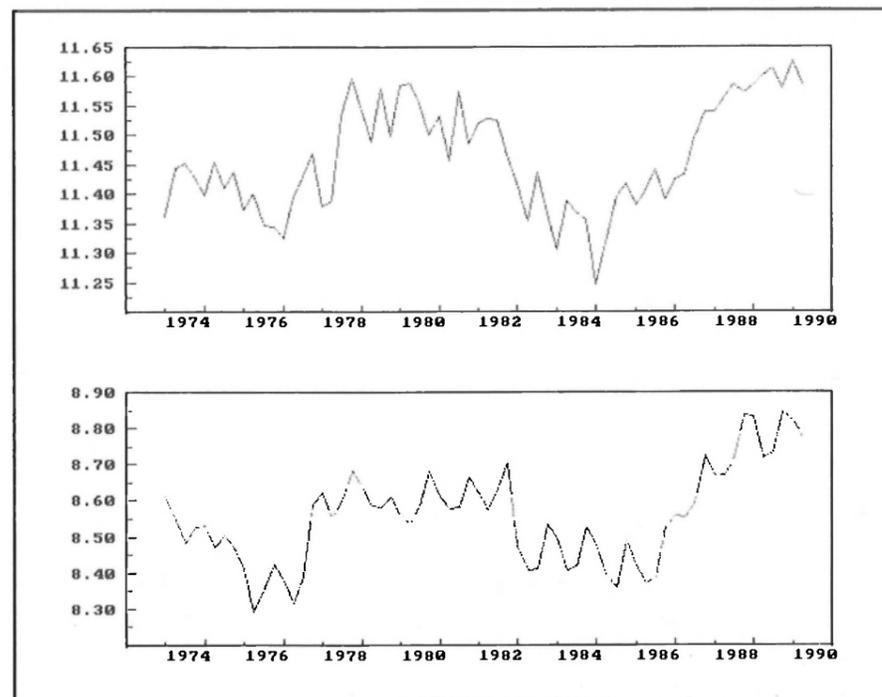
Since the early 1970s Kenya has been prone to a number of substantial external shocks which have significantly affected the macroeconomy. Starting with the 1973 oil shock the economy experienced its first economic contraction since Independence a decade earlier (see Figure 2), but this was rapidly overtaken by the effects of the 400% rise in coffee prices in 1976-7 which resulted in a 54% terms of trade improvement. The coffee boom was short-lived and, precipitated by a the second oil shock of 1979-80, was followed by a period of deep recession and economic crisis which lasted until late 1984. The re-establishment of fiscal and external balance in 1985 was followed by a further coffee boom in 1985 saw the economy recover markedly from 1985 to 1987, although towards the end of the period growth has once again stagnated.

Figure 2 (b) shows the path of narrow money stock in 1985 prices over the same period. Four distinct periods can be discerned starting with the period from 1973 where the economy experienced its first significant increase in prices since independence and a consequent erosion of real money balances. Accumulation of cash was rapid from 1976 to 1977 and by 1979 real balances had again recovered to their previous level. The delayed inflationary effect of the failure to sterilize the boom fed through the system in the early 1980s and saw the real value of money balances again fall through 1985, while the sharp fall in price inflation from 1986 to 1988 once again reversed the trend, leading to a rapid increase in real money balances through to the end of the period, although the re-emergence of inflation since late 1987 has moderated this

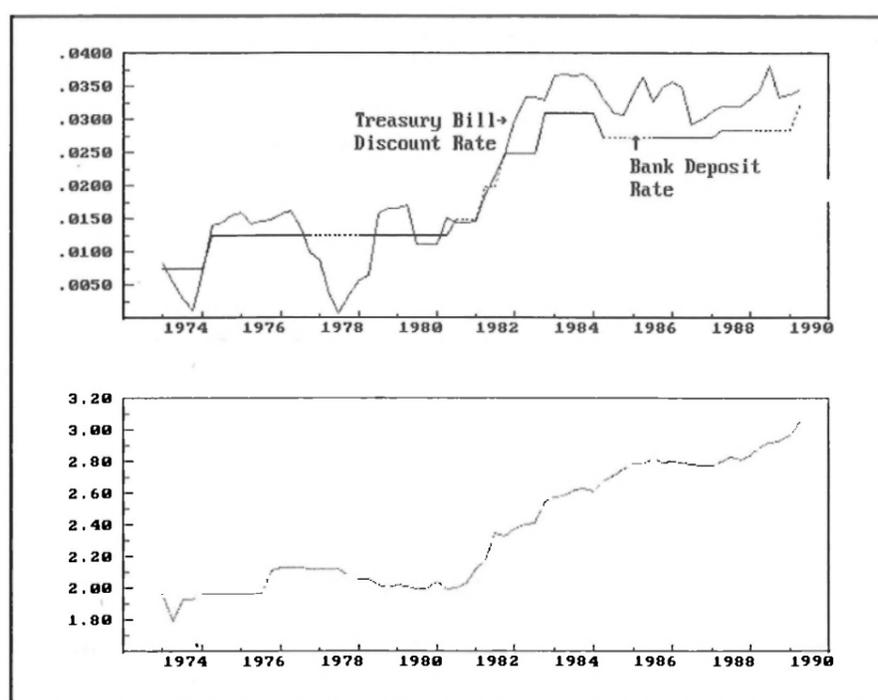
24 All the estimation in this section has been carried out using PC-GIVE Version 6.01 (see Hendry (1989)). All the data can be supplied on request to the author.

growth. As in the majority of Sub-Saharan African countries, however, domestic asset markets are thin, the number of financial instruments is low, and foreign asset holdings are illegal for the non-bank private sector. Interest rates have until very recently been set by central bank fiat (see Figure 3(a)), but have risen over the period and have been positive in real terms throughout most of the 1980s (see Killick and Mwega (1990) for a comprehensive review of monetary and financial conditions over this period).

**Figure 2(a)** Real gross national income (log)  
**Figure 2(b)** Real narrow money stock (log)



**Figure 3(a)** Quarterly interest rates  
**Figure 3(b)** KSh/US\$ exchange rates



### Modelling the demand for narrow money

Models of the aggregate demand for narrow money balances are generally built up from microeconomic models which view the demand for money in terms of the level of real income and transactions costs. Adding uncertainty to these models provides the basis for a transactions and precautionary demand for money as a function of income, transactions costs, the opportunity cost of holding cash as a nominally non-interest bearing asset. Work on the demand for money in developing countries has also emphasised potential currency substitution effects, such that the rate of return to holding foreign currency may also be a determinant of money demand.<sup>25</sup>

<sup>25</sup> See for example the papers by Judd and Scadding (1982), Akerlof and Millbourne (1980), Tanzi and Blejer (1982) and Domowitz and Elbadawi (1987).

Adopting the terminology from Part I of this paper we therefore define our vector of variables of interest,  $\mathbf{X}$  as

$$(2.1) \quad \mathbf{X} = ( (m-p), (y-p), r, r^*, \Delta p )$$

where  $m-p$  is real money balances,<sup>26</sup> defined as notes and coin in circulation outside the banking system,  $y$  is some measure of income,  $r$  is the domestic rate of interest,  $r^*$  a measure of the rate of return on holdings of foreign currency, and  $\Delta p$  the rate of inflation. If we let  $m-p$  be  $y$  and the other variables  $z$ , then, rather obviously, our conditional model can be expressed as

$$(2.2) \quad D(y_t | z_t, \mathbf{X}_{t-1}, \lambda).$$

Before estimation of the model (2.2) for the demand for money we commence by defining the data in detail and analyzing the time series characteristics of each series to be modelled.

