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MONETARY TRADE, MARKET SPECIALIZATION AND STRATEGIC BEHAVIOUR

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Abstract

This paper [1] looks at the role of money as a medium of exchange in a competitive set-up. Together with this we have explored why, historically speaking, monetary trade and market specialization always go hand in hand. The set-up taken up for the purpose is derived from the well-known frame-work of Kiyotaki and Wright (1989). Our frame-work extends the above set-up to incorporate exchanges through trading posts for different pairs of goods. Here each agent is trying to choose his optimal strategy for trade given the best strategies of the others. The exercise reveals how a monetized trading post set-up can manifest itself through the agent's optimizing behaviour.

[1]The author wishes to acknowledge many useful discussions with Dipankar Dasgupta.

1. Introduction

The theory of Walrasian equilibrium yields a set of prices at which the aggregate competitive demand for each commodity equals it aggregate competitive supply. Two important issues arise in this context. The first is concerned with discovering the laws which guide the behaviour of the many economic variables, but especially prices, when the system is out of equilibrium. Walras (1890) tackled this problem by providing an algorithm for price adjustment which is well-known as the tatonnemént scheme.

The other issue revolves around the function of an auctioneer as a clearing house for commodities. All agents are assumed to deposit their initial endowments with this auctioneer, who in turn reallocates them according to the pattern of excess demands. Thus, in the words of Starr (1972), "In a Walrasian pure exchange general equilibrium model, trade takes place between individual households and the *market*. Households do not trade directly with each other." Such an abstraction suppresses several important issues, in particular the problems of direct exchange between households due to a lack of mutual coincidence of wants even at market clearing prices. Transaction costs as well as a medium of exchange can become crucial in such cases. This paper is devoted to these issues.

When trade takes place between households in a decentralized fashion, it is likely that they would be restricted to those between pairs of agents. More importantly such pairwise meetings of a particular trader with different traders need to be separated in time. In the absence of a centralized agency, each agent going through such *sequential bilateral* trade will naturally insist on the value of his incomings to be at least as large as the value of his outgoings. In other words, trades should be bilaterally balanced in value terms after each meeting, or, equivalently, maintain a quid-pro-quo condition. However, in the absence of a perfect mutual coincidence of wants between the agents, this quid-pro-quo may have to be maintained by transferring a good to the creditor for which he has no Walrasian excess demand. The need for a medium of exchange in a competitive set-up can be best appreciated against this back ground, for as soon as an agent accepts a good for which he does not have excess demand, it takes the form of a medium of exchange.

The earliest recognition of this problem came in Menger (1892). There have been many subsequent attempts to look into the role of money as a medium of exchange, but it was Hicks (1969) who posed it in the context of a Walrasian equilibrium. The last couple of decades saw further progress in this branch of the literature. In fact it has been a shared view (Starr (1972) Ostroy & Starr (1979), Kiyotaki and Wright (1989)), that money as a medium of exchange is an indispensable tool for attaining ones desired allocation. One then naturally asks which particular good can emerge as a medium of exchange through the agent's own optimizing behaviour aimed at transactions cost reduction. The *time* needed to attain ones desired allocation starting from the initial endowments is universally considered as a transaction cost which every agent wishes to minimize.

However, in order to deal with the problem, these authors have considered an institutionally vacuous economy insofar as trades are assumed to take place in the absence of specialized markets. All agents are assumed to gather in one place and meet in pairs to explore trading possibilities. In this context, together with developing the role of money as a medium of exchange, we would explore why, historically speaking, monetary trade and market specialization always go hand in hand. How far is it advantageous from the point of view of transactions cost to have monetary trade coupled with the social institution of markets ?

The set-up taken up for the purpose is derived from the well-known framework of Kiyotaki and Wright (1989) where each agent is trying to choose his optimal strategy for trade given the best strategies of the others. Here, the optimization is done on an agent's life time utility, with respect to transactions cost. Several Nash equilibria can be derived, revealing in the process the different goods which may play the role of a medium of exchange.

