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SIMPLE COMPUTATIONS FOR THE LINEAR EXPENDITURE SYSTEM: RESULTS OF SOME EXPERIMENTS

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The linear expenditure system developed by Stone (1954) is an extremely useful tool of demand analysis. Its applicability appears to be restricted however by the somewhat cumbersome computations suggested so far in the literature for estimating the underlying parameters. The object of this note is to provide a simple computational scheme which permits the disaggregation of commodities into a fairly large number of groups but at the same time keeps the computational load within reasonable limits. No attempt is made here to study the theoretical aspects of the computational scheme such as the convergence of the suggested iterative procedure and possible biases in estimation; the method is applied however to two sets of data: consumer expenditure data relating to the United Kingdom during the period 1946 to 1965 and India during 1955 to 1969. The equation system estimated by the proposed method appears to produce excellent approximations to the observed data in both the cases.

THE PROPOSED SCHEME

The system of demand equations set out by Stone is given by

$$E_{it} = \alpha_i P_{it} + \beta_i \left(E_t - \sum_{j=1}^{k} \alpha_j P_{jt}\right), \quad i = 1,2 \ldots\ldots\ldots\ldots k \tag{1}$$

where, in any given time period t, E_{it} denotes the expenditure on the commodities in the i^{th} group, α_i and P_{it} standing respectively for the corresponding 'committed expenditure' and price. E_t is the total expenditure on all commodities: $\sum_{i=1}^{k} E_{it}$. The problem we are concerned with here is one of estimating the parameters of the system, α_i and β_i, on the basis of time series data: E_{it} and P_{it}, $t = 1,2,\ldots\ldots n$.

For a discussion of the properties of the system and its relevance to demand theory the reader may refer to Stone (1954 and 1964). The computational

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scheme suggested by Stone is a two-stage iterative procedure which requir first the estimation of β_i and then deriving α_i after eliminating β_i. The alternative we suggest here starts from rewriting (1) as (dropping the ti suffixes, for convenience).

$$E_i = \beta_i E + \alpha_i P_i - \beta_i \left(\sum_{j=1}^{k} \alpha_j P_j \right) \quad \ldots\ldots\ldots$$

or equivalent by

$$E_i = \beta_i E + \alpha_i (1-\beta_i) P_i - \beta_i \left(\sum_{j \neq i} \alpha_j P_j \right) \quad \ldots\ldots\ldots \quad ($$

Before we discuss the estimation procedure we must recognise that a free estimation of the parameters is not possible because they are subject to the following constraints.

$$0 \leq \beta_i \leq 1 \quad \ldots\ldots\ldots \quad ($$

$$\alpha_i \geq 0 \quad \ldots\ldots\ldots \quad ($$

$$\text{and} \quad \sum_{j=1}^{k} \beta_j = 1 \quad \ldots\ldots\ldots \quad ($$

While paying adequate attention to (4.3), Stone has not explicitly considered the problems posed by the non-negativity requirements in (4.1) and (4 In actual data analysis some of the α_i or β_i may well turn out to be negati unless the constrains are built into the estimation procedure.

The computational procedure we consider here is also an iterative one: Suppose we have initial estimates β_i^* and α_i^* of all the parameters. Then the second round estimates are obtained from the equations.

$$Y_i = E_i + \beta_i^* \left(\sum_{j=1}^{k} \alpha_j P_j \right) = \beta_i E + \alpha_i P_i \qquad i = 1, 2 \ldots\ldots k \qquad (5$$

which are equivalent to those in (2).

We can use ordinary least squares for estimating α_i and β_i from the regression of Y_i and E and P_i. Thus k sets of regressions, each involving only two variables on the right hand side, need to be estimated for deriving the second round estimates, which, in turn, can be used in the same way for obtaining the next round estimates.

For each time period t, since

$$Y_{1t} = E_{1t} + \beta_{1t}^{*}\left(\sum_{1}^{k} \alpha_j^{*} P_{jt}\right), \quad Y_{2t} = E_{2t} + \beta_2^{*} \left(\sum_{1}^{k} \alpha_j^{*} P_{jt}\right) \ldots .$$

the sum $\sum_{1}^{k} \alpha_j^{*} P_{jt}$ remains the same for all equations and accordingly it can be seen that equation (2) is more convenient than (3).

However, we found that initial values generated from

$$E_i = \text{Constant} + \beta_i E + \alpha_i (1-\beta_i) P_i \qquad \ldots\ldots \qquad (6)$$

lead to fairly rapid convergence of the estimates of the parameters. We must add that that equation (6) is not even a good approximation to (3) unless the prices P_{jt} do not vary much during the period of study.

In all the computational experiments we have performed thus far we found that there is a tendency for the procedure to yield negative values for some of the parameters in the initial stages. To overcome this we incorporated the following rule in the computations.

(R): If at any stage an estimated parameter is negative replace it by zero in the next round of computations.