### The time-series characteristics of the data

A number of important issues arise when considering the appropriate measures to be used for the variables of the model. The first concerns the choice of income series. Traditionally some measure of GDP is employed. However in the case of an open economy such as Kenya the volatility in the terms of trade mean that taking GDP as a measure of income will fail to accurately reflect the transactions demand for money. We there choose two alternative measures of income, gross national income (GNY), equal to GNP adjusted for changes in the terms of trade, and total final expenditure (TFE).<sup>27</sup> The second issue concerns the price series. Whereas most models of the demand for money in developed economies uses the GDP or GNP deflator, we choose to use the consumer price index (CPI). Though this choice is determined to an extent by the fact that the CPI is the only consistent quarterly price data series available for Kenya, using a CPI measure is acceptable if it is a true reflection of the real prices faced by consumers. Unlike many other African economies where the CPI weighting is inappropriate or where a significant proportion of total transactions are conducted at parallel market prices, evidence for Kenya

<sup>26</sup> The demand for real balances is assumed to be homogeneous of degree 1 in prices. This neutrality restriction can be tested and is accepted by the data for Kenya. However we do not assume that real balances are neutral in terms of inflation.

<sup>27</sup> Both series are reported only annually and an interpolation exercise was required to create suitable quarterly data. Details on the construction of these two series, and the analysis of the implied seasonality can be obtained from the author on request.

suggests that the CPI is a fair approximation of prevailing prices. Third, concerning interest rates we note that there is only a small number of interest-bearing assets to be held by the private sector. Throughout most of the period under examination domestic interest rates have been controlled by the authorities. However, all interest rates have generally been adjusted in a consistent manner over the period despite the absence of an active market mechanism through which interest rate changes are transmitted (see Figure 3(a)). Consequently using the discount rate for Treasury Bills by the Central Bank of Kenya is a reasonable approximation to the true interest rate facing the non-bank private sector. Finally on the issue of measuring the effect of currency substitution we face a number of quite serious conceptual problems, most of which we are unable to address in this paper. Ideally the appropriate measure for currency substitution in an economy such as Kenya where domestic agents are prohibited from holding foreign currency would be measured by the parallel market exchange rate. Though data do exist on black-market premia, these cover only a relatively short period. However for the period for which data do exist it is clear that in view of the nature of the exchange rate regime adopted in Kenya, the premium to holding foreign exchange is low and has been relatively constant. As a first approximation the premium is a constant and we therefore let  $r^*$  be the official exchange rate *vis a vis* the US dollar (Figure 3(b)). Further work may be required to improve on this first approximation.

Having established the vector of variables of interest we now consider the order of integration of each series using the Dickey-Fuller class of unit root tests and the Sargan-Bhargava DW test. The Dickey-Fuller method is a test on the size of the coefficient  $\rho$  in the equation

$$(2.3) \quad y_t = \alpha + \rho y_{t-1} + u_t; \quad u_t \sim (0, \sigma^2); \quad y_0 = 0$$

Normally this would be carried out using a traditional 't' test against the null  $H_0: \rho = 0$ . However, in unit root testing we are testing against the null  $H_0: \rho = 1$ , and the test statistic is not distributed as a standard 't'. Dickey and Fuller (1976) have tabulated the distribution of the statistic, which, however, varies depending on whether the model is estimated with a constant and trend. The test for the existence of unit roots (ie  $\rho = 1$ ) has the following critical values.<sup>28</sup>

28 The Dickey-Fuller Distribution is however non-constant depending on the form of the null. See Banerjee *et al.* (1991)

**Table 2(a)** Dickey Fuller test statistics  
Critical value for rejecting null in favour of  $\rho < 1$ <sup>29</sup> where null is  $y_t = \alpha + \rho y_{t-1} + u_t$

Sample size	Critical value 5%
25	-3.00
50	-2.93
100	-2.89
$\infty$	-2.86

Source: Banerjee *et al.* (1992)

**Table 2(b)** Sargan-Bhargava Test Statistics

Sample size	Critical value 5%
50	0.78
100	0.39
200	0.20

Source: Banerjee *et al.* (1992)

In terms of the tests performed in this paper the sample size is approximately 65 which would suggest a critical value for the statistic of around -2.25. One of the drawbacks of the Dickey-Fuller test, however, is that it necessarily assumes the DGP is an AR(1) process as above under the null. If it is not, then autocorrelation in the error term in (2.3) will bias the test. In order to overcome this problem the "Augmented" Dickey-Fuller test can be used. The ADF is identical to the standard DF test but is constructed within a regression model of the form

$$(2.4) \quad \Delta y_t = \alpha + \rho y_{t-1} + \sum \gamma_j \Delta y_{t-j} + u_t$$

<sup>29</sup> We reject the null in favour of  $\rho = 1$  (ie. the series is stationary) if the statistic is less than the critical value.

where the lag length  $j$  is set so as to ensure that any autocorrelation in  $\Delta y_t$  is absorbed, and the error term is distributed as white noise. Using quarterly data, we find that by initially setting  $j = 6$ , all residual autocorrelation is captured.

A second form of testing for the presence of unit roots is the Sargan-Bhargava Durbin-Watson Test. This test is based on the standard Durbin-Watson statistic, but the test is applied not to the residuals of the regression as usual, but on the level of each individual series. The SBDW test is defined as

$$(2.5) \quad DW(y) = \frac{\sum (y_t - y_{t-1})^2}{\sum (y_t - y)^2}.$$

Unlike the Dickey-Fuller tests, the test is against the null that the series is  $I(0)$ , in which case the value of the DW statistic will tend towards a value of 2. If the statistic is low then this is evidence of an  $I(1)$  series.<sup>30</sup> The relevant critical values for the statistic are given in Table 2(b), where values of the statistic less than the critical value indicate rejection of the null.

Tests for unit roots are frequently biased, and have low power in distinguishing stationary series, ie those where  $|\alpha|$  is close to but less than 1 and those series which genuinely have a unit root (see Banerjee *et al* (1986) for a discussion of biases in tests) and generally inference on the order of integration is drawn on the basis of more than one single test (even though they may be contradictory)<sup>31</sup>. It may be noted that one of the strong arguments for using the DF and SBDW statistics together is that in each case the null hypothesis is the same as the alternative hypothesis of the other, thus providing a cross-check for the test.

Table 3 reports the results of the Sargan Bhargava and Dickey-Fuller tests statistics for the order of integration of each series.<sup>32</sup>

The results reported here are typical of many problems in time-series econometrics, namely that of slightly ambiguous test statistics. Although the SBDW statistics strongly reject stationarity, the DF and ADF test statistics are relatively close to their critical values. However with the exception of the log of inflation ( $\Delta p$ ) which is clearly  $I(0)$ , we interpret these results as indicating that money, the two measures of income and all the interest rates are indeed  $I(1)$ , or non-stationary. The immediate implication flowing from this data set

<sup>30</sup> The correspondence with the normal interpretation of the DW statistic is clear. A low value of the traditional DW statistic is taken as evidence of positive first-order autocorrelation. Such high autocorrelation is exactly what defines a non-stationary series.

<sup>31</sup> Other tests include the Phillips and Peron and Said-Dickey tests. See Banerjee *et al* (1992).

<sup>32</sup> All series are in natural logs, with interest rates expressed as  $LR = \ln(1+RQ)$  where  $RQ$  is the rate of interest per quarter.

is that any dynamic specification of the model in the levels of the series (such as in terms of the partial-adjustment model frequently found in the literature) is likely to be inappropriate, and may be plagued by problems of spurious regression. However, we may be guided towards an error-correction form if the series of the model are cointegrated.

**Table 3** Unit Root Tests on Quarterly Data

Variable	SBDW	DF	ADF	Longest Lag	Order of Integration
(m-p)	0.3241	-2.28	-2.21	8	1
(gny-p)	0.3659	-2.29	-2.60	8	1
(tfe-p)	0.3367	-2.45	-1.79	1	1
$\Delta p$	1.3653	-4.99	-2.05	6	0
rtb	0.0596	-1.48	-0.84	-	1
ex	0.0192	0.29	-0.03	2	1 or 2

### Testing for Cointegration

In testing for cointegration both DF and SBDW tests are again used although in this case we apply the test to the residuals of the cointegrating regression, rather than the levels of the series. Following Engle-Granger we specify the cointegrating regression as

$$(2.6) \quad y_t = \alpha_0 + \alpha_1 Z_t + \varepsilon_t$$

The residuals of the equation,  $\varepsilon_t = (y_t - \alpha_0 - \alpha_1 Z_t)$  is simply the linear difference of the I(1) series. If the residuals from the linear combination of non-stationary series are themselves stationary then we can accept that the I(1) series are cointegrated. In the case of tests for cointegration the critical values for the tests differ according to the number of variables,  $n$ , in the cointegrating regression. The relevant critical values are reported in Table 4.

