A Model of A Dynamic Islamic Economy and the Institution of Interest
About the Book

Fourth in the series of Essays in Islamic Economic Philosophy published by the Pakistan Institute of Development Economics, this essay discusses the problem of *riba* (interest) in the framework of a dynamic Islamic economy. Since Islam is opposed to the institution of interest as a fixed rate of return on investment on which all modern economies operate, Muslim economists are faced with the problem of replacing interest with an alternative which is in accord with the moral and ethical principles of Islam.

The study concludes that, in general, what holds for any other dynamic economy is also true of an Islamic economy. We cannot abolish a positive rate of interest by simply ordering that the price of capital be zero in an Islamic economy nor can we hope to achieve the Islamic ideal by simply legislating that all financial investments ensuring a fixed rate of return on investment be replaced by those which promise only variable rates of return.

An essential feature of any set of viable policy alternatives to the institution of interest is that, in order to fulfil the Islamic ethical requirement of *al 'Adl*, steps are taken to ensure inter-generational equity, and that adequate provision is made, in all ‘states of the economy’, to safeguard the economic interest of the least-privileged class in the society. It follows that, from both economic and ethical points of view, much dubiousness attaches to financial reforms that have the effect of consolidating the hold of big capitalists on the financial market while crowding out persons of small means — i.e. those who cannot afford to take any risks whatsoever. Indeed, such ‘reforms’, far from taking us to the Islamic paradise, will land us in a capitalistic hell!
A Model of A Dynamic Islamic Economy and the Institution of Interest

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"It is not righteousness that ye turn your faces towards East or West; but it is righteousness to believe in Allah and the Last Day, and the Angels, and the Book, and the Messengers; to spend of your substance, out of love for Him, for your kin, for orphans, for the needy, for the wayfarer, for those who ask, and for the ransom of slaves; to be steadfast in prayer, and practise regular charity; to fulfil the contracts which ye have made . . ."

— *al Qur'an* (2:177)
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INTRODUCTION TO THE SERIES

In the unitary Islamic perspective, economic activity is seen as only a subset of a wider human effort to usher in a just society based on Divine ethical principles, which are immutable because "There is naught that can change His words" (6:116). These principles are required to weave the kaleidoscopic 'facts' of life into a recognizable pattern and provide a formula for orderly social change based on justice. The Divine Law of al 'Adl symbolizes a set of Islamic principles, which acts as the norm to evaluate the justice or otherwise of existing social institutions. Being the opposite of Zulm, the Divine Law rules out the acceptance of social injustices even as a pretext for economic and social progress. Within the framework of a relatively absolute freedom of the individual, which rests on Man's theomorphic character, the Islamic commitment to maximizing social welfare is total. Any resistance to social change coming from the vested interest has been unequivocally condemned in the Holy Qur'an: "And leave Me (Alone to deal with) Those in possession of The good things of life, who (yet) deny the Truth..." (73:11)\(^1\)

That ethics must be brought into the ‘picture’ explicitly should be clear from the fact that Islam’s is a philosophy of the ‘right’ — i.e. it evaluates ‘what is’ with reference to ‘what ought to be’. The real challenge that Muslim economists must face lies in taking this ethical principle as the point of departure for a systematic and scientific enquiry into the fundamental rules of economic behaviour in an Islamic economy. The present series on “Islamic Economic Philosophy” seeks to provide a forum for a non-apologetic and scientific debate, which, by focusing on the interface of ethics and economics, works out the ‘rules of the game’ that economic agents must ideally follow in the pursuit of Islamically defined social bliss.

The objective of this debate should be to evolve gradually the outlines of an Islamic economic system, with reference to which the veracity of specific statements about such a system can be verified. That such search must obey certain constraints of Islamic legitimacy must be clearly recognized by the researchers in this area. However, to make any progress at all it is important to define a minimal set of such binding constraints; or else there will be not too many “degrees of freedom” left for scientific enquiry. In this search after the truth, Muslim economists will have to separate the fundamental from the subsidiary principles and the objectives from the policies designed to achieve them. The contributors to this series will be expected to make such distinctions, even if these sometimes appear to be only hair-splitting. This will be a lot better than a light-hearted cataloguing of what cannot be done.

Editor

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INTRODUCTION
Chapter I

INTRODUCTION

One of the most ticklish problems in Islamization schemes pertains to replacement of the institution of interest by a set of alternative viable policy instruments. While there is unanimity among Muslim scholars that there is no place for interest in the Islamic vision of socio-economic process, opinions differ about the alternative set of institutions with which to replace it, about its viability, and about its compatibility with the ideals of an Islamic economic system.

The purpose of the present Essay is to expand and elaborate the theme presented in Naqvi [3] — that the problem of interest in an Islamic economy is a problem in dynamic optimization. Such an analytical framework has the advantage that, by its very composition, it formally incorporates the ethical assumptions underlying intertemporal investment decisions, e.g. the concept of intergenerational equity, and allows a rigorous analysis of the meaning and consequences of Islam’s unequivocal insistence on the primacy of ethics over economics. Furthermore, this

1 To avoid controversy, interest rate is taken, throughout this Essay, as synonymous with riba. For a comprehensive discussion of the meaning and purpose of riba, based on original sources, see Ziaul Haq [1].

analytical framework discounts the oft-repeated suggestion about the inevitability of the institution of interest and, instead, it states the minimal set of functions that must be adequately performed within the framework of a dynamic, capital-scarce economy, including an Islamic economy. We are thus led to view the novelty of the problem posed by Islam not in terms of the rules of the game of a dynamic economy, but in terms of the policy instruments that must be devised to play the 'game' in a manner consistent with Islam's ethical perception.

In this context, an essential feature of any set of viable policy alternatives to the institution of interest is that, in order to fulfil the Islamic ethical requirement of al 'Adl, steps are taken to ensure intergenerational equity, and that adequate provision is made, in all 'states of the economy', to safeguard the economic interest of the least-privileged class in the society. One characteristic of the members of this class is that they are risk-avers, i.e. if they are allowed to choose between alternative financial arrangements, they would unambiguously prefer the one that guarantees them a fixed rate of return on their investment (saving). It follows that, from both economic and ethical points of view, much dubiousness attaches to financial reforms that have the effect of consolidating the hold of big capitalists on the financial market while crowding out persons of small means.

\(^3\)In the context of long-run equilibrium positions – i.e. stationary states – capital scarcity is defined as a state of affairs in which an additional amount of resources permits an alternative stationary state, which allows a higher consumption of some commodities and no lower consumption of any other commodity, in all future periods. If this state of affairs does not exist, we get the state of capital saturation, which is perfectly consistent with a zero rate of interest. For a discussion of this, see Koopmans [2, pp. 122-123]. However, the Islamic injunction against interest does not necessarily require capital saturation.
i.e. those who cannot afford to take any risks whatsoever. Indeed, such 'reforms', far from taking us to the Islamic paradise, will land us in a capitalistic hell! This is a situation that even capitalistic countries do not look forward to for their economic salvation. It must be clearly understood that risk-taking is not an absolute good in itself. On the contrary, to the extent that such an 'activity' is undertaken in an Islamic economy, its 'value' must be minimized.

Once the problem is seen in this way, two important conclusions immediately follow: (i) interest rate cannot be wished away by an administrative fiat, without making far-reaching arrangements in the financial and real sectors for an effective performance of the set of functions required for dynamic equilibrium; and (ii) it is a non sequitur that, since interest rate is bad, any policy which replaces it must ipso facto be good. The optimality of the alternative set of policies replacing the institution of interest must be established independently in terms of its superior merits in achieving the stated objectives of economic policy in an Islamic economy. In this connection, we examine and reject the notion that the problem can be solved by legislating a substitution of interest by profits throughout the economy. We hold the view that such a substitution is neither a necessary, nor a sufficient condition for the establishment of an Islamic economic system. This is especially the case if such a financial reform is taken to mean a replacement of a regime of fixed and variable rates of return on investment with one that promises only variable rates of return. In other words, there is no guarantee that once such a reform has been carried out, any real-life economy will thereby be transformed, as if by the flourish of a magic wand, into an Islamic
economy. Indeed, one can easily think of model economies where investible resources are generated by activities of variable-returns type, e.g. those based on gambling, betting, etc., which are definitely not Islamic. The challenge facing Muslim economists and policy-makers is not that of forcing all classes of investors (savers) into the procrustean bed of an economy where only variable rates of return on investment are available, but rather that of devising financial instruments which, in the absence of interest, safeguard the interests of all classes of investors (savers), both risk-averters and risk-takers.