It can be easily seen that rule (R) becomes ineffective if a negative β_i does not change sign in the next round. For, if β_i^* is negative, it is replaced by zero in (5) and β_i and α_i, the next round estimates are obtained from the regression.

$$E_i = \beta_i E + \alpha_i P_i \qquad \ldots\ldots \qquad (7)$$

and if β_i again turns out to be negative, equation (7) (i.e. the regression of E_i on E and P_i) is repeated in the iteration producing the same pair of values β_i (negative) and α_i all the time. Experimentally, we found that this hinders the convergence of the estimates. In the computations that we did, this problem was solved by combining the commodity group that gives rise to this phenomenon with a neighbouring group. Since the computations can be started with a fairly large number of commodity groups, this is, perhaps a heavy price to pay.

We found that the α_i do not pose a similar problem - although the proc may yield negative values for some of them in the initial stages - for, even if a particular negative α_i^* is replaced by zero in (5), other α_i^*s contr bute to a change in the next round estimates. But if a negative value pe we can take recourse to regrouping of the commodities, as in the case o negative β_is

Finally, we found that the constraint (4.4) is automatically satisfi in our experiments, by the values to which the estimates converge. The co tions, which will ensure this in the general case, needs further investigation, of course.

TWO ILLUSTRATIVE EXAMPLES

Table 1 gives the estimated values of parameters, obtained by the procedure described in the last section, corresponding to the data on consu mer expenditure in the United Kingdom during the period 1946 to 1965. The values of the product-moment correlation between the observ expenditure levels and those approximated by the linear expenditure system by seen to be fairly high.

Table 1
Estimated Values of the Parameters of a Linear Expenditure System: Consumer Expenditure in United Kingdom, 1946-1965

	odity Group	α_i (£100M)	β_i	Correlation Coefficient Between Observed and Expected Values
1.	Food	0.365	0.141	0.998
2.	Alcoholic Drink	0.084	0.051	0.985
3.	Tobacco	0.091	0.037	0.993
4.	Housing	0.129	0.079	0.998
5.	Fuel and Light	0.056	0.059	0.996
6.	Clothing	0.119	0.105	0.993
7.	Motor Cars and Motor Cycles	0.014	0.102	0.939
8.	Furniture etc.	0.050	0.084	0.984
9.	Household textiles, hardware etc.	0.024	0.023	0.964
10.	Matches and cleaning materials	0.016	0.006	0.954
11.	Books and recreational goods	0.042	0.038	0.997
12.	Chemists' and other goods	0.031	0.043	0.988
13.	Public travel and communication	0.062	0.019	0.993
14.	Vehicle running costs	0.014	0.093	0.992
15.	All other services	0.192	0.114	0.968

These estimates are based on data on consumer expenditure in current and constant prices collected from Feinstein (1972). Price data were derived by deflating each expenditure figure in current prices by that measured in 1958 prices. The expenditure groups are the same as in Feinstein, except that Domestic services, Catering and Other services had to be combined to make one single group, designated as 'other services'; this was necessary because of the persistence of a negative β for Domestic service in an earlier round of computations.

Table 2 gives similar estimates in respect of data on per capita consumer expenditure in urban India.

Table 2

Estimates of Parameters of a Linear Expenditure System: Per Capita Consumer Expenditure, Urban India, 1955 to 1969

Commodity Group	α_i (Rupees)	β_i	Correlation Coeffici Between Observi and Expected Va
1. Foodgrains	0.0386	0.1908	0.992
2. Milk and milk products	0.0107	0.1073	0.976
3. Edible oil	0.0053	0.0217	0.967
4. Meat, fish and eggs	0.0027	0.0467	0.965
5. Sugar and gur	0.0027	0.0361	0.968
6. All other food	0.0369	0.0022	0.981
7. Non food	0.0238	0.5914	0.989

The estimates in Table 2 are based on data given in the study by Radhakrishnan (undated). The original data had to be regrouped in the above manner because of the persistence of negative values for groups such as 'salt' and 'clothing'. The 'Non-food' group above includes the groups clothing, fuel and light and 'Miscellaneous', usually given separately in the National Sample Survey estimates, on which Radhakrishnan's study is based. These survey data have been subjected to a similar analysis for a different period earlier by Paul and Rudra (1964) - but with a different grouping scheme.

REFERENCES

FEINSTEIN, C.H., *National Income, Expenditure and Output of the United Kingdom, 1855-1965,* Cambridge University Press, 1972.

PAUL, PUSHPAM and RUDRA, ASHOK., Demand Elasticity for Foodgrains, *The Economic Weekly*, Vol. XVI, No.48, 1964, pp.1893, 1894

STONE, RICHARD., Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand, *The Economic Journal*, Vol.LXIV, 1954, pp.511-527

-do- *Mathematical Models of the Economy and Other Essays*, Chapman and Hall, 1970, Chapter VIII, pp.75-74.

RADHAKRISHNAN, P.N., Trends in Private Consumer Expenditure in India— 1954-55 to 1968-69, *Perspective Planning Division, Planning Commission,* (Unpublished and undated)

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