Though the literature suggests that a cointegrating (or long-run) relationship may be expected to be found between money holdings and the level of income (the pure transactions model), we may also expect to find a similar cointegrating relationship between measures of the opportunity cost of holding money balances relative to interest bearing assets, or relative to the real rate

of return in the economy, as proxied by the rate of inflation. Similarly the currency substitution literature suggests the presence of a long-run relationship between holdings and the return on foreign currency holdings. We therefore are obliged to examine all these possibilities. The order of integration tests indicate that focus should be on the I(1) series for money, income interest rates and the exchange rate, but since  $\Delta p$  seems to be a stationary series then by definition it will not be cointegrated with the other series. However even though the statistics seem to conclusively reject non-stationarity for inflation we initially allow for the possibility that there are biases in the statistics and include  $\Delta p$  in the cointegration analysis. Note that this does not mean that inflation does not play a rôle in the determination of actual money holdings, but rather that at least in terms of this aggregate it has no long-run relationship.

**Table 4** 5% Critical Values for Cointegration Tests

n	T	SBDW	DF	ADF
2	50	0.78	-3.67	-3.29
	100	0.39	-3.37	-3.17
3	50	0.99	-4.11	-3.75
	100	0.55	-3.93	-3.62
4	50	1.10	-4.35	-3.98
	100	0.65	-4.22	-4.02
5	50	1.28	-4.76	-4.15
	100	0.76	-4.58	-4.36

Source: Banerjee *et al.* (1991) Table 5.1

We choose to test for the possibility of cointegrating relationships between the variable by running a series of bivariate cointegrating regressions. In this section we shall conduct our analysis of cointegration by adopting the methods developed by Engle and Granger (1987) and discussed in the *Oxford Bulletin of Economics and Statistics* Special Issue on Cointegration in (1986). More recently cointegration analysis has tended to concentrate on the methods developed by Johansen and Juselius (1990) which provide a more detailed

analysis of multiple cointegrating relationships between series. The Johansen and Juselius method is, however, somewhat complex and although it has been applied to Kenyan monetary data with consistent results, they are not presented here. The results of the cointegration analysis are reported in Table 5<sup>33</sup> where the coefficients on the constant and seasonal dummies are omitted, and *t* statistics are in parenthesis.

The first step is to address the issue of which income measure is the more appropriate for this analysis which we do simply by examining the bivariate cointegrating regressions of *m-p* first on the gross national income series, (*gny-p*) and then on the total final expenditure series (*tfe-p*). With *T* = 64 the critical value for the SBDW test (against a null that the residuals are stationary) is approximately 0.78. Thus we cannot reject the null for the (*gny-p*) equation, but do so for the (*tfe-p*) equation. Similarly on the basis of the Dickey Fuller tests we reject the null of non-stationarity only in the case of the (*gny-p*) equation (the appropriate critical values for the DF and ADF tests are -3.67 and -3.29 respectively). We thus conclude that the gross national income measure is *statistically* more appropriate for this data and sample period. Taking (*gny-p*) as the only income variable we next test for cointegration between the I(1) series. The results are conclusive: amongst the I(1) series in the model only two are cointegrated, namely money and income. In all other cases the SBDW statistics are less than 0.78 and the DF and ADF statistics greater than -3.67 and -3.29 respectively, indicating that for each pair of series the residuals from their regression (in other words their linear combination) is itself non-stationary.

Applying the Granger-Engle Representation Theorem we can now proceed to an error correction dynamic specification of the form

$$(2.7) \quad \Delta y_t = \alpha_0 + \alpha_1 \Delta z_t - \alpha_2 (y - z)_{t-1} + \varepsilon_t$$

where *z* is the vector of variables as before and *z<sub>t</sub>* is the income term. Before doing so we however must note that though the Granger-Engle Two-Step theorem asserted that the coefficients of static regression were "super-consistent",<sup>34</sup> more recent Monte Carlo evidence suggests that, using the DF and SBDW tests, cointegration may be accepted too readily, and that the estimates of the static regression may be biased. An initial rule-of-thumb indication of the presence of the bias is in the value of *R*<sup>2</sup>. Low *R*<sup>2</sup> values for the static regression may indicate the acceptance of cointegration when it does not in fact exist. An alternative consistency check is to compare the value of

33. Note that in the bivariate case cointegration is symmetric, so that if *y* is cointegrated with *x* then *x* will be cointegrated with *y*. We therefore only report each pair once.

34. See Banerjee *et al.* (1991)

the coefficients of the static regression with those of the long-run solution of the equivalent over-parameterized autoregressive distributed lag (ADL) model of the form

$$(2.8) \quad A(L)y_t = \gamma_0 + B(L)z_t.$$

If the coefficients of the static cointegrating regression are insignificantly different from those of the ADL, then we accept that the cointegrating relationship is valid. Thus we seek to test the restrictions that  $\gamma_0/A(L) = \alpha_0$  and  $B(L)/A(L) = \alpha_1$ . The two equations are given below with standard errors in parentheses.

**Table 5** Cointegrating Regression and Test Statistics

	LR Coefficient	SBDW	Residual DF	ADF
(m-p) on (gny-p)	1.14 (10.9)	1.055	-3.76	-3.12
(m-p) on (tfe-p)	1.26 (6.3)	0.386	-2.34	-1.90
(m-p) on rtb	0.79 (0.57)	0.136	-1.33	-1.75
(m-p) on dp	-1.38 (1.64)	0.163	-1.30	-1.02
(m-p) on ex	0.103 (2.49)	0.151	-1.68	-2.03
rtb on (gny-p)	-0.007 (0.470)	0.057	-1.45	-0.75
rtb on ex	0.028 (15.59)	0.323	-2.46	-1.86
ex on (gny-p)	0.546 (1.04)	0.028	-0.28	-0.53

It is clear, even without any formal testing of the cross equation restrictions, that these two equations are indistinguishable, and we can therefore take the residuals from the cointegrating regression as a valid error-correction term, ECM, which is then built into the error-correction model. This variable is plotted as Figure 4, and, as required, if money and income are cointegrated it is clearly  $I(0)$ .

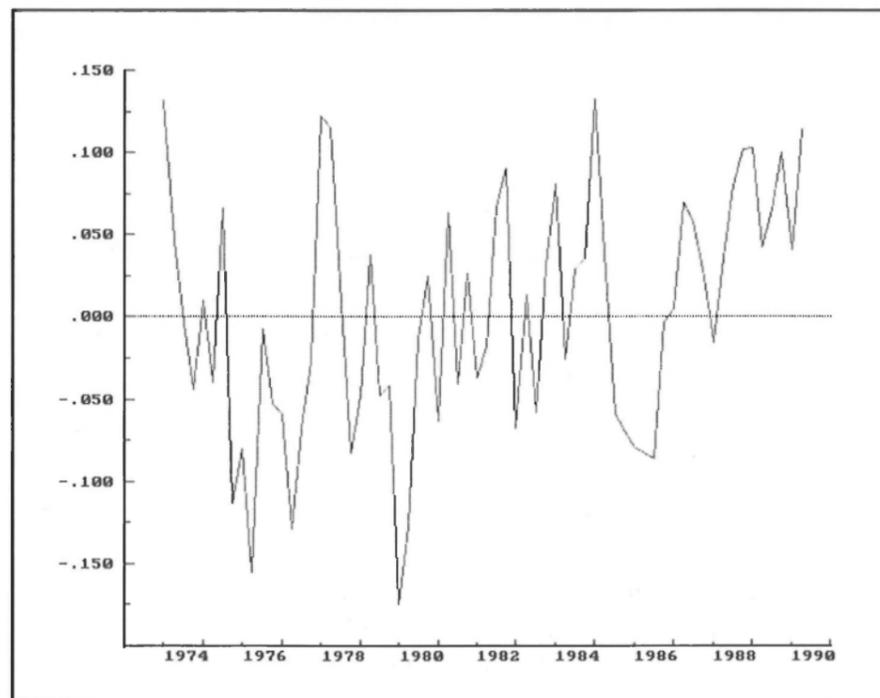
**Table 6** Consistency of solution to cointegrating regression (from Table 5)