The present Essay is spread over seven chapters. The second chapter presents the model of a dynamic economy. It features a dynamic constrained-optimization model, which is used to 'separate' the institution of interest from its 'functions'. It will be shown that when a fully dynamic analysis is carried out, interest rate in dynamic equilibrium must be equal to a sum of three terms: (i) a positive rate of social time-preference (i.e. present income being systematically preferred to future income); (ii) the social cost of using capital for productive purposes; and (iii) the secular decline in the utility (or increase in the disutility) of saving owing to its growth over time. Chapter 3 presents some characteristics of a model Islamic economy along with a somewhat modified version, which is due to Ramsey, of the model outlined in Chapter 2. The modified version differs from the basic model by the absence in it of positive (private) rate of time-preference. It is shown in these chapters that the model presented in Chapter 2 is an adequate representation of an Islamic economy. It is also shown that uncertain situations can be handled by the model, which uses
Hamiltonian equations, by ‘reducing’ them to ‘as if certain’ situations.

Consistent with the general approach of this Essay, Chapters 4 to 6 examine the economic non-viability of alternative ‘solutions’ to the problem. We have examined and rejected the possibilities of abolishing interest by an administrative order, or by legislating a mechanical shift from an interest-based economy to a profit-based economy. Chapter 5 offers a fairly complete analysis of the problem posed by the existence of risk and uncertainty. It is shown that in any Islamic solution financial instruments guaranteeing a fixed rate of return must coexist with those promising variable rates of return. This proposition runs counter to the position taken by many Muslim economists — namely, that an Islamic economy will feature only variable rates of return. Chapter 6 shows that a dynamic Islamic economy must make adequate arrangements to perform all the three functions noted above. An active role of the government is noted in this context. The final chapter draws together the threads of the main argument of the Essay.

REFERENCES


A MODEL OF A DYNAMIC ISLAMIC ECONOMY
Chapter 2

INTEREST RATE AND ITS FUNCTIONS

In this chapter and the next we present an outline of a dynamic Islamic economy and its main characteristics. These chapters specify the problem of abolishing — or, more accurately, replacing — the institution of interest as a problem in dynamic optimization: an appropriate utility functional, incorporating Islamic ethical norms, is maximized subject to the constraint of maintaining optimal economic growth over a specified time interval. The solution of this problem yields, among other things, the desired ‘separation’ of the institution of interest from its functions in dynamic equilibrium. This is done in the present chapter. The next chapter will bring out other interesting insights that the model provides into the characteristics of a dynamic Islamic economy.

It is interesting to recall in this context that a necessary condition for dynamic equilibrium is that the rate of interest must equal the marginal productivity of capital [1]. This apparently simple rule requires making complicated inter-temporal adjustments in the basic parameters of the economy. A clear recognition of these parameters is, therefore, the main step in effecting an analytical ‘separation’
between the institution of interest and its essential functions. The analysis in this Essay is conducted primarily in terms of a ‘real’ dynamic economy. This procedure avoids complicated mathematics, without losing much of the generality of results obtained. For a positive (real) interest rate is essentially a relative price which equals the marginal rate of transformation between the present price of capital and the future price of capital, which, being a ratio, is independent of the unit of account in which it is expressed. Furthermore, in determining the movement of key economic magnitudes – e.g. level of saving, extent of investment, degree of employment – it is the real rate of interest that counts.¹

The next analytical step is to introduce money into the system and analyse the consequences of it in terms of the essential functions that interest performs. A full analysis of the problem is a complicated matter. However, at the level of generality of the present Essay, a highly ‘stylized’ treatment of the problem should be enough. The outcome of such an analysis, presented at the end of this chapter, is that for a distortion-free economy in dynamic equilibrium money does not matter.

INTEREST RATE IN A DYNAMIC ‘REAL’ ECONOMY

A positive rate of interest arises in a dynamic real economy because the society, at a given point of time, must make an explicit choice between the amount it wishes to consume now and that which it chooses to consume in the

¹ For instance, to increase (personal) savings in an inflationary period, the money rate of interest must be kept higher than the rate of inflation to keep the real rate of interest positive. The reverse chain of events is warranted in a deflationary period.
future. Since investment is essentially postponed consumption, such a choice entails a decision regarding the optimal intertemporal allocation of investment. The problem is made difficult by the fact that, depending on the initial conditions, (e.g. the initial level of capital stock, in the present context) dynamic equilibrium is consistent both with too little saving and with too much saving; and a variety of feasible time paths of consumption satisfy the requirements of the steady-state growth. Hence the need for finding an optimal growth path — in a loose sense, the 'best' among the many 'good' ones.

Complications arise because the society must simultaneously examine the discounted sum of the utility of consumption over the entire planning period and keep the economy growing at the planned rate by making adequate provision for capital formation over the same period. In mathematical parlance, the problem is to maximize a current-value utility functional, subject to the constraints of technology and restrictions on initial conditions relating to the size of the pre-existing level of capital stock.

To bring out the role of interest rate in the above-mentioned dynamic constrained-maximization problem,

2 The treatment in this section has drawn on Arrow [1] and Dorfman [3].

3 A functional is a mapping from a set of functions into a set of functions, just as a function is a mapping from a subset of real numbers into real numbers. As such, it is distinct from a function of a function, which, given a real number, yields a real number. Given a function (or curve), a functional yields another function (or curve). For this reason, we deal with functionals rather than with functions in dynamic analysis which is the subject-matter of the calculus of variations and Pontryagin's Maximum Principle. For an exposition of the latter, see Pontryagin et al. [6].
consider a simple 'real' economy with the following characteristics.\(^4\) The production function is given by

\[ Y = f[K, L], \ldots \ldots \]  

(1)

where \(Y\) is output, \(K\) is capital, and \(L\) is labour. It is assumed that the production function is 'well-behaved' and that the labour force is growing exponentially at a rate \(\gamma\), so that

\[ L(t) = L_0 \exp(\gamma t), \ldots \ldots \ldots \]  

(2)

When full employment is assumed, the discounted utility of per capita consumption, \(x\), for labour force is given by

\[ u[x] = \int_0^T \left[ \exp(\gamma - \alpha)t \right] \phi[x] dt, \ldots \ldots \ldots \]  

(3)

where \(\alpha\) is the social rate of time-preference, which is independent of time, and \(\phi[x]\) is the utility of per capita consumption.\(^5\) (The finite time-horizon problem can be converted to an infinite time-horizon problem by taking the limit \(T \to \infty\).)

The aim of public policy should be to maximize \(u[x]\) relative to the 'state' variable, namely capital per capita. This is done by manipulating the 'control' variable — namely per capita consumption — subject to a dynamic constraint. The dynamic constraint is given by

\(^4\)The model presented in the text makes the restrictive assumption of 'malleability', i.e. capital can be shifted costlessly from one line of production to another in order to secure an optimal capital/labour ratio.

\(^5\)The variable \(x\) is a function of time, as are the variables \(\nu\) and \(p\), introduced later. To avoid cluttering of symbols, we drop the explicit representation of time-dependence of these variables, e.g. \(\phi[x(t)]\) is written as \(\phi[x]\).
\[ \dot{v} = \psi[v] - x - (\alpha + \beta)v, \ldots \ldots \ldots \quad (4) \]

where the 'state' variable \( v \) is the capital per capita, \( \psi[v] \) is the productivity functional of the capital per capita, \( \beta \) is the rate of capital depreciation, and \( \dot{v} \) denotes \( dv/dt \).

Note the chain of causation. Starting at time \( t = 0 \) and the initial value of 'state' variable, \( v_0 \), an optimal initial value of \( x > x_0 \) is chosen. This gives a positive \( \dot{v} \) in Equation (4). In this way the time-path is traced both of the 'state' variable \( v \) and of the 'control' variable \( x \).

Equations (3) and (4) constitute a typical constrained-optimization problem, which is solved with the use of Pontryagin's Maximum Principle, involving the Hamiltonian

\[ H[x, v; p] = u[x] + p\dot{v} \]

\[ = \exp \left\{ (\gamma - \alpha)t \right\} \phi[x] + p \left\{ \psi[v] - x - (\beta + \gamma)v \right\} \ldots \ldots \ldots \quad (5) \]

where \( p \), the Lagrange multiplier, is a dynamic 'shadow' price in the sense of giving the marginal contribution of the 'state' variable, \( v \), to total utility, i.e. \( \frac{\delta \phi}{\delta v} \). (Note that \( \frac{\delta \phi}{\delta v} \) signifies functional differentiation with respect to \( v \).) To maximize the objective functional, the Hamiltonian must satisfy the Hamilton equations:

\[ \frac{\delta H}{\delta v} = -\dot{p}, \]

\[ \frac{\delta H}{\delta p} = \dot{v}, \quad \ldots \ldots \ldots \ldots \quad (6) \]

\[ \frac{\delta H}{\delta x} = 0. \]
Also, the Hamiltonian must be extremal with respect to the 'control' variable. For the Hamiltonian given by Equation (5), these conditions yield

\[ \exp\left[(\gamma - \alpha)t\right] \cdot \phi'[x] = p, \quad \ldots \quad \ldots \quad \ldots \quad (7) \]

\[ p\left[\psi'[v] - (\gamma + \beta)\right] = -\dot{p}, \quad \ldots \quad \ldots \quad \ldots \quad (8) \]

where \(-\dot{p}\) is the time rate of capital depreciation and \(\phi'\) and \(\psi'\) denote the functional derivatives of the respective functionals relative to the corresponding functions. We rearrange Equation (8) and eliminate \(p\) by computing the time derivative of \(p\) in Equation (7), to get

\[ \psi[v] = \alpha + \beta - \frac{\phi''[x]}{\phi'[x]} \dot{x}. \quad \ldots \quad \ldots \quad \ldots \quad (9) \]

This equation gives the rule for optimal capital accumulation.