Our framework however extends the above set-up to incorporate exchanges through trading posts for the different pairs of goods. The exercise reveals how a monetized trading post set-up can manifest itself through the agents' optimizing behaviour.

More precisely, when the cost of establishing and running a market is not very high as against a marketless trading arrangement, the social institution of markets can reduce the transaction costs (through a reduction in time cost) to the extent of dominating all possible equilibria.

The next section reviews some of the earlier work in the literature. Section 3 describes the basic frame-work under consideration. The penultimate section looks into the equilibrium strategies and makes a comparison between a trading post and a market less set-up. The concluding section sums up the findings.

2. A brief review of the literature

Once the role of money as a medium of exchange had been emphasized, the literature began to ask which commodity could be a "good" choice as a medium of exchange, i.e., which commodity, if used as a medium of exchange, could lead the agents to their desired allocation within a reasonable *time span.* An attempt to characterize the commodity led Ostray and Starr to impose the following condition on the money good.

$$\sum_{c \neq M} p_c [z_{kc}]^+ \leq p_M w_{kM}$$

where c is the index for commodities, M the index for the money good, z_{kc} the excess demand of agent k for good c with $z_{kc} > 0 (< 0)$ if k is an excess demander of c (if k is an excess supplier of good c),

$$[z_{kc}]^+ = \max[0, z_{kc}] \text{ and}$$

w_{kM} the initial endowment of good M for agent k.

Thus, this condition implies that if there exists a commodity such that the initial endowment of it is large enough for each agent of the economy for them to back up all their desired purchases with this good, then its use as a medium of exchange allows agents to attain the equilibrium allocation for **any** competitive economy in at most *one round*; that is, after each agent visits each other agent once and only once.

Clearly, the above condition imposes a strong restriction on the initial endowment of the money good. However, it has been shown later by Starr (1976) that if one removes all the conditions on the money good and allows every good to be a means of payment, the time requirement can become unboundedly high.

In order to keep a balance between the two extremes of Ostroy and Starr (1974), Starr (1976), a need to look for a more plausible condition has been felt. In this respect we have shown that if a good is (finally) demanded in positive quantities (however small) by all the agents of the economy then using that good as a medium of exchange, equilibrium can be attained in *finite* time (see Dasgupta, D. and M. Rajeev (forthcoming)).

To emphasize the role of the social institution of markets in monetary trade, it has been further shown in the context of Ostroy-Starr (1974) framework (see M. Rajeev (1996)) that the time requirement to attain equilibrium allocation in any competitive economy is bounded above irrespective of the size (number of agents) of the economy, when trades take place through trading posts. Conversely, this time cost, in the absence of markets can become indefinitely high as population grows. This reveals the advantages of market specialization for large economics. The following discussion is going to address the same issues in a game theoretic framework.

3. The frame-work

As with Kiyotaki and Wright (1989) we consider a competitive economy in a state of equilibrium, consisting of three types of infinitely lived agents (type 1, 2 and 3) each specialized in consumption and production. Every type consists of an equal number of agents producing one unit of a specific good. There are three indivisible commodities, viz., goods 1, 2 and 3. Type k agents derive utility from the consumption of good k only and are able to produce k^* ($\neq k$). In our model we assume $1^* = 2$, $2^* = 3$, $3^* = 1$. As soon as a type k agent acquires good k he consumes it and produces one unit of k^*. Each good can be stored at a cost, but an agents capacity to store is restricted to one unit only. Let b_{kc} denote the cost (in terms of instataneous disutility) to the type k agents of storing good c. It is assumed that $0 < b_{k1} < b_{k2} < b_{k3}$. For a type k agent, let u_k denote the instantaneous utility from consumption of good k net of disutility of producing k^* and $\beta \in (0, 1)$ the common discount factor. An economy with these features is denoted by E.

For the economy E we consider two types of trading arrangements viz., the marketless arrangement and the trading post set-up. In a marketless arrangement (Kiyotaki and Wright (1989), Aiyagari and Wallace(1991)), the agents meet each other randomly in pairs (irrespective of the goods they want to trade) and exchange of goods takes place when it is mutually agreeable.