## Cointegrating Regression

$$(2.9) \quad (m-p) = -4.48 + 1.14 (gny-p) \\ (1.19) \quad (0.10)$$

## Long Run Solution to ADL (with 6 lags)

$$(2.10) \quad (m-p) = -3.79 + 1.09 (gny-p) \\ (2.79) \quad (0.24)$$

**Figure 4** Granger-Engle error correction

### Developing a parsimonious error correction model

Having established the extent and form on cointegrating relationships between the variables of the model we now proceed to estimate an over-parameterized error correction model. Though the Engle-Granger Theorem establishes the encompassing power of the error-correction model over other forms of dynamic specification when cointegration exists it does not reveal any information about the nature of the dynamic processes *around* the embedded long-run solution. Consequently the initial specification of the error-correction model should set the lag length on all variables to as long as the data will permit.<sup>35</sup> Equation (2.11) reports the initial over-parameterized error correction model, where, in general,  $x_t$  stands for  $x_{t-1}$ .

At this stage the over-parameterized model is difficult to interpret in any meaningful way: its main function is to allow us to identify the main dynamic patterns in the model and to ensure that the dynamics of the model have not been constrained by a too short lag length. To check the latter we calculate a series of F tests against information from each lagged period. The tests are against the null that the information at period  $t-i$  (ie the value of the vector  $X_{t-i}$ ) is not significant in determining the current period value of the dependent variable (Table 7).

**Table 7** Tests on the Significance of each LAG

LAG	F Statistic	Value	Probability
5	F [5, 28]	0.429	.825
4	F [5, 28]	1.801	.145
3	F [5, 28]	1.536	.211
2	F [5, 28]	0.766	.582
1	F [5, 28]	5.716	.001

On the basis of these F tests we can conclude that the significant information in the model is contained principally at the first lag and to a less extent at the 4th lag, but the information content of the data falls off

<sup>35</sup> Note that the ECM term *must* be lagged one period since the current value of the ECM is not a valid regressor (its inclusion will render the model singular).

dramatically beyond the fourth lag. This pattern not only accords with our expectations for a model of quarterly money demand, but allows us to accept the equation as an effectively unrestricted model.

We can now proceed to simplify the model into a more interpretable, and certainly more parsimonious, characterization of the data. Our simplification and reduction strategy is guided by a number of considerations, most of which are principally statistical rather than economic. The purpose initially is to ensure data admissibility and then consider whether the model is consistent with theory - and if not, why not. Since we are dealing with a linear model then *any* linear transformation of the variables is a valid re-parameterization of the model. Thus, in simplifying the model we use a combination of parameter restrictions (principally setting certain parameters to zero) and transformations, the validity of which can be assessed and tested at each stage. In terms of the overall validity of the reduction sequence we seek to maximize the goodness of fit of the model with the minimum number of variables: this is essentially the definition of parsimony. The Schwarz Information Criterion provides a guide to parsimonious reductions and is defined as

$$SC = \ln\sigma^2 + k\ln T/T,$$

where  $\sigma^2$  is a degree-of-freedom adjusted equation standard error,  $k$  the number of parameters, and  $T$  the sample size.

The Schwarz Criterion is increasing in  $\sigma^2$  and in  $k$ : thus a fall in SC is an indication of model "parsimony".<sup>36</sup> In addition, at each stage of the reduction we can re-assess the model in terms of the diagnostic tests (for autocorrelation, heteroscedasticity) and for parameter stability. We shall return to this below.

Initially we attempt to reduce the size of the model by imposing zero coefficients on those lags where the 't' statistic is low in equation (2.11). The result is given in Eq (2.12)

<sup>36</sup> Note that SC can only be used for the reduction sequence for an individual model. Since it is constructed on model-specific values ( $\sigma$ ,  $k$  and  $T$ ) it cannot be used for comparison across models.

EQ(2.11) Modelling  $\Delta(m-p)$  by OLS : sample 1974 (3) to 1989 (2)

Variable	Coefficient	Std error	H.C.S.E.	t-Value	Partial $r^2$
$\Delta(m-p)1$	.3133	.1492	.1769	2.0992	.1449
$\Delta(m-p)2$	.4927	.1527	.1779	3.2254	.2858
$\Delta(m-p)3$	.2475	.1375	.1168	1.7989	.1107
$\Delta(m-p)4$	.5278	.1556	.1657	3.3922	.3068
$\Delta(m-p)5$	.1656	.1852	.1830	.8944	.0299
$\Delta p$	-1.8316	.3031	.3590	-6.0417	.5840
$\Delta p$ 1	.1108	.3747	.3092	.2958	.0034
$\Delta p$ 2	.2595	.3508	.3624	.7397	.0206
$\Delta p$ 3	-.2045	.3967	.4479	-.5156	.0101
$\Delta p$ 4	1.2627	.3880	.4224	3.2539	.2894
$\Delta p$ 5	-.7062	.4192	.3733	-1.6844	.0984
$\Delta(gny-p)$	-.0886	.1270	.1559	-.6975	.0184
$\Delta(gny-p)1$	-.3163	.1678	.1508	-1.8841	.1201
$\Delta(gny-p)2$	-.1623	.1498	.1492	-1.0829	.0432
$\Delta(gny-p)3$	.0749	.1189	.1312	.6303	.0151
$\Delta(gny-p)4$	-.0129	.1094	.1002	-.1180	.0005
$\Delta(gny-p)5$	-.1063	.1018	.0959	-1.0438	.0402
$\Delta rtb$	-1.5854	1.8933	2.1393	-.8373	.0263
$\Delta rtb$ 1	-4.4065	1.8422	1.8156	-2.3919	.1804
$\Delta rtb$ 2	-2.0053	1.6989	1.5691	-1.1803	.0509
$\Delta rtb$ 3	-1.2925	1.7617	1.9139	-.7336	.0203
$\Delta rtb$ 4	2.8946	1.9108	1.3876	1.5149	.0811
$\Delta rtb$ 5	.4372	1.8009	1.8032	.2428	.0023
$\Delta ex$	.3141	.1412	.1709	2.2237	.1598
$\Delta ex$ 1	.0715	.1503	.1374	.4760	.0086
$\Delta ex$ 2	.2807	.1203	.1372	2.3323	.1730
$\Delta ex$ 3	.0300	.1122	.0914	.2678	.0028
$\Delta ex$ 4	.2448	.1016	.0817	2.4087	.1824
$\Delta ex$ 5	-.0429	.1053	.1218	-.4077	.0064
ecm 1	-.4492	.1143	.1142	-3.9275	.3724
CONSTANT	.1120	.0239	.0239	4.6757	.4568
Q 1	-.1671	.0279	.0291	-5.9777	.5788
Q 2	-.1600	.0334	.0360	-4.7788	.4676
Q 3	-.0523	.0250	.0233	-2.0920	.1441

$R^2 = .9485$      $\sigma = 0.0266$      $F(33, 26) = 14.52 [ .0000]$      $DW = 2.059$   
 Schwarz Criterion  $SC = -5.769159$

Notes: HCSE = heteroscedastic consistent standard error  
 't' Statistics are calculated on the basis of the unadjusted 't' statistic.  
 The F statistic is against the null that all coefficients = 0

EQ(2.12) Modelling  $\Delta(m-p)$  by OLS : sample is 1974(3) to 1989(2)