It may be noted that this rule is limited to selecting the 'best' from a set of feasible steady-state positions corresponding to pre-assigned initial conditions; it does not necessarily prescribe the most preferred course of action, given any arbitrary initial position.

Now, recall the basic result for a growing economy in a steady state:

\[ i(t) = \psi'[v]. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10) \]

In words, interest rate, \(i(t)\), at any given point in time, equals the marginal productivity of capital.\(^6\) This makes

\(^6\)This rule follows directly by differentiating the production function, assumed to be linear homogeneous, with respect to capital (\(K\)).
sense because, as pointed out by Arrow [1], investment can be regarded as a short-term loan of an arbitrarily short duration. Substituting Equation (10) into Equation (9), we get

\[ i(t) = \alpha + \beta \left( \frac{\phi''[x]}{\phi'[x]} \right) \dot{x}. \ldots \ldots \ldots (11) \]

To be in dynamic equilibrium, the rate of interest is the price that the society must pay to offset a positive rate of social time-preference, and capital depreciation, and to provide for the increasing marginal disutility of saving owing to its secular growth. It must be noted that Equation (11), reflecting as it does the future movements of the profitability of investment throughout the economy, underscores the highly important fact that resource allocation of today is determined by our perception of the future. As noted by Frisch [4], the first and third terms in Equation (11) are Bohm-Bawerk's second and first grounds for the existence of a positive interest rate: a positive \( \alpha \) reveals 'the systematic under-valuation of future consumption', while the third term gives 'the decrease in the marginal utility of consumption owing to its growth'.

**INTEREST RATE IN A DYNAMIC MONEY-ECONOMY**

Let us now introduce money into the system. This complicates the picture considerably because money, in effect, introduces a second good into a single-commodity economy, which we have been assuming so far.

\footnote{It is interesting to note in this connection that the Keynesian rule, which compares the marginal efficiency of investment now with the rate of interest now, is not correct because new investment is marginal to capital. [See 2, Ch. 3.] This is an important point which has been missed by those authors who mistakenly quote Keynes to justify, on economic grounds, Islam's prohibition of interest rate.}
A simple, though entirely legitimate, way of translating Equation (11) into an optimal rule for a dynamic money-economy is to assume that the shadow (relative) price of capital is also the market price of capital. This is a reasonable assumption because in a steady state there will be no 'distortion' in the system — i.e. the marginal rate of substitution (in a closed economy) will equal the marginal rate of transformation. By Equation (7) the term $\dot{p}(t)/p(t)$ will indicate the rate of inflation, once we make the additional assumption that while $\dot{p}(t)/p(t) > 0$, i.e. the relative price of capital is rising, all other relative prices are either constant or also rising. With this additional assumption, an increase in the relative price of capital will lead to an increase in the general price level.

It may be noted that, with inflation rate defined as $\dot{p}(t)/p(t)$, we do not get any additional terms in the fundamental Equation (11). This may be checked by differentiating Equation (7) with respect to time. The reason is that $i(t)$ appears only through $\psi'[\nu]$ in Equation (8). (The implications of differentiating Equation (7) will be discussed in Chapter 4.) In other words, within the framework of the assumptions made in this section, the money rate of interest, $i_m(t)$, in dynamic equilibrium will be equal to the real rate of interest, so that Equation (11) holds in a money-economy as well, i.e.

$$i_m(t) = i(t) = \alpha + \beta \left[ \frac{\phi' \cdot [x]}{\phi' \cdot [x]} \right] \hat{x} \quad \ldots \quad \ldots \quad (11')$$

A full analysis of the consequences of introducing (fiat) money into the model is, of course, a more complicated matter than is indicated by Equation (11). As has been
shown by Jovanovic [5], inflation tends to reduce welfare by lowering optimal per capita consumption below the level indicated by the Golden Rule of Accumulation. The latter result flows from the fact that inflation increases the cost of holding money-balances. How does this result affect Equation (11)? It should be obvious that only the size of the third term of the equation is affected, though not necessarily its (negative) sign, the other terms being constants. Hence, there is a possibility that $i_m(t) < i(t)$. However, for the moment we rule out this possibility by allowing “inside money” into the model, which, as Jovanovic shows, will drive out fiat money and ensure that Equation (11') strictly holds.

REFERENCES


Chapter 3
BASIC CHARACTERISTICS OF
A DYNAMIC ISLAMIC ECONOMY

The fulcrum on which must rest the lever that is to move an Islamic economy is the Divine Law of al 'Adl.\(^1\) The Divine Law connotes a grand balance of forces, or (normative) equilibrium, at the level of the Absolute, in the Universe and on the plane of social existence. ‘Rational’ individual behaviour must conform to this Law to ensure social justice. This conformity of individual behaviour to the dictates of social welfare is brought about, in accordance with the principle of al 'Adl, by enjoining a balance between an individual’s freedom and his social responsibility. This conformity manifests itself in an explicit recognition of the “rights” of the “deprived” and the “oppressed” in the wealth of the rich so that the needs of the least privileged in the society are adequately met in all ‘states of the economy’. By making economic behaviour subservient to the overriding dictates of ethical norms, the Divine Law enjoins a restructuring of the basic institution of the society in such a manner that these “rights” of the oppressed and the deprived are met. Since the Divine Law is timeless, it must be satisfied at a given point in time as well as intertemporally. Any

\(^1\) For a detailed discussion of the assertions made in this paragraph, see Naqvi [3, 4].
model of an Islamic economy must reflect this basic property of the Divine Law.

In the present chapter we examine some basic characteristics of the model presented in Chapter 2 with reference to the existence of a positive time preference in the model. Also, the suitability of the model in situations of risk and uncertainty will be investigated.

**BASIC CHARACTERISTICS OF THE MODEL**

In a dynamic context, the ethical principle of al 'Adl will require that equity in consumption be maintained not only within a generation but also intergenerationally. The model outlined in Chapter 2 provides an appropriate framework within which to incorporate the Islamic ethical principles. The utility functional $u[x]$ can easily be interpreted as incorporating the requirements of the 'target' groups whose consumption requirements must be met at all times; for the dynamic-equilibrium position, defined by Equation (6), represents a state of the economy in which as additional resources become available with economic growth, the consumption of the least privileged in the society will be higher while the consumption of all other income groups will be no lower [see footnote 3, Ch. 1]. An explicit identification of the target group in the utility functional will considerably complicate the mathematics. But there is no conceptual difficulty in carrying out such an exercise.

The solution procedure outlined in Chapter 2 should also be noted in this context. It will be recalled that the

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2See Jan Tinbergen [6], where he advocates the use of ethical principles as an integral part of welfare economics to resolve the problem of interpersonal comparison of utilities without the need to measure them.
problem is solved by manipulating the ‘control’ variable $x$ (per capita consumption). It can easily be stipulated that $x$ is manipulated in a manner prescribed by Islam. Such a stipulation will ensure that both the rate of growth of the economy and the optimal allocation of investment between generations will also be determined in the prescribed manner. Remember that the rate of investment, which signifies the movement through time of the ‘state’ variable (capital stock), depends on the size of the capital stock and on the allocation at any given time between per capita consumption and the capital requirements of a growing labour force and capital depreciation.

(i) The Problem of Positive Preference

It may also be noted in this connection that a positive $\alpha$ in the utility functional should pose no problem from an Islamic point of view because discounting future consumption is essentially a satisfactory method of comparing utilities associated with different levels of per capita consumption, without having to measure them explicitly. To bring out this point, let us consider an alternative formulation of $u[x]$, due to Ramsey [5], which explicitly postulates, on ethical grounds, a zero rate of time-preference and is written as

$$
u[x] = \int_0^\infty [B - \phi[x]] \, dt.$$. \hspace{1cm} (12)

The functional $u[x]$ is the difference between the “bliss” level of consumption ($B$) and the utility of the actual level of per capita consumption, $\phi[x]$. To make this utility functional Islamically acceptable, it can, again, be stipulated that the bliss level of consumption makes an adequate provision for the consumption of the poor. As before, the
problem is defined as maximizing Equation (12), subject to the constraint given by Equation (4), with $\gamma = \alpha = 0$.

\[
\dot{v} = \psi[v] - x - \beta v. \quad \ldots \quad \ldots \quad \ldots \quad (13)
\]

The solution of the problem is obtained, as outlined in Chapter 2, by applying the Maximum Principle,$^3$

\[
\phi'[x] = p \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)
\]

\[
p \left( \psi'[v] - \beta \right) = -\dot{p} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)
\]

Using Equation (14) to eliminate $p$ from Equation (15), and keeping in view Equation (10), we get

\[
i(t) = \beta - \left( \frac{\phi''[x]}{\phi'[x]} \right) \dot{x} \quad \ldots \quad \ldots \quad \ldots \quad (16)
\]

It may be noted that Equation (16)$^4$ differs from Equation (11) only by the absence of $\alpha$ in the latter. This is because, according to Ramsey [5], the government, representing the society, should not discount the future because, unlike individuals, it is not naturally myopic. However, authors like Marglin feel that it is undemocratic for governments not to discount the future when individuals do so. After all, democratic governments must reflect the preferences of the individuals in the society.