On the other hand in a trading post set-up there exists three different markets to deal with good 1 against 2, good 1 against good 3 and good 2 against good 3. By the (c, c') trading post we refer to the market where good c is exchanged against good c'. Agents wishing to trade good c against good c' visit the (c, c') trading post where buyers and sellers (of c against c' can identify each other and meet and trade). It appears therefore, that trading post set up would be able to avoid meetings between agents who are unlikely to benefit from trade. However, though there is a saving of time cost in the economy, one needs to incur additional costs (above the storage costs) for the setting up and maintanance of a market system. More precisely, let $\gamma_{cc'}$ be the per period cost to be incurred by an agent trading in the (c, c') trading post to run the market (it includes eg. tax payable, electricity charges etc.). [2]

We would consider two types of alternative relations amongst

[2]Here we have made these costs market specific. Similar exarcise can be carried out if one makes these costs agent specific or alternatively dependent on the good one wants to *sell* in that market.

the costs to be incurred in a trading post set-up.

$$CASE I \quad b_{k2} + \gamma_{12} < b_{k1} + \gamma_{31} < b_{k3} + \gamma_{23}, \gamma_{12} \leq \gamma_{31} \leq \gamma_{23}.$$

$$CASE II \quad b_{k2} + \gamma_{12} > b_{k1} + \gamma_{31} > b_{k3} + \gamma_{23}, \gamma_{12} \geq \gamma_{31} \geq \gamma_{23}.$$

All other possible relationships can be considered and delt with similarly. It is assumed that the net utility u_k is large enough compared to the costs (measured in terms of instantaneous disutility) so as not to induce any agent to drop out of the market economy. This may be ensured through the following sufficient condition.

$$u_k - \frac{b_{kk^*} + \gamma_{ck^*}}{1-\beta} \geq -\frac{b_{kc} + \gamma_{cc'}}{1-\beta}, \forall c \text{ and } c'.$$

In a set-up with trading posts a type k agent has two pure stategies: either to go for direct barter i.e., to exchange k against k^* directly or to go for indirect trade by exchanging k^* against some good c and then c against k. In the next section we would examine the possible equilibrium strategies for such a scenario.

4. Equilibrium Strategies:

4.1 Set-up of complete marketisation

Here we look for the steady state Nash Equilibrium strategies [3] for Cases I and II separately under the assumption that trades can be carried out *only* through the trading posts, i.e., marketless trading is not permitted. In both these situations fundamental strategies (see Kiyotaki and Wright (1989)) can be shown to be the equilibrium strategies. More precisely, it means that the type of agents for whom the storage cost of the goods they produce plus the running cost of the markets

[3] A steady state Nash equilibrium is a set of trading strategies S_k one for each type k, together with a steady state distribution $\bar{p}$ which gives the proportion of type k agents with good c, that satisfies
(i) each individual k chooses S_k to maximize his expected utility given the best strategies of others and the distribution $\bar{p}$;
(ii) given S_k, $\bar{p}$ is the resulting steady state distribution.
The exercise holds even when we introduce an additional it once for all fixed establishment cost which satisfies the condition that an agent's expected life time utility covers the cost.

relevant for their direct barter trade is the highest, would opt for indirect trade by using a good with lesser transactions cost (a good which they neither produce nor use for final consumption) as the medium of exchange. Naturally, the trading post that would have been relevant for their direct barter will not function. Thus, we have:

Proposition 1: Under CaseI, the fundamental strategies in a trading post set-up forms a set of equilibrium strategies under the following sufficient condition:

$$\gamma_{13} - \gamma_{12} < \frac{\beta}{2} u_3$$

Proof : See Appendix.

The above condition ensures that the discounted net utility gain from direct barter trade for a type 3 agent exceeds the additional cost he has to incur to run the (1, 3) market.

In an exactly similar manner one can establish,

Proposition 2 : Under Case II, the fundamental strategies in a trading post set-up form a set of equilibrium strategies for all parameter values.