Variable	Coefficient	Std error	H.C.S.E.	t-value	Partial $r^2$
$\Delta(m-p)$ 1	.1800	.1086	.1300	1.6569	.0575
$\Delta(m-p)$ 2	.3235	.1046	.1023	3.0923	.1753
$\Delta(m-p)$ 4	.3493	.1349	.1450	2.5894	.1297
$\Delta p$	-1.3992	.2302	.2338	-6.0759	.4507
$\Delta p$ 4	.7092	.2847	.2555	2.4903	.1211
$\Delta(gny-p)$ 1	-.0776	.1038	.0995	-.7475	.0123
$\Delta rtb$ 1	-3.8306	1.5506	1.7759	-2.4703	.1194
$\Delta rtb$ 2	-3.0842	1.6490	1.7736	-1.8703	.0721
$\Delta ex$	.1913	.1168	.1275	1.6374	.0562
$\Delta ex$ 4	.2450	.0978	.0998	2.5040	.1223
$ecm$ 1	-.3085	.0777	.0930	-3.9693	.2593
CONSTANT	.0940	.0221	.0249	4.2494	.2864
Q 1	-.1372	.0198	.0189	-6.9161	.5153
Q 2	-.1384	.0278	.0275	-4.9768	.3550
Q 3	-.0383	.0177	.0205	-2.1609	.0940

$R^2 = .8857$   $\sigma = .0301$   $F(14, 45) = 24.92$  [ .0000]  $DW = 2.127$

Schwarz Information Criteria:  $SC = -6.268027$

Eq (2.12) allows for a clearer interpretation of the dynamic process. The first feature to notice is the well defined error-correction term,  $ecm_{t-1}$  which indicates a feedback of approximately 30% of the previous quarter's disequilibrium from the long-run income elasticity of demand. The strong significance of the coefficient on  $ecm$  supports our earlier conclusion that money and gross national income are indeed cointegrated. Second, we note the virtual absence of any short-run income effect on money holdings (other than through the error-correction effect), and that all the price effects have the expected signs: current period inflation has a sharply negative effect on money balances in the current period, but a smaller offsetting lagged effect. These two results are consistent with the "buffer-stock" models of money holdings popularised by Akerlof and Millbourne (1980) and Cuthbertson (1988) which are characterised by low or zero short-run income elasticities of demand and a high inflation elasticity. Finally, we note that the two terms capturing the rate of return on competing assets are correctly signed<sup>37</sup> with both suggesting that there is a degree of substitution from non-interest bearing Kenyan notes and coin into interest-bearing Kenyan financial assets and into illegal holdings of foreign currency.

<sup>37</sup> Note that a rise in the exchange rate is a devaluation.

As a result of this initial simplification we have managed to produce a more parsimonious and interpretable model. Though the  $R^2$  has fallen slightly, and the equation has risen from 2.7% to 3.0% this has been offset by a halving in the number of explanatory variables, so that the SC has fallen from -5.77 to -6.28. It is however possible to achieve even greater parsimony from this model as a number of testable restrictions on the structure of the model immediately present themselves. Note first that the coefficients on the domestic interest rate are virtually the same at both lags,  $\Delta rtb1$  and  $\Delta rtb2$ . Clearly we can re-express these terms as  $\gamma_1 rtb_{t-1} - \gamma_1 rtb_{t-2} + \gamma_2 rtb_{t-2} - \gamma_2 rtb_{t-3}$  and if the data accepts the restriction that  $\gamma_1 = \gamma_2$  then the dynamic interest rate effect can be captured by the second difference term  $\Delta^2 rtb_{t-1} = rtb_{t-1} - rtb_{t-3}$ . In other words, the demand for money balances is a function of the bi-annual growth in nominal interest rates.

Similar reasoning can be used to examine possible transformation of the three lagged dependent variable terms  $\Delta(m-p)1$ ,  $\Delta(m-p)2$ , and  $\Delta(m-p)4$  which can be approximated by a four-period moving average, which we denote  $MA4\Delta(m-p)$  and define as  $\sum \Delta(m-p)/4$   $i=0...3$ . Again this transformation is intuitively plausible: the fact that it is derived from the data as an aspect of the model design reflects our initial observations about the relationship between the role of economic theory in informing notions of equilibrium behaviour and the data describing the behaviour of agents around that equilibrium. Finally we also consider a further re-parameterization of the effect of the exchange rate change by taking the annual depreciation of the exchange rate,  $\Delta^d ex$ . With the exception of the short run income elasticity this term is the least well supported by the data, and it is likely that a more detailed specification of the currency substitution effects may be expected to improve on the specification.

### Diagnostic testing and stability analysis

Imposing these restrictions leads us to our final version of the model which purports to provide a "tentatively adequate characterization of the data". It is reported as equation (2.13). Examination of the coefficients of equations 2.12 and 2.13 confirms the validity of these restrictions, while the  $\sigma$  and Schwarz Criterion indicate a resulting improvement in parsimony. The model tracks the data well over the sample period (see Figure (5)). The array of further diagnostic tests<sup>38</sup> indicates that the model is consistent with the data. There is no evidence of first or higher-order autocorrelation in the equation errors while the other statistics support the view that the distribution of the error term

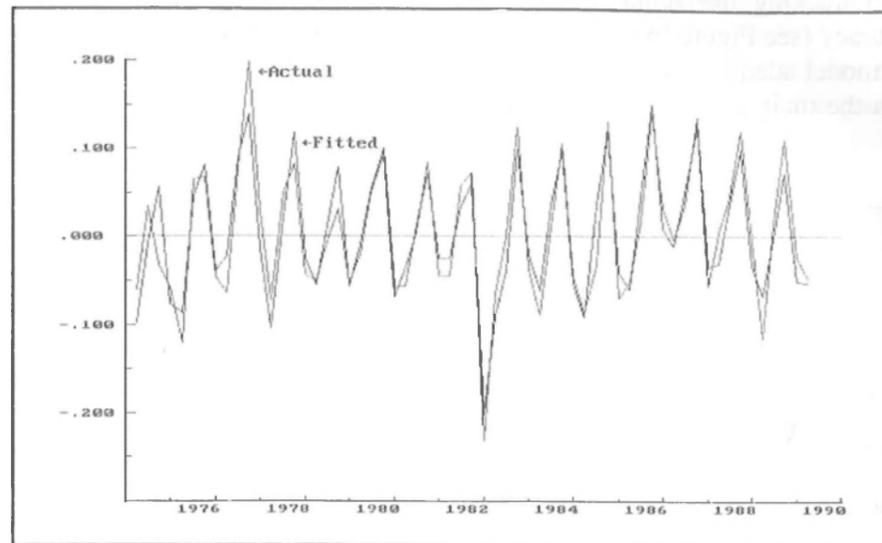
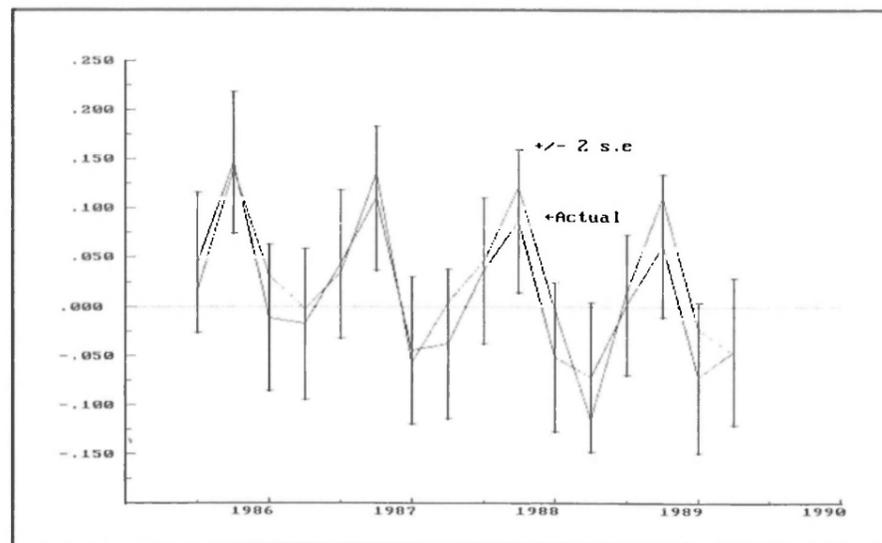
<sup>38</sup> The details of these tests are explained in the Appendix.

is independently and homoscedastically normal. Evidence of within-sample forecast accuracy further supports this hypotheses with the model estimated to 1985 tracking the actual data from 1985 to 1989 with a high degree of accuracy (see Figure (6)). We can conclude, therefore, that for the full sample the model adequately capture the salient features of the data and is consistent with the main implications of economic theory.