The difficulty with such a view is that in order to reflect the preferences of the individuals, the government must find

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$^4$ As pointed out in Chapter 2, Equation (16) also holds in money economy since $i_m(t) = i_0(t)$, where $i_m(t)$ is the money rate of interest.
a way of averaging them, which will require positing axioms based on an irreducible value judgement. This is because, according to Arrow's Impossibility Theorem, no such rational deduction is possible from democratic premises. One of the tasks of Muslim economic policy-makers will, therefore, be to devise axioms, based on the teachings of Islam, reflecting the ethically-conditioned preference-ordering of individuals in an Islamic society. This is important because, unlike a communistic regime in which individuals do not matter, an Islamic society does not neglect the role of individual initiative. It also seeks a non-dictatorship solution to the problem of maximizing social welfare.

In our view, it is more plausible to assume, ethically and economically, that myopic discounting should find a place in an Islamic economy, as in other economies, because a zero time-preference amounts to imposing a too heavy consumption sacrifice on the present generation for the sake of posterity — in violation of the Islamic tenet of al 'Adl. It follows that, strictly on ethical grounds, $\alpha$ should be positive in an Islamic economy.

The next question is: Granted that $\alpha > 0$, how high should it be in an Islamic society? While we cannot give an authoritative answer, there is some presumption that it will, in general, be low. For Islam denounces in no uncertain

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5 However, this point should not be used as a justification for the capitalistic mentality which glorifies egoistic behaviour and for which also Islam has little room. Islam allows for only constrained freedom, where social responsibility dictates commitment, rather than egoism, as the ideal form of the individual's economic behaviour. See Naqvi [3].

6 It may be noted that when the rate of interest is zero — i.e. the cost of transferring future income to present consumption is zero — a positive (negative) time-preference implies negative (positive) saving. A zero or neutral time-preference will, therefore, result in zero saving. Thus a policy of reducing $\alpha$ to zero will only reduce the level of dissaving in the economy owing to this factor, when it is positive and high. It may also be noted that, with a positive time-preference, dissaving will occur at low rates of interest.
terms the squandering away of wealth on vain and conspicuous consumption. However, to make such a presumption operational, the basic parameters of the underlying Islamic economic model will have to be spelt out clearly. Furthermore, this presumption requires that the Islamically legitimate levels of consumption be defined and systematically enforced through a conscious government policy.

(ii) The Problem of Uncertainty and Risk

It has been objected⁷ that the model outlined in Chapter 2 is unsuitable for an analysis of the problem in an Islamic context; for the Hamiltonian equations are based on the assumption of perfect certainty, while a model of an Islamic economy should feature uncertainty as a prominent part of the economic universe.

This argument is based on a misunderstanding of the analytical technique used in this Essay; for the Hamiltonian equations can be used to comprehend uncertain situations as well. Note that we can write the ‘uncertain’ element in \( i(t) \) as a ‘certain’ \( i_c(t) \) plus a stochastic term \( e(t) \). The ‘uncertainty’ in \( i(t) \) is, then, just the scatter given by

\[
\delta = \frac{\int_{t_1}^{t_2} e^2(t) \, dt}{t_2 - t_1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)
\]

Thus \( i(t) \) can be statistically expected to be \( i_c(t) \pm \alpha \) in the time interval \( (t_1, t_2) \). Provided that \( \delta \) is sufficiently small, we can deal with \( i_c(t) \) instead of \( i(t) \). This procedure is adopted, for example, in quantum mechanics where uncertainties have to be included in all calculations.

⁷See Anas Zarqa [7].
A theorem, due to Ehrenfest,\(^8\) shows that the expectation values of all quantum quantities follow the classical laws,

\[
\begin{align*}
\frac{d}{dt} \langle q_i \rangle &= \langle \frac{\delta H}{\delta p_i} \rangle, \\
\frac{d}{dt} \langle p_i \rangle &= \langle \frac{\delta H}{\delta q_i} \rangle,
\end{align*}
\]

where \(q_i\) and \(p_i\) are the canonically conjugate variables, \(t\) is time and \(H\) is the Hamiltonian. In economics, the canonically conjugate variables can be interpreted as "quantity" \((q_i)\) and the "shadow price" \((p_i)\). As in quantum mechanics, in economics, too, there is no guarantee of certainty in individual cases but we are nevertheless very sure of the average outcome. In other words, we can talk of an uncertain situation as a certain situation on the average, even though we cannot do so for individual cases.

The upshot of the discussion is that uncertain situations can be dealt with if we convert their average values, through risk minimization, into 'as if certain' situations. It may be noted that the requirement of minimizing uncertainty (and risk) is the only position consistent with Islamic economic philosophy. If this were not so — that is, if uncertainty was a good in itself — then gambling, betting and speculative practices, in which the element of risk is very large, would not have been declared haram (prohibited) by Islam.

\(^8\)The effect of the fluctuation denoted by \(\delta\), called "quantum fluctuation" in quantum mechanics, would have to be taken into account. This point is considered in Chapter 5, where risk-taking is discussed in more detail. For a discussion of Ehrenfest's theorem, see [2].
REFERENCES


REPLACING THE INSTITUTION OF INTEREST
Chapter 4

ABOLISHING THE INSTITUTION OF INTEREST IN AN ISLAMIC ECONOMY

The analysis presented in Chapter 2 was limited to separating analytically the institution of interest from the minimal set of the essential functions that it performs to keep the economy in dynamic equilibrium. The next question is: exactly what policy instruments will be required, in the absence of the institution of interest, to perform these functions in an Islamic economy? It is proposed in this chapter to discuss briefly the consequences of abolishing the institution of interest by an administrative order, without providing an alternative mechanism for pricing capital in an Islamic economy.

ADMINISTRATIVE FIAT

To replace the institution of interest, one possible policy option is to abolish it by an administrative order. Our analysis has made one thing absolutely clear: in a capital-scarce economy, interest rate can never be abolished by an administrative fiat without making provision for alternative financial and real institutional arrangements to ensure that its
essential functions, identified above, are performed effectively. If such arrangements are not made, all capital will depreciate without replacement through unrestricted capital consumption. This observation can be easily proved.

Theorem 1: The utility of per capita consumption increases exponentially over time if interest rate is set equal to zero.

Proof: Putting interest rate equal to zero in Equation (11), we have

\[
\frac{\phi''(x)}{\phi'(x)} \dot{x} = \alpha + \beta. \quad \ldots \quad (19)
\]

Integrating both sides of the equation yields

\[
\ln \phi'[x] = \ln A + (\alpha + \beta)t, \quad \ldots \quad (20)
\]

where \( \ln A \) is a constant of integration. Clearly, \( A > 0 \), as \( \ln A \) is not defined for \( A \leq 0 \). Since the integration is over time, \( A \) cannot be a function of time. Further, since \( x \) is a function of time, \( A \) cannot be a function of \( x \). Thus, on exponentiating,

\[
\phi'[x] = A \exp [(\alpha + \beta)t], \quad \ldots \quad (21)
\]

Multiplying both sides by \( \dot{x} \), we obtain

\[
\dot{x} = A \exp [(\alpha + \beta)t] \dot{x}. \quad \ldots \quad (22)
\]

Now, clearly, \( \dot{x} \) must be a non-decreasing function of time; for we cannot let per capita
consumption fall over time. Since $\alpha, \beta < 0$, the time rate of change of $\phi[x]$ is at least exponentially increasing. For this to be true $\phi[x]$ must, at least, be an exponentially increasing function of time.

Q.E.D.

The result is intuitively clear. Making interest rate zero by administrative fiat will result in a maximal marginal utility of consumption or – which is equivalent – a maximal marginal disutility of saving. Since saving must equal investment in dynamic equilibrium, it follows that in this case investment will be minimized. This cannot possibly be the objective of an Islamic economy, particularly as Islam does not encourage profligate squandering of wealth.

Next consider the consequences of abolishing interest through an administrative fiat when the possibility is allowed for a change in price level.¹

Theorem 2: If interest is zero, the (market) price of capital, $p(t)$, will increase exponentially.

Proof: Using Equations (8) and (10), we can write

$$\frac{\dot{p}(t)}{p(t)} = -[i(t) - (\gamma + \beta)]. \quad \ldots \ (23)$$

Integrating Equation (23) over time yields

¹For a statement of the conditions under which a rise in the market (relative) price of capital leads to a rise in the general price level, see the last section of Ch. 2 of the present Essay.
\[ \ln[p(t)] = -\int i(t) dt + (\gamma + \beta)t + \ln p_o, \quad (24) \]

where \( \ln p_o \) is a constant of integration.