In the equilibrium under Case I of Proposition 1, the type 2 agents would go for indirect trade by using good 1 as a medium of exchange whereas in the equilibrium of Proposition 2, the type 1 agents would act as the intermediaries.

Let us now define the welfare derived by a type k agent as

$$WF_k = (1-\beta)\sum_c p_{kc}V_{kc}$$

where, p_{kc} is the proportion of type k agents with good c in the steady state and V_{kc} is the utility derived by a type k agent by acquiring good c.

If we were to compare the steady state welfare levels (see Kiyotaki and Wright (1989)) of the equilibrium of Proposition 1 with that of corresponding fundamental equilibrium in a marketless economy we arrive at the following result :

Proposition 3 : For the economy E defined above the welfare of every agent is higher under the fundamental strategies in a trading post set-up as compared to that of the marketless trading arrangement, if the following conditions hold :

$$\frac{\beta u_1}{3} > \gamma_{12}, \frac{\beta u_3}{3} > \gamma_{13}, \frac{\beta u_2}{3} > \frac{1}{2}(\gamma_{12} + \gamma_{13})$$

Proof : See Appendix.

Thus, if u_k's are sufficiently large (and the discount rate is not very small) as compared to the cost of running a market, a trading post set-up would always dominate a marketless arrangement.

Kiyotaki and Wright have also shown that none of the equilibrium strategies in a marketless arrangement are Pareto-optimal. For, a nonimplementable strategy (to be called S) which directs every pair of agents to exchange the respective goods they possess (whenever they meet) can be (welfarewise) Pareto superior if u_k's are sufficiently high. But left to themselves, the traders would never opt for this strategy and hence it would not constitute an equilibrium. However, if $\gamma_{cc'}$'s are not very high as compared to the u_k's, fundamental strategies in a trading post set-up can even dominate S with respect to welfare.

4.2 Mix of trading post set-up and marketless arrangement

The above results (in particular, Propositions 1 and 2) are derived under the assumption that trades are to be carried out necessarily in the respective trading posts. This need not always be desirable especially if the cost of establishing and running a market is prohibitively high. Therefore, a natural question arises : If the option of trading in a marketless set-up is available together with a trading post set-up, will trading without some markets be preferred to trading through them, resulting thereby in the coexistence of marketless trading and exchange through a network of trading posts ? Under some restrictions on the parameter values, the answer to this question is in the affirmative.

Thus, consider the following set of strategies :

S_{k1} : type k agents go for direct barter through a trading post.

S_{k2} : type k agents go for indirect trade through a trading post.

S_{k3} : type k agents go for direct barter through marketless trade.

S_{k4} : type k agents go for indirect trade through marketless set-up.

S_{k5} : type k agents go for indirect trade first in a market and then in a marketless set-up.

S_{k6} : type k agents go for indirect trade by trading first in a marketless set-up and then through a trading post.

Thus we have the following result :

Proposition 4 : Under Case I, the strategy profile (S_{11}, S_{26}, S_{33}) constitutes a set of steady state Nash equilibrium strategies if the following conditions hold

$$p_1^{12}u_1 - \gamma_{12} > 0 \text{ and } p_{31}\{\beta u_2 - (b_{21} + \gamma_{12})\} \geq b_{23} - b_{21}$$

where, p_1^{12} is the steady state probability of meeting an (type 2) agent with good 1 in the (1, 2) market by a type 1 agent and p_{31} is the probability of meeting a type 3 agent with good 1 in the marketless set-up.

Proof : See Appendix.

Under the strategy profile (S_{11}, S_{26}, S_{33}) the type 1 agents would go for direct barter in the (1,2) trading post and the type 3 agents would opt for direct trade in a marketless set-up. It is the type 2 agents who would act as the intermediaries by exchanging good 3 against good 1 in a marketless set-up and then buy good 2 for good 1 in the (1,2) market.