EQ(2.13) Modelling  $\Delta(m-p)$  by OLS: sample is 1974 (2) to 1989 (2)

Variable	Coefficient	Std error	H.C.S.E.	t-value	Partial $r^2$
$\Delta p$	-1.3588	.2311	.2214	-5.8781	.4039
$\Delta p$ 4	.5773	.2420	.2215	2.3855	.1004
ecm 1	-.2920	.0709	.0676	-4.1171	.2495
$\Delta^2 rtb$ 1	-3.2928	1.0627	1.3568	-3.0983	.1584
$\Delta^4 ex$	.0908	.0493	.0439	1.8407	.0623
MA $\Delta(m-p)$ 1	.6796	.2395	.2225	2.8368	.1363
CONSTANT	.1105	.0123	.0147	8.9428	.6106
Q 1	-.1389	.0124	.0116	-11.1940	.7107
Q 2	-.1494	.0115	.0126	-12.9323	.7663
Q 3	-.0689	.0117	.0143	-5.8712	.4033
R <sup>2</sup> = .8599		$\sigma$ = .0295	F( 9, 51) = 34.78 [ .0000]	DW = 2.017	

Schwarz Information Criteria: SC = -6.418636

**Figure 5** Equation 2.13 actual vs fitted**Figure 6** Forecast stability 1985 to 1989

**Diagnostic Tests**

Jarque - Bera Test for Error Distribution

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 Chi-Square(2) = 2.279
 

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Residual Correlogram: [Order 7]

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LAG	1	2	3	4	5	6	7
COEFF.	.1096	-.0513	-.1132	-.0434	.0813	.0259	-.1647
S.E.s	.1610	.1624	.1643	.1645	.1512	.1515	.1492

F( 7, 37) = .38 [ .9070]

Lagrange Multiplier Serial Correlation Tests

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F ( 1, 50) =	.04 [ .8488]
F ( 2, 49) =	.05 [ .9550]
F ( 3, 48) =	.49 [ .6912]
F ( 4, 47) =	.37 [ .8307]

---

Lagrange Multiplier Autoregressive Conditional Heteroscedasticity

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F (1, 49) =	.03 [ .8572]
F (2, 47) =	.35 [ .7040]
F (3, 45) =	.43 [ .7308]
F (4, 43) =	.28 [ .8875]

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Quadratic Heteroscedasticity (White's)

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F (15, 35) =	.4648 [ .9429]
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## Regression Specification [RESET]

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F (1, 50) =	.002 [.9610]
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Note values in [ ] indicate probability

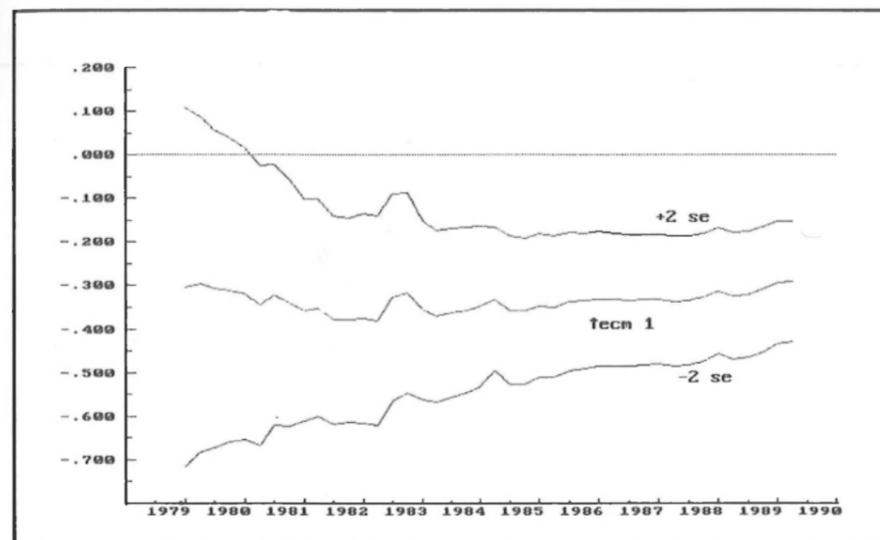
The foregoing tests have, however, concentrated exclusively on the full-sample characteristics of the model and its parameters. In interpreting this equation we are implicitly assuming that the full-sample coefficients have remained stable and at these levels throughout the entire period under investigation. It is entirely possible that this has not been the case and thus inferences drawn on the strength of the full-sample estimates may be invalid. Moreover non-constancy of the parameters over the period may indicate that the implicit exogeneity conditions of the model are in fact not valid. We finally turn our attention to the recursive stability of the model. We therefore re-estimate the model using the Recursive Least Squares estimator (see Appendix II) to test whether there has been any significant change in the value of the coefficients of model throughout the period by estimating the model over the period from 1973(1) to 1977(4) and then recursively thereafter quarter by quarter. The resulting series of recursive estimators can then be analyzed for their stability. Though recursive estimates are calculated for every variable in the model we plot the recursive estimates for only two variables, the error correction term, and the current period inflation term  $\Delta p$ . In the case of the former (Figure (7)) the plot is very steady and any movements in the value have not been significant.<sup>39</sup> In other words, at no point in the period did the feedback to the long run equilibrium differ significantly from 0.30. On the other hand the plot for the inflation term (Figure (8)) shows a marked, but only just significant break in 1982 where the value of the coefficient fell from approximately -1.00 to -1.30. In the case of all the other coefficients the plots revealed a strong degree of stability, while as Figures 9 and 10 indicate neither the recursive residuals of the equation, which indicate that at no point was the one period equation error statistically significant, while the one-step Chow tests for the entire sample also indicate that over the period the model never failed to explain changes in real money balances.

Before leaving these results however, there remains a concern that in the light of Figure 7 that the slight instability of the  $\Delta p$  coefficient values may be

<sup>39</sup> At no point in the sample does the value of the coefficient lie outside the narrowest point of the confidence interval.

due to the endogeneity of inflation in the demand for money model and hence represent a violation of the weak exogeneity of inflation in the model. We therefore conclude this paper by re-estimating Equation 2.13 by the method of Generalised Instrumental Variable Estimation (GIVE), and allowing for  $\Delta p$  to be an endogenous variable. Equation 2.14 reports the results of the structural equation for money demand, where SEASONS captures all three seasonal dummies, and LRUSD is the US domestic interest rate.

Figure 7 RLS estimates ecm 1



Comparison of Equations 2.13 and 2.14 indicate that though there are some differences in the coefficients<sup>40</sup> these differences are not statistically insignificant, and we can therefore accept that for this data set that prices are indeed weakly exogenous with respect to the demand for narrow money, and that the slight parameter instability revealed in the Recursive plots is not significant. Taken together with the full-sample results this recursive analysis indicates a highly acceptable level of coherence with the data.