Exponentiating Equation (24) gives

\[ p(t) = p_o \exp\left[(\gamma + \beta)t\right] \exp\left[-\int i(t) dt\right] \]

\[ \ldots \quad \ldots \quad \ldots \quad (25) \]

Now, if \( i(t) = 0 \) in Equation (25) we see that

\[ p(t) = p_o \exp\left[(\gamma + \beta)t\right] \quad \ldots \quad \ldots \quad (26) \]

Q.E.D.

This result implies that if interest rate is arbitrarily set equal to zero, the price level will increase exponentially over time at a rate which is the sum of the rates of capital depreciation and labour growth.

The analysis presented here clarifies the simple, though often neglected, point that the Islamic position is not so much the abolition of interest, as it is its replacement by an appropriate set of policies.\(^2\) This is because, in a capital-scarce economy, the shadow price of capital can never be zero. However, it does not imply that an administrative order will not be involved in replacing the institution of interest.

\(^2\) For a clear statement of this important 'clarification', see M.S. Khan [1] and Naqvi [2; Ch. 7].
REFERENCES


Chapter 5

FIXED AND VARIABLE RATES OF RETURN IN AN ISLAMIC ECONOMY: THE PROBLEM OF RISK AND UNCERTAINTY

The observations made in the preceding chapters show that the answer to the problem at hand is not to abolish interest by any means whatsoever. The set of policies replacing interest must satisfy two basic conditions: (i) they must meet the requirements of a dynamic economy; and (ii) they must be consistent with the Islamic requirement of al'Adl.

In this chapter we ask the question that lies at the centre of the debate about a model Islamic economy: can an economy-wide replacement of interest by profit meet these two conditions? Since there is no difference of opinion among Muslim economists about the question of replacing the institution of interest, we rephrase the question to ask whether a model Islamic economy is an economy based on fixed and variable returns on investment or whether it is one which is based only on variable rates of return. Our answer to the question is that an economy based only on variable rates of return is not viable. It is also not ethically acceptable since it violates the dictates of the Divine Law.

To understand the reasoning behind these assertions, let us consider two extreme examples of an institutional reform
according to which investments (savings) yield only variable rates of return but which does not satisfy the dictates of Islamic legitimacy. First consider a hypothetical economy in which corporate retained earnings (profits) are the only source of financing capital formation. In such an economy, the processes of saving and investment are brought together, so to speak, under one roof. A positive rate of interest is not required here to directly\(^1\) promote saving or to redirect it into alternative investment channels. The consumer in effect ‘grants’ an interest-free loan, through the market, to the capitalist who then invests these funds directly into productive channels. Under this arrangement, saving originates in the corporate sector as a result of a combination of fiscal, price and wage policies — all tending to swell undistributed corporate profits, which are then reinvested.\(^2\) This is done on the premise that it is more efficient to divert investible resources into the hands of those who can use them effectively, though not necessarily optimally. Such an arrangement, which induces a highly unequal redistribution of income and wealth, is obviously un-Islamic in spirit because it runs counter to the principle of \(\text{al 'Adl.}\)

Another extreme example of an economy that promises only variable rates of return on investment (savings) is one that relies completely on such activities as gambling, betting, etc., to generate all, or most, of its investible resources. This is obviously not a shining example of an Islamic reform.

\(^1\) However, the interest rate does enter \textit{indirectly} into the process of generating savings. The constellation of policies required to create the economic scenario outlined in the text includes low interest rates. But for the purposes of the argument in the text, let us \textit{assume} that this is not the case.

Imagine Monaco and Las Vegas as models of an Islamic economy! At any rate, such activities are explicitly prohibited in Islam.

These two extreme examples serve to highlight the central proposition which we prove in this chapter — namely, the purpose of Islamic reform is not, as is often accepted uncritically by most Muslim economists, to replace, throughout the economy, financial instruments ensuring both fixed and variable rates of return with those promising only variable rates of return on investment. In a complex modern society there is a continuum of risk-takers, and its financial structure must take care of the preferences of all classes of risk-takers, including those who do not want to take risk — i.e. the risk-averters who 'strictly' prefer an investment which guarantees a fixed rate of return.

The following discussion, which is quite general in its scope, presents an account of the complexity of the decision-making process in an Islamic economy under conditions of uncertainty and risk.

A Mathematical Formulation

Let the variable rate of return on investment be expressed as $P(t)$ and the fixed rate of return as $i(t)$. $P(t)$ can

More specifically, it is suggested that Islamic financial reform should aim at substituting a financial system based on equity financing alone for the existing capitalistic system based on equity and debt. See, for instance, Umer Chapra [2]. The undesirable consequences of such a 'reform' are brought out most clearly in Case III considered later in this chapter.

More accurately, of course, since the population is finite, and in any case denumerable, we do not have a continuum. However, we are anyhow regarding the population as sufficiently large to be approximated by a continuum so that calculus can be applied to its analysis.

$P(t)$ is to be distinguished from $P(t)$, which was used in previous chapters to denote the shadow price of capital.
be expressed as the sum of its expected value, \( < P(t) > \), and a stochastic term \( \epsilon(t) \) so that

\[
P(t) = < P(t) > + \epsilon(t), \quad \ldots \quad \ldots \quad (27)
\]

where \( < \epsilon(t) > = 0 \) by definition.

**Case I**

The first case to consider is one in which uncertainty tends to zero — i.e. \( \epsilon(t) \to 0 \) in Equation (27). From the fundamental equation, Equation (11), such an assumption implies that \( P(t) \) would tend to \( i(t) \) — i.e.

\[
\lim_{\epsilon(t) \to 0} P(t) = i(t) = \alpha + \beta - \left\{ \frac{\phi^{''}[x]}{\phi'[x]} \right\} \hat{x}
\]

\[
(28)
\]

In words, investors, when confronted with a choice between investment alternatives, would prefer an arrangement that guarantees fixed rates of return — call it \( P(t) \) or \( i(t) \).

It may be noted that such a substitution of \( i(t) \) by \( P(t) \) satisfies Equation (11), but it does not meet the condition of Islamic legitimacy if this condition is interpreted to mean, incorrectly in our view, that \( \epsilon(t) \neq 0 \) always.

**Case II**

Let us now consider the more general case in which \( \epsilon(t) \neq 0 \), i.e. the variance of uncertainty, \( < \epsilon^2(t) > \), which is also a measure of the risk involved in investment, has a finite value. In this case, \( P(t) \neq i(t) \). Here the investor would no longer be indifferent, as in Case I, to the choice between \( i(t) \) and \( P(t) \). It is interesting to see that here we have a case in which the alleged Islamic assumption (that \( \epsilon(t) \neq 0 \) is
satisfied, but the resulting financial arrangement \( P(t) \neq i(t) \) does not satisfy Equation (11).

Let us analyse this case in detail. It is shown below that the following fundamental theorem must hold for investors to prefer variable rates of return on their investment.

**Theorem 3:** A variable-return investment will be strictly preferred to a fixed-return investment, \( i(t) \), if the rate of return has an expectation value

\[
<\mathcal{P}(t)> < \frac{i(t)}{2} \left[ \sqrt{\frac{1 + 4\kappa <\epsilon^2(t)>}{i^2(t)}} + 1 \right],
\]

\[\ldots \ldots \ldots \] (29)

where \( \kappa \) is a measure of risk-aversion.

**Proof:** When \( <\epsilon^2(t)> \) is finite, let the rate of risk premium be \( r(t) \), so that

\[
<\mathcal{P}(t)> = i(t) + r(t).
\]

\[\ldots \ldots \ldots \] (30)

Now the risk premium must be proportional to the risk involved, which is measured by \( <\epsilon^2(t)> \), which, in turn, must be inversely proportional to \( <\mathcal{P}(t)> \). Hence we have

\[
r(t) = \kappa \frac{<\epsilon^2(t)>}{<\mathcal{P}(t)>}, \ldots \ldots \ldots \] (31)

Since \( \epsilon(t) \) is measured in the same units as \( r(t) \) and \( <\mathcal{P}(t)> \), \( \kappa \) is independent of units and is, therefore, a pure number. The risk premium is higher for someone who likes to avoid risk than for someone who does not mind taking risk.
\( \kappa \) is a measure of the degree of risk aversion, and its value is higher when the preference for risk aversion is greater. It will be noted that, apart from individual preferences, \( \kappa \) depends also on the size of the capital (saving) available in the economy. Thus \( \kappa < 1 \) if the capital stock is sufficiently large and \( \kappa > 1 \) if the capital stock is relatively small.