This equilibrium, however, will be Pareto non-comparable with the one of Proposition 1. This is because type 1 agents are going to be worse off in this new equilibrium as their complementary trading partners (i.e., the type 2 agents) are now going through a more time consuming trading process, whereas the type 3 agents would be better off if the running cost of the market relevant for them, i.e, γ_{31}, is sufficiently high. Thus we have

Proposition 5 : The equilibrium derived in Proposition 4 is Pareto non-comparable with that of Proposition 1 if the running costs of some markets (viz., (1,3) and (2,3)) are sufficiently high. However, if $\gamma_{cc'}$'s are sufficiently small, in particular $\gamma_{cc'}$ goes to 0 for all (c, c') and the welfare levels are positive, the equilibrium under complete marketization (of Proposition 1) is welfarewise Pareto superior to the equilibrium derived in Proposition 4.

Proposition 4 establishes our intution that the utility of trades through monetized markets cannot be dominated by monetized trade in the absence of markets. However, under reasonable assumptions, the former would in fact be superior.

5. Conclusion

This paper looks into the possibility of trade through a trading post set-up vis-a-vis a marketless trading arrangement. In this context, several interesting steady state Nash equilibria are derived and the steady state utility levels are compared. However, we are concerned here only with commodity money. The use of fiat money in the process of exchange is an important issue which needs detailed study too.

The role of fiat currency in a search theoretic marketless framework has been discussed in Kiyotaki and Wright (1993, 1991, 1989).

In Kiyotaki and Wright (1989), *fiat money* is introduced as a commodity with least (in particular 0) storage cost. Due to the indivisibility of the real commodities and one unit storage space availability, it is not possible to hold fiat currency and a real commodity simultaneously. It is then shown how the lack of faith in fiat money makes it an unusable medium of exchange in Nash's sense. However, if on the contrary every one believes that others will accept fiat money then fundamentals (storage cost) and marketability both acting favourably together makes it in equilibrium a medium of exchange.

Let P units of fiat money be required to buy one unit of each of the real commodities and real balance is defined by $R = \frac{M}{P}$. Steady state utility level or welfare for a type i agent can be shown to be $\frac{\partial W_i}{\partial R}]_{R=0} > 0$, $\forall i$, as long as U_i's are not too large. Using fiat money reduces inefficient storage of real commodities. However, since introduction of fiat money in turn reduces real commodities, which can have an unfavourable effect on the frequency of consumption, welfare improvement cannot hold unconditionally.

In this context it would be interesting to examine how these conditions on U_i change with the introduction of the social institution of *markets* and its effect on the velocity of circulation of a fiat money.

One can also try to extend this framework to study the effect of money illusion. For example, if an agent holds a part of the medium of exchange as wealth for his future use the amount of money available in the system would get reduced. This in turn would effect the transactions costs. It would be interesting to see whether in the steady state equilibrium welfare would increase or decrease.

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Appendix

Proof of Proposition 1

Let V_k^D and V_k^I denote respectively the (expected, discounted, lifetime) utility derived by a type k agent by going through direct and indirect trades respectively. We want to show $V_1^D > V_1^I$, $V_2^D < V_2^I$ and $V_3^D \geq V_3^I$

Let us first consider a type 1 agent. As soon as he decides to go through direct barter he has to pay the costs $b_{12} + \gamma_{12}$. Next period he would meet a type 2 agent with good 1 with probability p_{21} and attain net utility u_1 and the entire process starts again. With probability $(1 - p_{21})$ he has the option of choosing V_1^D or V_1^I whichever is larger.

$$V_1^D = -(b_{12}+\gamma_{12})+\beta[p_{21}(u_1+max(V_1^D, V_1^I)]+(1-p_{21})max(V_1^D, V_1^I)]$$

When he decides to go through indirect trade, he would visit $(2, 3)$ market and for entire life time would not meet any complementary trading partner given $V_2^D \leq V_2^I$ and $V_3^D \geq V_3^I$.

Hence

$$V_1^I = -\frac{b_{12}+\gamma_{23}}{1-\beta} \Rightarrow V_1^D > V_1^I$$

Similarly, one can show that V_2^I and V_3^D are also optimal strategies (under the condition on the parameters stated above). It can be easily checked that in the steady state, $p_{21} = \frac{1}{2}$.