40 Note, however, that due to the presence of lagged instruments the GIVE estimation is over a slightly shorter period.

EQ(2.14) Modelling  $\Delta(m-p)$  by GIVE : Sample 1977( 1) to 1989( 2)

1 endogenous and 7 exogenous variables with 17 instruments

Variable t-value	Coefficient	Standard error	
$\Delta p$	-1.16799	.38004	-3.0733
$\Delta p$ 4	.46326	.23214	1.9956
ecm 1	-.24437	.07105	-3.4394
$\Delta^2 rtb$ 1	-2.99989	1.02307	-2.9323
$\Delta^4 ex$	.07617	.05476	1.3908
MA4 $\Delta(m-p)$ 1	.58345	.25887	2.2539
CONSTANT	.11067	.01213	9.1259
SEASONS	1.00823	.06942	14.5226

New instruments used: LRUSD LRUSD 1 LRUSD 2 LRUSD 3 LRUSD 4

Structural  $\sigma$  = .0277740 DW = 1.959  
 Reduced Form  $\sigma$  = .03915686  
 Specification  $CHI^2( 9)/ 9$  = 1.07

Figure 8 RLS estimates dp

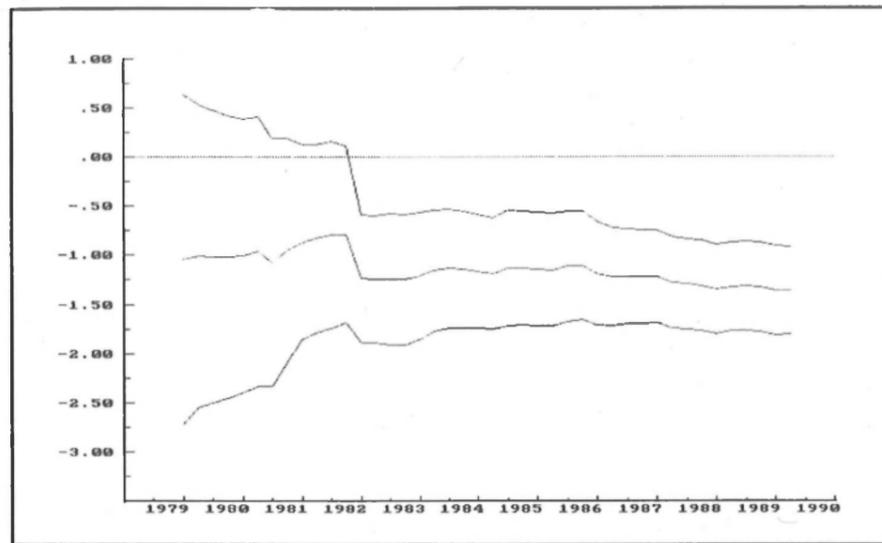
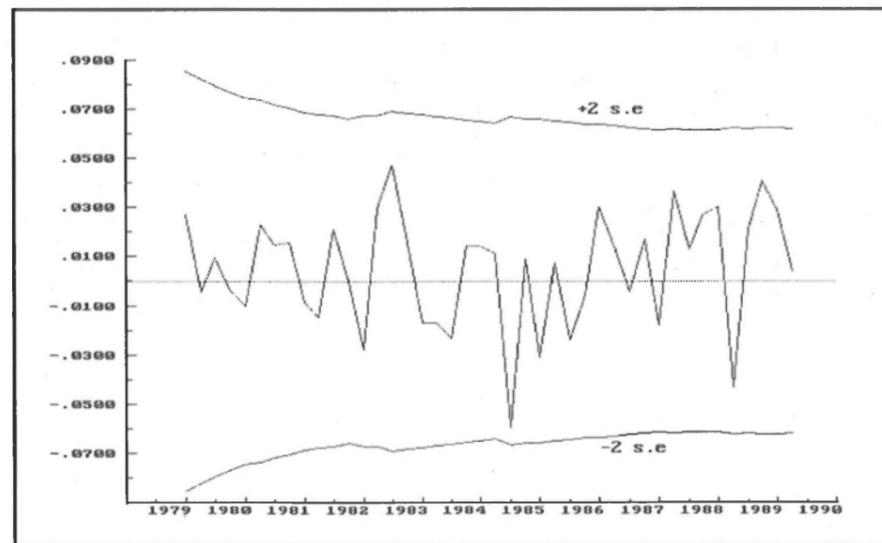
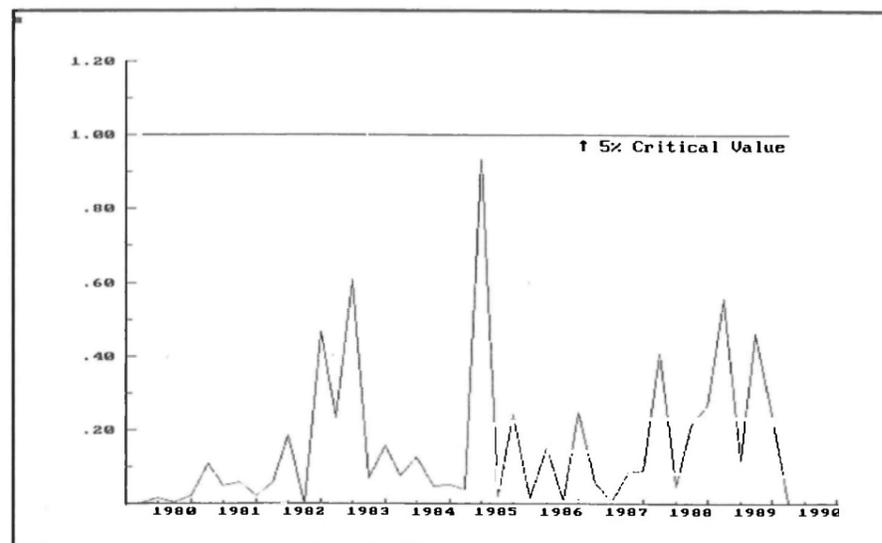


Figure 9 Recursive residuals



**Figure 10** One-step Chow tests

### Conclusions

In the spirit of the methodology described here we conclude by leaving the model a hostage to fortune. The adequate performance of this specification against such a battery of diagnostic tests is a necessary, but not sufficient condition for accepting the model. These tests are simply design criteria, and ultimately the model can be maintained only to the extent that it continues to explain out-of-sample data and to encompass rival models. The passage of time will provide the data for out-of-sample testing; rival specifications must, however, come from other researchers in the field.

## Annex I

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### Diagnostic Tests

The purpose of this Annex is to provide an intuitive interpretation of the main tests employed in the PC-GIVE package (which has been used in this paper), and to direct the reader towards the main references which explain the theory behind the tests in more detail.

#### *The LM autocorrelation test: AR(m)*

Unless all regressors are strongly exogenous, and unless the error process is strictly first order, (ie AR(1)), the standard Durbin Watson statistic is not a sufficient test for autocorrelation. The LM test is a general test for autocorrelation allowing for the case where higher order lagged dependent variables are included as regressors, and when the error process is AR(m) where m can be of any value (PC-GIVE allows for up to 13th order error processes). The test is distributed as  $\chi^2$ , although a sample-adjusted F form test is also reported. In small samples (ie less than 50 to 100 observations), the latter form is preferred (see Kiviet (1985)).

#### *The LM ARCH test: ARCH (m)*

ARCH stands for Autoregressive-Conditional Heteroscedasticity. An ARCH process is one where the errors have a zero mean, but where the errors are of the form

$$(A1.1) \quad h_t = \alpha_0 + \alpha_1 \varepsilon_t + \alpha_2 \varepsilon_{t-1}^2 + \dots + \alpha_m \varepsilon_{t-m}^2,$$

where  $h_t$  in this case is a first-order ARCH process. The ARCH test is a test of the null  $H_0: h_t = \alpha_0$  against an alternative that  $h_t$  is a first- or higher-order ARCH process. The ARCH(m) test is distributed in the same way as the AR(m) test, and again the F form is preferred in small sample models.

In the case of the LM tests the presence of significant AR or ARCH behaviour in the error term<sup>41</sup> indicates that there is some systematic information in the DGP which the model fails to pick up. Thus where previously the presence of autocorrelation (regardless of its order) may induce the researcher to "correct" for it by using the Cochrane Orcutt method, current thinking would suggest that this is inappropriate. Presence of autocorrelation (or other systematic behaviour) would induce the researcher to return to the specification of the model and to re-parameterize the model in a more efficient way.

#### *Jarque-Bera normality test*

This test is a direct test of the distribution of the error terms (recall that the efficiency and consistency of OLS is based on the normal distribution of the error terms). The JB test uses the first four moments of the distribution (mean, standard deviation, skewness, and excess kurtosis) along with the minimum and maximum values of the series, to construct a distribution. This is then compared against the equivalent value produced by the standard normal distribution. The difference is distributed as a  $\chi^2$  statistic. It may be noted that the JB test is of relatively low power, and visual analysis of the residuals is a necessary complement to the test.