Inserting Equation (31) into Equation (30) yields the quadratic equation

\[
\langle P(t) \rangle^2 - i(t) \langle P(t) \rangle - \kappa \langle e^2(t) \rangle = 0,
\]

\[\ldots \ldots (32)\]

for the minimum value of the expected rate of variable returns. This equation has two solutions

\[
\langle P(t) \rangle = \frac{[i(t) \pm \sqrt{i^2(t) + 4 \kappa \langle e^2(t) \rangle}]}{2}.
\]

\[\ldots \ldots (33)\]

Since the lowest root is negative, which implies an economically unacceptable negative variable rate of return in the face of a positive fixed rate of return, the lower root does not give the least acceptable value for the latter. Hence,

\[
\langle P(t) \rangle = \frac{i(t)}{2} \left[ \sqrt{1 + \frac{4 \kappa \langle e^2(t) \rangle}{i^2(t)}} + 1 \right].
\]

\[\ldots \ldots (34)\]

Thus, for investment promising a variable return to be strictly preferred, the rate of return must be greater than that given by Equation (34).

Q.E.D.
Clearly, \( <P(t)> \), given by Equation (34), is not less than \( i(t) \). In fact, if there is any risk aversion (however small) and any risk involved (however small), \( <P(t)> \), given by Equation (34), must be strictly greater than \( i(t) \). In other words, the investors (savers) must be able to anticipate, with complete confidence, that a regime of a variable rate of return will always give them a higher reward on their investment than a regime of a fixed rate of return. This is a very important result.

Let us discuss the implications of Equation (34) in different ‘risky’ situations.

(a) High-Risk Case: Consider, first, the case in which risk is extremely high, i.e. \( <e^2(t)> \rightarrow \infty \). In this case, clearly, \( <P(t)> \rightarrow \infty \) as \( \sqrt{<e^2(t)>} \rightarrow \infty \). In words, \( P(t) \) would have to become infinite for investment to be made at all. And since there are only two factors of production in our model, capital and labour, this case also requires that wage rate \( (w) \) should tend to zero, i.e. \( w(t) \rightarrow 0 \).

These considerations drive home the point that since increasing of risk without bound would cause investment (and wages) to vanish, risk cannot be maximized. Hence risk-taking cannot be regarded as good in itself. However, given the inevitability of positive risk in any dynamic economy, including an Islamic economy, it can only be minimized.

\(^6\)Compare this result with the observation made at the end of Chapter 3 that uncertain situations can be handled by converting them into 'as if certain' situations. Here it is shown that this 'conversion' must occur.
(b) **Low Risk-Aversion Case**: We next consider the very low risk-aversion case, \( \kappa < 1 \). This situation will be characterized by

\[
\langle P(t) \rangle > i(t) + \kappa \frac{\langle e^{2}(t) \rangle}{i(t)} . \quad \ldots \quad \ldots
\]  

(35)

(c) **‘Normal’ Risk-Aversion Case**: This is the case when \( \kappa = 1 \). In this case we have

\[
\langle P(t) \rangle > i(t) + \frac{\langle e^{2}(t) \rangle}{i(t)} . \quad \ldots \quad \ldots
\]  

(36)

(d) **High Risk-Aversion Case**: On the other hand, for very high risk-aversion, \( \kappa >> 1 \)

\[
\langle P(t) \rangle > \sqrt{\langle e^{2}(t) \rangle} \sqrt{\kappa} . \quad \ldots \quad \ldots
\]  

(37)

Comparing these four cases, (a) – (d), it may be noted that for high risk-taking we get a result similar to that for high risk-aversion: \( \langle P(t) \rangle \) must be large and should increase without bound as the risk involved in making investment decisions increases. On the other hand, for low risk-taking or low risk-aversion, \( \langle P(t) \rangle \) must be greater than \( i(t) \). In any case, \( \langle P(t) \rangle \) must be strictly greater than \( i(t) \), as pointed out above.

**Case III**

Another important case to be considered is the one in which \( i(t) = 0 \). Putting \( i(t) = 0 \) in Equation (33), we get

\[
\langle P(t) \rangle > \sqrt{\langle e^{2}(t) \rangle} \sqrt{\kappa} . \quad \ldots \quad \ldots \quad \ldots
\]  

(38)
Here $\kappa$ is generally greater than unity, because there is no positive $i(t)$ to "cushion" any losses. Again, if $\kappa = 1$, we have the requirement that

$$<P(t)> > \sqrt{<e^2(t)>}. \quad \ldots \quad \ldots \quad (39)$$

As such, only capitalists would have $\kappa$ approaching unity. (Only gamblers could, in this case, have $\kappa << 1$.) Clearly this situation is highly unstable since only those who have a lot of money can afford to invest. As such, $\kappa$ would be impossibly large for the bulk of the population, and the general tendency would be for a population of the poor people to work to support a very small group of capitalists with $\kappa = 1$. The risk-aversers and the moderate risk-takers will be crowded out of the financial market. This is hardly an example of an Islamic solution.

**Case IV**

It follows that in any viable Islamic economy, $\frac{\kappa <e^2(t)>}{i^2(t)}$ must be kept sufficiently low to let Equation (35), rather than Equation (37), hold. Since $\kappa$ will be large for the bulk of the population, $<e^2(t)>$ must be kept much less than $i^2(t)$; or

$$\sqrt{<e^2(t)>> < i(t)}. \quad \ldots \quad \ldots \quad (40)$$

Equation (40) may be read in reverse as giving a condition in which the fixed rate of return must be more than the square root of the variance. Only then would the small savers be encouraged to save. For $\kappa < 1$ we should have investment available with higher risk, but we still require that

$$\sqrt{<e^2(t)>} < i(t). \quad \ldots \quad \ldots \quad (41)$$
CONCLUDING REMARKS

We thus conclude that an economy which promises variable rates of return alone is only feasible if all classes of investors are guaranteed a higher return than would obtain otherwise. This is just another way of saying that such a regime is not feasible without such a guarantee. But an Islamic economy can provide such a guarantee only if the financial instruments that offer a fixed rate of return coexist with those giving variable rates of return. This conclusion is logically impeccable and economically reasonable. It is also 'earth-shaking' because it turns the table against those Muslim economists who justify a regime which promises only variable returns on investment on the ground that, according to them, it is Islamically just and economically viable. Our discussion shows that this cannot be the case: such a regime, if it exists, will have no place for small investors (savers). Hence, such a regime is unacceptable from an ethical point of view. However, if efforts are made to make room for small investors, such a regime must cease to exist!

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Chapter 6

REPLACING THE INSTITUTION OF INTEREST IN AN ISLAMIC ECONOMY

We know now what kind of reforms will not accomplish the goal of replacing interest in an Islamic economy. In particular, we can see clearly that the replacement of a regime of fixed and variable rates of returns by one which promises only variable rates of return is neither economically viable nor ethically justifiable. The central question that must now be answered is: if the many policy options analysed in the last two chapters do not provide viable answers, then what else does? Our answer is that we must look for policy alternatives which ensure that each of the three terms unveiled by Equation (11) — namely, $\alpha$, $\beta$ and $\frac{\phi''[x]}{\phi'[x]} \cdot \dot{x}$ — remain positive. To establish this point, let us consider in this chapter the reasons why each of these terms must remain positive. Exactly what policies will ensure the positivity of these three terms is a matter of detail, which we discuss only briefly. Although it is not central to our argument, there is an a priori case for an increasing role of government in the achievement of this result. This is especially true of countries where capital markets are imperfect.
SOCIAL TIME-PREFERENCE (\(\alpha\))

It was noted in Chapter 3 that \(\alpha\) will continue to be positive in an Islamic economy. Theorem 5, discussed below, provides a rigorous demonstration of the inevitability of a positive \(\alpha\) when we allow in our model, as we must, for positive growth of the labour force. Here, as in Chapter 3, we continue to discuss the proposition in general terms.

At a moral level, habits of simpler living as enjoined by Islam would probably keep \(\alpha\) low. But this is not the same thing as asserting that individuals will divert, without limit, future income to present consumption as \(\alpha \to 0\) under the force of moral persuasion, since this implies that individuals will systematically start preferring future income to present income just because of moral restraint. This is not an economically acceptable proposition. A positive time-preference derives its plausibility from the hard economic fact that while the present income can be deployed to take advantage of profitable investment opportunities as long as marginal productivity is positive and large, future income cannot be so used. However, note that even with \(\alpha=0\), while all dissaving ceases, net saving will still be zero! To generate positive saving it is essential that \(\alpha\) should be negative. In a capitalistic system, a positive rate of interest generates positive saving by increasing the cost of diverting future income to present consumption — that is by making \(\alpha\) negative.\(^1\)

How can this task of turning a positive \(\alpha\) into a negative \(\alpha\)

\(^1\)Alternatively, one can also think of a positive rate of interest as enhancing the future income of savers by making it possible to lend present income for future consumption. In the case noted in the text, a positive rate of interest plays the role of decreasing the amounts of future income, which can be transferred to present consumption by borrowing.
be accomplished in an Islamic society, in the absence of a positive rate of interest as a policy instrument? In other words, how can positive savings be generated in the face of a positive (though much lower) $\alpha$?