Proof of proposition 3 :

For a marketless set-up (see Kiyotaki and Wright (1989))

$$WF_1 = \frac{\beta u_1}{6} - b_{12},\ WF_2 = \frac{\beta u_2}{6} - \frac{1}{2}(b_{21}+b_{23}),\ WF_3 = \frac{\beta u_3}{6} - b_{31}$$

For a trading post set-up

$$\overline{WF_1} = (1-\beta)V_1^D = -(b_{12}+\gamma_{12}) + \frac{\beta}{2}u_1 \text{ and } \overline{WF_3} = -(b_{31}+\gamma_{31}) + \frac{\beta}{2}$$

For a type 2 agent let V_{21} and V_{23} denote respectively the indirect utilities of acquiring good 1 (by visiting the (1, 3) trading post) and of acquiring good 3 (by visiting (1, 2) trading post i.e. by acquiring good 2 and then producing good 3). We have $p_{21} = p_{23} = \frac{1}{2}$.

$$V_{23} = -(b_{23} + \gamma_{13}) + \beta V_{21},$$

$$V_{21} = -(b_{21} + \gamma_{12}) + \beta(u_2 + V_{23})$$

$$\overline{WF_2} = (1-\beta)\left(\frac{1}{2}V_{21} + \frac{1}{2}V_{23}\right) = \frac{\beta u_2}{2} - \frac{(b_{23} + \gamma_{13}) + (b_{21} + \gamma_{12})}{2}$$

Comparing $\overline{WF_k}$ with WF_k we get the result.

Let WF_k^* denote the welfare derived by a type k agent under the strategy profile S. It can be shown that (see Kiyotaki and Wright (1989)) :

$$WF_k^* = \frac{\beta u_1}{3} - \frac{1}{3}(b_{13} - b_{12}) - b_{12}, \; WF_2^* = \frac{\beta u_2}{3} - \frac{b_{23} + b_{21}}{2} - \frac{1}{6}$$

$$WF_3^* = \frac{\beta u_3}{3} - \frac{1}{3}(b_{23} - b_{31}) - b_{31}$$

Thus, if

$$\frac{\beta u_1}{6} + \frac{1}{3}(b_{13} - b_{12}) - \gamma_{12}, \; \frac{\beta u_2}{6} - \frac{\gamma_{12} + \gamma_{13}}{2} + \frac{1}{6}(b_{23} - b_{21})$$

and $\frac{\beta u_3}{6} + \frac{1}{3}(b_{32} - b_{31}) - \gamma_{13}$ quantities are non-negative we get $WF_k^* \leq \overline{WF_k}, \forall k$. As can be seen if $Lt\ \gamma_{cc'} \to 0, \ \forall c, c'$ then $WF_k^* < \overline{WF_k}$ holds unconditionally.

Proof of Proposition 4 :

Let U_{ki} denote the (expected, discounted, lifetime) utility derived by a type k agent by adopting the strategy $S_{ki}(i = 1, 2, \ldots, 6)$.

$$\begin{aligned} U_{11} = -(b_{12} + \gamma_{12}) \quad &+ \quad \beta[p_1^{12}(u_1 + max(U_{ki}, i = 1, 2, \ldots, 6)) \\ &+ \quad (1 - p_1^{12}) max(U_{ki}, i = 1, 2, \ldots, 6)] \end{aligned}$$

where p_1^{12} is the probability of meeting a trader with good 1 in the (1, 2) trading post. $U_{12} = -\frac{b_{12}+\gamma_{23}}{1-\beta}$, $U_{13} = -\frac{b_{12}}{1-\beta}$

$$\begin{aligned} U_{14} = -b_{12} \quad &+ \quad \beta[p'_{23}\{-b_{13} + p'_{31}(u_1 + max(U_{ki}, i = 1, 2, \ldots, 6)) \\ &+ \quad (1 - p'_{23}) max(U_{ki}, i = 1, 2, \ldots, 6)] \end{aligned}$$

where p'_{23} is the steady state probability of meeting a type 2 agent with good 3 in a marketless set-up when a type 1 agent adopts S_{14}, p'_{31} is the steady state probability of meeting a type 3 agent with good 1 in a marketless set-up when a type 1 agent adopts S_{14}, v_{13} is the indirect utility of acquiring good 3 by a type 1 agent.