#### *Heteroscedasticity tests*

PC-GIVE provides two additional heteroscedasticity tests, both of which are more general than the ARCH test. Thus both test the null  $H_0: \text{var}(y_i/x_i) = \sigma^2$ , ie that the standard error is independent of  $x_i$ , against a more general alternative that  $\sigma^2_i = g(x_i)$ , where  $g$  is some, undefined, function. White's (1980) tests the null of homoscedasticity against the alternative hypothesis that the squared residuals from the OLS estimated equation are a function of the squares of the regressors. The test has F Distribution. This test (the results of which are also reported as HCSE (heteroscedastic conditional standard errors) in the main equation output, can be complemented by a slightly more specific test which tests the null against the alternative of heteroscedasticity where  $g(x_i)$  is specified as a function of the cross-product of all regressors,  $X_i X_j$ , for all  $i, j$ .

41 PC-GIVE provides significance values for each statistic in the F Distribution.

*RESET test*

The RESET (or regression specification test) is used to detect mis-specification due to non-linearities in the model. Suppose the null model is  $y_t = X_t'\beta$ . The RESET test tests this model against the alternative of the form

$$(A1.2) \quad y_t = X_t'\beta + \gamma_1 y_t^2 + \gamma_2 y_t^3 + \dots + \gamma_m y_t^{m+1} + \varepsilon_t.$$

The null  $H_0: \gamma_1 = \gamma_2 = \dots = \gamma_m = 0$  is tested against  $H_1: \gamma_i \neq 0; i=1..m$ . As above, the RESET test is reported as an F test.

*Omitted variables tests*

The standard omitted variable test is of the following form. Given the null  $H_0: y_t = X_t'\beta$ , an alternative is constructed as  $H_1: y_t = X_t'\beta + \gamma Z(L)_t$ , where  $Z$  is some other variable and  $L$  the lag operator. The null  $H_0: \gamma = 0 \forall t$ , and is distributed as an F test. It is important to note that significant values for the omitted variable test indicate invalid marginalization of the GDP and require the researcher to return to the original ADL model at re-commence the general-to-specific search.

*Encompassing tests*

There are two main forms of encompassing test, the nested and non-nested. Models are said to be nested when one model is a special case of the other. Nested encompassing tests can be mimicked by using the F test statistics for omitted variables (see above). The non-nested encompassing tests presented in PC-GIVE are based on two competing models

$$(A1.3) \quad M_1: y = X\beta_1 + v_1 \quad v_1 \sim (0, \sigma_1^2)$$

$$(A1.4) \quad M_2: y = X\beta_2 + v_2 \quad v_2 \sim (0, \sigma_2^2)$$

PC-GIVE estimates these two models and reports their standard error along with the standard error from the model formed by the joint linear nesting model formed from the combination of  $M_1$  and  $M_2$ , called the joint model. Under the null that  $M_1$  encompasses  $M_2$  the Cox test is distributed normally. A significant value means a failure to encompass. If both models fail to encompass each other, then the Cox statistic is significant for both  $M_1$  vs  $M_2$  and  $M_2$  vs  $M_1$  (Note that for two models to fail to encompass each other is

valid and is evidence of two weak models). The Ericsson and Sargan tests are used to evaluate the validity of the instruments and more relevant to IV estimation, while the F test is a test of each model against the joint nesting model.

## Annex II

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### The recursive least squares estimator

Standard OLS estimation techniques implicitly assume that the parameters of the model

$$(A2.1) \quad y_t = \beta' X_t + \varepsilon_t \quad t = 1 \dots T$$

i.e.  $\Theta = (\beta, \sigma^2)$  are invariant over time. If, however, the processes generating  $y_t$  and  $X_t$  are from non-identical distributions then we must admit the time dependence of  $\Theta$ , i.e.  $\Theta = (\beta_t, \sigma_t^2)$ . Standard approaches to testing whether  $\Theta$  is time dependent have focused on the use of split sample Chow-type tests. This is a relatively indirect test and is highly dependent on the choice of break-point. More direct evidence can be gained through the use of a recursive estimation method.

If an estimator  $\beta$  is calculated on the first  $k$  observations it can be updated as new observations are added to the data set. There are two important advantages of this technique: i) RLS allows the OLS estimators to be tracked over time and allow the researcher to assess the stability of individual model parameters; ii) since the residuals of the RLS estimation process are well-behaved this allows a richer set of tests to be constructed to evaluate the model.

#### *Updating formula - the RLS estimator*

Consider the model  $y_t = X_t' \beta + u_t \quad t = 1 \dots T$

From this model the OLS estimate  $\beta_t$ , based on the first  $t$  observations, can be written in the normal way as

$$(A2.2) \quad \beta_t = (X_t' X_t)^{-1} X_t' y_t$$

where  $X_t = (x_1, x_2, \dots, x_t)'$  and  $y_t = (y_1, y_2, \dots, y_t)$

Suppose the estimator for  $\beta$  were computed on the first  $t-1$  observations (where  $t-1 \geq k$ ), such that

$$(A2.3) \beta_{t-1} = (X'_{t-1} X_{t-1})^{-1} X'_{t-1} y_{t-1}$$

Once the  $t$ th observation is available,  $\beta_t$  may be obtained from  $\beta_{t-1}$  and  $y_t$  without going through the matrix inversion process by using the following formula.

$$(A2.4) \beta_t = b_{t-1} + (X'_{t-1} X_{t-1})^{-1} x_t (y_t - x'_t \beta_{t-1}) / f_t$$

where  $f_t = 1 + x'_t (X'_{t-1} X_{t-1})^{-1} x_t$

The term  $(y_t - x'_t \beta_{t-1})$  is the one period-ahead prediction error, and contains all the new information required to update the estimator  $\beta_{t-1}$ . Note that the final estimator  $\beta_T$  obtained by the RLS method is identical to that obtained by the full-sample OLS method.

The RLS estimator produces graphical as well as numerical representation of the value of the estimate of each coefficient from  $t=k$  to  $t=T$  together with the one-step (2 standard error) confidence interval. The graphical output provides a visual assessment of the implicit null hypothesis  $H_0: \beta_1 = \beta_2 = \beta_3 \dots = \beta_t = \dots = \beta_T$ , which can be interpreted in terms of the stability of the coefficient plot, its significance relative to zero, and its efficiency (the extent to which the confidence interval reduces over time).

Suppose the estimator for  $\beta$  were computed on the first  $t-1$  observations (where  $t-1 \geq k$ ), such that

$$(A2.3) \beta_{t-1} = (X_{t-1}' X_{t-1})^{-1} X_{t-1}' y_{t-1}$$

Once the  $t$ th observation is available,  $\beta_t$  may be obtained from  $\beta_{t-1}$  and  $y_t$  without going through the matrix inversion process by using the following formula.

$$(A2.4) \beta_t = \beta_{t-1} + (X_{t-1}' X_{t-1})^{-1} x_t (y_t - x_t' \beta_{t-1}) / f_t$$

where  $f_t = 1 + x_t' (X_{t-1}' X_{t-1})^{-1} x_t$

The term  $(y_t - x_t' \beta_{t-1})$  is the one period-ahead prediction error, and contains all the new information required to update the estimator  $\beta_{t-1}$ . Note that the final estimator  $\beta_T$  obtained by the RLS method is identical to that obtained by the full-sample OLS method.

The RLS estimator produces graphical as well as numerical representation of the value of the estimate of each coefficient from  $t=k$  to  $t=T$  together with the one-step (2 standard error) confidence interval. The graphical output provides a visual assessment of the implicit null hypothesis  $H_0: \beta_1 = \beta_2 = \beta_3 \dots = \beta_t = \dots = \beta_T$ , which can be interpreted in terms of the stability of the coefficient plot, its significance relative to zero, and its efficiency (the extent to which the confidence interval reduces over time).

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