The key to an understanding of the problem lies in remembering that while private time-preference will continue to be positive, there is no need for the public time-preference to be positive as well. The government is not hampered by 'optical illusion' with respect to future possibilities that individuals suffer from. Hence, generating 'net' public saving really amounts to more than offsetting the private $\alpha$, which is positive, by a government $\alpha$, which must be made negative. It, therefore, appears that the share of public saving will have to increase substantially in total saving. This is especially true of developing countries where capital markets are imperfect. But the basic point to grasp is that financial institutions, in both public and private sectors, have to be created to offset individual positive time-preference in order to generate positive saving.

**DEPRECIATION OF CAPITAL ($\beta$)**

We have already seen that an important role of a positive rate of interest is to provide for depreciation of capital. Hence, in the absence of the institution of interest, public policy must aim at making enough provision for this purpose.

This is simple enough. What else must government policy do towards this end? The answer is that, within the context of the model outlined in Chapter 2, public policy must require that $i(t)$ be not less than $\beta$. This requirement is
important. We, therefore, state it formally. The basic point here is the positivity of $i(t)$, which is defined in Chapter 5 as any financial instrument which ensures a fixed rate of return on investment.

**Theorem 4**: The function of a positive $i(t)$ is to provide for capital depreciation, $\beta$, and the increasing scarcity of capital due to the rate of increase of the labour force, irrespective of the values taken by $a$ and $\phi(x)$.

**Proof**: Notice that if we take a constant value of $i(t)$, given by

$$i(t) = \beta + \gamma, \quad \ldots \quad \ldots \quad (42)$$

the price of capital, $p(t)$, given by Equation (23), remains constant over time. Note further that Equation (42) implies that in Equation (11)

$$\gamma = a - \left\{ \frac{\phi'[x]}{\phi[x]} \right\}, \quad \ldots \quad \ldots \quad (43)$$

so that $p(t)$ does not depend on $a$ and $\phi(x)$. A (positive) constant $i(t)$ is required to offset capital depreciation and increase in labour force. If $i(t)$ falls below this minimal value, the price of capital would start to rise sharply, making positive investment increasingly costly.

Q.E.D.

It should, therefore, be clear that $i(t)$ must be kept positive through active government policy, even if its sole
purpose is to provide for depreciation of capital and an increase in the labour force.²

**MARGINAL UTILITY OF CONSUMPTION (DISUTILITY OF SAVING) OVER TIME**

\[
\left\{ \frac{\phi''[x]}{\phi'[x]} \right\} \dot{x}
\]

It should be clear that the third term in Equation (11), \( \left\{ \frac{\phi''[x]}{\phi'[x]} \right\} \dot{x} \), must be positive in an Islamic economy. To see this, we need to show that the third term has the same sign as \( \dot{x} \).

The proof of the above statement is, however, non-trivial. We will need to prove, first, a theorem which will be used to prove the above-mentioned assertion.

**Theorem 5:** Given \( \alpha, \beta \), and \( \gamma \), the utility of per capita consumption is a unique function of time for a given initial utility and rate of growth of per capita consumption.

**Proof:** We start the proof by noticing that

\[
\left\{ \frac{\phi''[x]}{\phi'[x]} \right\} \dot{x} \ (t) = \left[ \ln \phi[x(t)] \right]' \dot{x}. \quad \ldots (44)
\]

Inserting Equation (42) into Equation (11) gives

\[
i(t) = \alpha + \beta - \left[ \ln \phi'[x(t)] \right]' \dot{x}. \quad \ldots (45)
\]

To ensure dynamic equilibrium, Equation (42) must be satisfied.

²Note that, according to Theorem 5 discussed below, under such circumstances \( \alpha \) must also be positive.
Using this value,

\[ \left[ \ln \phi \left[ \dot{x}(t) \right] \right]' \dot{x} = (\alpha - \gamma). \quad \ldots \quad (46) \]

Integrating this equation over time yields

\[ \ln \phi \left[ x(t) \right] = \ln A + (\alpha - \gamma)t, \quad \ldots \quad (47) \]

Where \( \ln A \) is a constant of integration. Exponentiating this equation yields

\[ \phi \left[ x(t) \right] = A \exp \left[ (\alpha - \gamma)t \right]. \quad \ldots \quad (48) \]

We need to integrate, functionally, this equation with respect to the function \( x(t) \). Since the right side of the equation contains time, we must regard time as a function of per capita consumption. This is valid only if \( x(t) \) is a monotonic function. Now \( x(t) \) must be non-decreasing for any economy so that people's standard of living should not drop continuously. If \( x(t) \) were constant, \( \phi' [x(t)] \) would have to be independent of time (with \( \alpha = \gamma \)). This is a special case of a linear function. Otherwise, \( x(t) \) must be monotonically increasing. Thus

\[ \phi \left[ x(t) \right] = \int A \exp \left[ (\alpha - \gamma)t(\bar{x}) \right] d\bar{x}. \quad \ldots \quad (49) \]

To replace the functional integral with an ordinary integral, we need to integrate with respect to time. Thus we must write
\[ d\bar{x} = \ddot{x}(\overline{t}) \, d\overline{t}. \]  \(\ldots\) \(\ldots\)  \(50\)

Using the value in Equation (49), we can rewrite it as

\[ \phi[x(t)] = \int_{t}^{\overline{t}} A \exp\left[ (\alpha - \gamma) t \right] \ddot{x}(\overline{t}) \, d\overline{t}. \]  \(\ldots\) \(\ldots\)  \(51\)

We can write this expression as a definite integral by adding a constant.

\[ \phi[x(t)] = \int_{0}^{t} A \exp[(\alpha - \gamma)] \ddot{x}(\overline{t}) \, d\overline{t} + C. \]  \(\ldots\) \(\ldots\)  \(52\)

Now, setting \(t = 0\), we can evaluate \(C\). Hence

\[ \phi[x(t)] = \int_{0}^{t} A \exp[(\alpha - \gamma) \overline{t}] \ddot{x}(\overline{t}) \, d\overline{t} + \phi[x(0)]. \]  \(\ldots\) \(\ldots\)  \(53\)

This proves the theorem.

\[ \text{Q.E.D.} \]

To appreciate the economic significance of this result, it is useful to consider particular choices of the variation of the control variable, \(x(t)\), over time. If \(x(t)\) is a constant, then \(\phi[x(t)] = \phi[x(0)]\). From Equation (46), this requirement entails that \(\alpha = \gamma\), i.e. along the optimal path if \textit{per capita consumption is held constant, the social rate of time preference must equal the rate of growth of labour.}\n
However, if \(x(t)\) increases linearly over time at a rate \(\delta\), then
\[
\phi[x(t)] = \left[ \exp \left\{ (\alpha - \gamma)t \right\} - 1 \right] \frac{A\delta}{(\alpha - \gamma)} + \phi[x(0)] \quad \ldots \quad (54)
\]

Thus, if \( \alpha > \gamma \), \( \phi[x(t)] \) is an exponentially increasing function of time. Clearly, if \( \alpha < \gamma \), \( \phi[x(t)] \) approaches a constant

\[
\phi[x(\infty)] = \phi[x(0)] - \frac{A\delta}{(\gamma - \alpha)} \quad \ldots \quad (55)
\]

If \( x(t) \) increases exponentially at a rate \( \delta \), then

\[
\phi[x(t)] = \left[ \exp \left\{ (\alpha - \gamma + \delta)t \right\} - 1 \right] \frac{A\delta}{(\gamma - \alpha + \delta)} + \phi[x(0)], \quad \ldots \quad \ldots \quad (56)
\]

provided \( \delta \neq \gamma - \alpha \). The behaviour is similar to that given by Equations (54) and (55) according as \( \delta \geq \gamma - \alpha \). In case \( \delta = \gamma - \alpha \),

\[
\phi[x(t)] = A\delta t + \phi[x(0)]. \quad \ldots \quad (57)
\]

We see that, generally, if per capita consumption increases (decreases) over time, so does its utility, provided that the rate of increase is greater (decrease is less) than the difference between the rate of growth of labour and social rate of time-preference. Thus, we are guaranteed a positive marginal utility of consumption over time for a sufficiently high rate of per capita consumption. Dynamic equilibrium is maintained by taking social rate of time-preference equal to the rate of growth of labour.

We are now in a position to prove the claim made at the outset of this section.
Theorem 6: The marginal utility of per capita consumption over time \( \left\{ \frac{\phi''[x]}{\phi'[x]} \right\} \) has the same sign as the rate of change of per capita consumption \( \dot{x} \).

Proof: Differentiating Equation (53) with respect to time yields

\[
\phi[x(t)] = A e^{(\alpha-\gamma)t} \dot{x}(t). \quad \ldots \quad (58)
\]

Now, \( A \) is positive and the exponential function is always positive for a finite argument. Hence, the only dependence on sign is due to the time rate of change of per capita consumption, \( \dot{x}(t) \). Hence the result.

Q.E.D.

Notice that if the social rate of time-preference equals the growth rate of labour, the marginal rate of per capita consumption is directly proportional to the time rate of change of per capita consumption with no other time dependence entering in. Clearly, then, this would be a natural choice for the social rate of time-preference, \( \alpha \).