$$U_{15} = -\frac{b_{12} + \gamma_{23}}{1 - \beta}$$

$$U_{16} = -b_{12} + \beta[p''_{23}(-\frac{b_{13}+\gamma_{13}}{1-\beta}) + (1-p''_{23})max(U_{ki},\ i = 1,2,\ldots,6)]$$

p''_{23} is the probability of meeting a type 2 agent with good 3 when the type 1 agents opt for S_{16}.

Now given the optimal strategy of the type 2 agents, in the steady state $p'_{23} = 0$. Thus, U_{11} will be optimal if $-(b_{12} + \gamma_{12}) + \beta p_1^{12} u_1 > -b_{12}$

$$\Rightarrow \beta p_1^{12} u_1 - \gamma_{12} > 0.$$

Proceeding in a parallel fashion it can be shown that U_{26} would be optimal if $p_{31}\{\beta u_2 - (b_{21} + \gamma_{12})\} \geq b_{23} - b_{21}$. Similarly optimality of U_{33} can be shown.

Steady State Probability Distributions :

Let $\frac{N_1}{N}$ and $\frac{N_2}{N}$ be the steady state proportions of the type 2 agents in a marketless arrangement and in a trading post set-up respectively. Thus, in the steady state (for the equilibrium of Proposition 4) we get :

$p_1^{12} = \frac{N_2}{N}$ and p_{23} = probability of meeting a type 2 agent with in the marketless arrangement (by a type :

$$= \frac{N_1}{N + N_1}$$

p_{31} : probability of meeting a type 3 agent with good 1 in the marketless arrangement (by a type 2 agent) $\Rightarrow p_{31} = \frac{N}{N+N_1}$

Also in the steady state $N_1.p_{31} = N_2$. Using these relations we get

$$p_{31} = \frac{\sqrt{5}-1}{2}, \quad p_1^{12} = \frac{2\sqrt{5}-4}{\sqrt{5}-1}, \; p_{23} = \frac{3-\sqrt{5}}{2}$$

Proof of Proposition 5 :

We define welfare in an exactly similar manner as that of proposition 3. Let $\overline{\overline{W}}_k$ be the welfare derived by a type k agent under the equilibrium strategy of Propostion 4.

Then

$$\overline{\overline{W}}_1 \simeq -(b_{12} + \gamma_{12}) + (.38197)\beta u_1$$

$$< -(b_{12} + \gamma_{12}) + \frac{1}{2}\beta u_1 = \overline{W}_1, \; p_1^{12} = \frac{2\sqrt{5}-4}{\sqrt{5}-1} \simeq .38197$$

$$
\begin{aligned}
\Rightarrow \bar{\bar{W}}_3 &\simeq -(b_{31}) + \beta(.38197)u_3 \\
&= -b_{31} + \frac{1}{2}\beta u_3 - .11903\beta u_3 \\
\Rightarrow \bar{\bar{W}}_3 &< \overline{W}_3 \; if \; .11903\beta u_3 - \gamma_{31} > 0
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{\bar{W}}_2 &= \frac{N_1}{N}V_{21} + \frac{N_2}{N}V_{23} \\
V_{21} &= \frac{-[b_{23} + \beta p_{31}(b_{21} + \gamma_{12})] + \beta^2 p_{31} u_3}{1 - \beta^2 p_{31}} \\
V_{23} &= -\frac{(\beta b_{23} + b_{21} + \gamma_{12}) + \beta u_2}{1 - \beta^2 p_{31}}
\end{aligned}
$$

It can be checked in a straight forward manner that if

$\gamma_{cc'} \rightarrow 0, \forall (c, c')$ and $\bar{\bar{W}}_k > 0, \forall k$ then $\bar{\bar{W}}_k < \overline{W}_k, \forall k.$