We, therefore, reach the (obvious) result that, in order to keep the third term in Equation (11), positive government policy in an Islamic economy must aim at a positive \( \dot{x}(t) \) to ensure that in the process of economic growth per capita consumption also rises.\(^3\) Furthermore, as pointed out in Chapter 3, the functional \( u[x] \) in Equation (3) will have to

\(^3\) Also, note that in the process of proving this important proposition, it was also established that \( \alpha \) must be positive in dynamic equilibrium to provide enough investible saving for rising labour force.
be constructed in a model of an Islamic economy in such a manner that the consumption requirements of the least-privileged groups in the society receive the highest priority. The need for an active government policy thus becomes even more compelling.
CONCLUSIONS
Chapter 7

CONCLUDING OBSERVATIONS

The main contribution of this Essay has been to bring into the unified framework of optimization theory certain diverse problems of ethics and economics — that is, problems relating to intergenerational equity, efficiency and growth. We maximize, by an appropriate choice of a set of policy instruments, a utility functional which incorporates the essential Islamic ethical norms, subject to the constraint of optimal economic growth. It turns out that the solution of this dynamic optimization problem also yields the necessary condition to 'separate' the institution of interest from the three (additive) essential functions it performs, viz. $\alpha$, $\beta$ and $\gamma$. We use Pontryagin's Maximum Principle to solve the problem in hand. With a few additional theorems we also establish that each of these three functions must be performed to keep (any) capital-scarce economy in dynamic equilibrium. We conclude that, in general, what holds for any other dynamic economy also holds for an Islamic economy.

That done, we can easily show what policies cannot help to 'solve' the problem: we cannot abolish a positive rate of interest by simply ordering that the price of capital be
zero in an Islamic economy, nor can we hope to achieve the Islamic ideal by legislating that all financial instruments ensuring a fixed rate of return on investment be replaced by those which promise only variable rates of return.¹ What we have done is to prove that the last-mentioned financial ‘reform’, proposed by many Muslim economists, can be made economically (and logically) plausible only by guaranteeing that the rate of reward on investment (saving) after the reform will always be higher than the corresponding rate before the reform. However, if such a guarantee is Islamically acceptable, then financial instruments bearing fixed rates of return must also be acceptable. By establishing this last result we hope to have cleared up a major source of confusion both about the nature of the problem and about the ways in which it cannot be solved. Such an analysis also gives us an inkling of fruitful ways of solving the problem.

A legitimate question arises at this point: if, as we show, the basic laws of motion are the same for all economic systems, then are we not by the same token arguing against the need for an Islamic economic system? The answer is that these two statements are not necessarily connected with each other. The need for establishing an Islamic economic system may be established independently on

¹Using a different analytical framework and a different set of arguments, Khan [2] shows that an economy based on variable rates of return alone will have to bear net “information costs” which are minimized at present by financial instruments bearing a fixed rate of return. However, it should be noted that in so far as all information gets quickly capitalized in the price of asset, it has a zero value. The recognition of this point forms the basis of “random walk” theories of financial market behavior. But, in a world characterized by “random walk”, the distribution of wealth will be highly skewed; and once inequalities in the distribution of wealth are created, there is no mechanism in such a world to correct them. This is the point we have emphasized in the text.
ethico-religious grounds, without implying that the basic laws of motion of a dynamic economy either are wrong or need a fundamental revision. Assuming, then, that the basic laws of economic dynamics are the same across different economic systems, the actual allocation of investment resources over time will be different depending on the 'weight' that is assigned to each of these functions in any specific economic system. And such systemic differences require an explicit specification of a set of institutions and a set of social policies which, in turn, may reflect distinctive ethical principles.²

This is a fundamental point because the solutions most relevant in the context of one type of economic framework may be entirely irrelevant in the context of another type of economic framework. Like mathematical structures, which are completely defined by their fundamental assumptions and the logical consequences of these assumptions, economic systems are also 'self-sufficient' entities. The truth of one economic system may become false in another economic system. In particular, the choice of policy instruments is constrained by those which are 'permissible' in a given economic framework. For instance, under capitalism the three essential functions which Equation (11) has pinpointed are performed by a positive rate of interest, while in a socialistic economy a complete socialization of all means of production makes interest a mere arithmetical discounting-device to determine the rank-ordering of different investment portfolios. An Islamic society prohibits interest mainly on ethical grounds. It is, therefore, a basic question,

²This point has been emphasized by Tinbergen [9].
and a non-trivial one at that, as to how to translate this ethical principle into a set of policy instruments that are economically viable and Islamically acceptable.

In this connection, we take the view that while the prohibition of interest in Islam is total and unalterable, the exact manner of enforcing this commandment in modern societies is an open question on which opinions can legitimately differ. This is what the present Essay has attempted to do: to pose the problem in an analytically meaningful way; to reject arguments that are economically, logically and ethically unacceptable; and to set up guideposts along which future work on the subject must be undertaken.

This general point has a direct bearing on the widely advocated policy alternative of making a total shift to a profit-based economy. In the opinion of the present authors, this is a question on which there is all the room for debate and discussion. We are of the opinion that no orthodoxy should be created to support the naive notion that profits are somehow sacrosanct. Nothing could be more dangerous for systematic thinking on Islam’s economic philosophy. For profits can be, as they are, as exploitative and morally reprehensible as interest. This is not to justify the

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3 At a philosophical level, this is what has been termed by Frithjof Schuon [7] as the 'human margin', which permits human beings to organize their social and economic relations in keeping with the exigencies of time and place. In particular, there is more than one sequential strategy to translate a Divine Message which is both perfect and timeless into imperfect, time-bound human institutions.

4 It is interesting to recall that Karl Marx made profits, not interest, the primary target of his criticism. According to him, profits symbolized the exploitation of labour by capitalists. See [6, Ch. 3].
institution of interest, but to show that a profit-based regime, in which only variable rates of return on investment are allowed, is not necessarily the only alternative to it.

The need or justification of Islamic economic reforms cannot be based just on the choice between the fixity or variability of the rates of return on investment. It will, instead, depend upon its success in inducing a restructuring of the economy in conformity with the Islamic principle of al 'Adl; and that would require making changes in the structure of economic power, involving a redistribution of income and wealth in the society. If the Islamic economic system is seen as a special type of "optimum regime", with a set of parameters, objectives and policy instruments, then a move from the existing economic system must of necessity involve a transfer of resources from the rich to the poor. We have shown that an economy-wide shift from interest to profit will not bring about such structural changes; for the unintended changes in the distribution of income and wealth that such a shift is most likely to induce are totally unacceptable to the Islamic principle of al 'Adl.

This observation has a direct bearing on the question of the role of government in an Islamic economy for the simple reason that individual initiative cannot be relied upon to carry out a programme of structural change. Although we have not discussed this question in any detail, the basic

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5 Tinbergen [10] observes that "as a rule the optimum regime requires income transfers" (p. 302). In the same vein is the observation made in the Willy Brandt Commission Report: "Where assets are distributed more fairly in the first place, sustained economic growth can provide jobs and better conditions for the poor" [1].
point to remember in this context is that the role of government in an Islamic society will be decided by the functions that it is called upon to perform. Being representative of the social conscience, it must do what the society requires it to do. In reaching out for a full-fledged Islamic economic system, a truly representative government will have to play a central role. The crux of the problem is how to set up alternative ways of solving the problem, including many combinations of individual autonomy and public control. It is important that not many doctrinaire exclusions are made in thinking about an Islamic economic system. For instance, pointing out that public control of economic processes is Islamically illegitimate is both unhelpful and meaningless. If public control is un-Islamic just because it reminds us of socialism, would it be logical to justify, in the name of Islam, measures that logically lead to private capitalism? All that can be said, at the level of generality at which we are operating, is that an ideal Islamic society will abhor an extremist solution: it will reject *laissez faire* private capitalism as well as a complete control by the State of all the means of production. Since the Divine Law of *al 'Adl* applies both to the State and to individuals, there is no room in an Islamic society for either State capitalism, or for communism or for private capitalism.7

It may be argued that abolition of interest is a Divine commandment and so is also the respect for every individual in view of his theomorphic nature. It may then appear that

6 Several Muslim economists, especially Baqir-ul-Sadr, have advocated a predominant role of the State in an Islamic society. See M. Nejatullah Siddiqui's wide-ranging essay [8, esp. pp. 18-23].

7 See Naqvi et al. [5].
pointing out an inconsistency between abolition of interest and freedom of the individual amounts to saying that there is an inconsistency in the Divine Message. This is false reasoning. For Islam, by rejecting unbridled individualism, subjects individual action to social responsibility which rests on moral constraints. However, even at the level where such an inconsistency may appear in practice, it can be explained in terms of the greater social benefits that a curtailment of individual freedom will entail.\(^8\) True, the Divine Will demands perfection which leaves no room for any inconsistencies; yet, at the plane of human existence, this perfection remains an ideal to be approximated, in stages, with varying degrees of success and with the imperfections that characterize all human solutions.

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\(^8\) See Naqvi [3;4].


CONSOLIDATED REFERENCES


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