SEMI-FEUDALISM AND PERPETUAL INDEBTEDNESS
— A COMMENT

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I. INTRODUCTION

In this note we examine the implications of removing some of the restrictive assumptions of a model of semi-feudalism recently formulated by A.Bhaduri.[2]

Bhaduri's model, set in a specific historical context, is essentially an investigation into the conditions under which the relations of production act as a barrier to the introduction of improved technology. The argument hinges on the fact that under forms of semi-feudalism in which two modes of exploitation, viz., usury and extraction of surplus through the right of property, are combined stagnation (i.e. absence of technical progress) may result if the gains from increases in productivity do not outweigh the concomitant losses in income from usury.

In Bhaduri's analysis, (1) perpetual indebtedness is regarded as a state of equilibrium in which semi-feudal relations force a tenant to borrow the same amount of grain year after year, (2) by assumption, the system is not allowed to generate a surplus for the tenant in any given year, i.e. after repaying the last year's debt with interest what remains is assumed to be insufficient for his current consumption needs — thus the consumption loan is treated as an essential characteristic of semi-feudalism while in reality it may be a consequence of

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this mode of production; moreover, the share of the tenant is assumed to be large enough to repay his past debts — thus ruling out the possibility of a negative balance for the tenant entailing debt accumulation, (3) the impact of changes in output are evaluated only in the context of an equilibrium, and (4) variations in output caused by exogenous factors are ruled out.

In contrast ours is mainly a disequilibrium analysis in which (1) perpetual indebtedness is defined in a more general way and is independent of equilibrium considerations, (2) the possibility of the system generating a surplus or a negative balance for the tenant is not ruled out, and (3) output fluctuations are allowed; these may arise from uncertainties as well as technical change.

This general framework enables us not only to analyse the contradiction between 'relations of production' and 'forces of production' in a more realistic way but also allows us to investigate the precise nature of the 'trap' that perpetuates a consumption loan. We also pay attention to the conditions under which exploitation need not necessarily stand in the way of technical progress.

II. A REFORMULATION OF THE MODEL

While freeing the model of its historic-specificity we retain the basic structure of semi-feudalism as postulated by Bhaduri; relations between tenant and landlord are characterised by the following assumptions:

(a) Share cropping: The landlord leases out land to the tenant and the harvest of grain is shared between them.

(b) <u>Landlord as the lender of consumption-loan</u>: When the tenant's share is not sufficient to meet his current consumption needs he takes a grain loan from the landlord.

(c) <u>Inaccessibility to the market</u>: The tenant has no access to the credit market either as a borrower or as a lender (in case of a surplus accruing to him in a given year).

Assumptions (b) and (c) guarantee the tenant's being 'tied' to his landlord.

We use the following notation (the suffix t refers to time period t; the length of the production cycle of grain is treated as the unit of time).

X_t = net grain harvest of the tenant

C_t = consumption of grain by the tenant

Y_t = available balance of the tenant

i = own rate of interest per period on the consumption loan of paddy, $i > 0$

α = legal share of the tenant in the <u>net harvest</u>, $0 < \alpha < 1$

We begin by assuming that

$$C_t = \lambda + \beta Y_t \quad \text{for } Y_t > 0 \qquad (1.1)$$

$$= \lambda \qquad \text{for } Y_t \leq 0 \qquad (1.2)$$

where $\lambda > 0$ and $0 < \beta < 1$. Let us write

$$D_t = C_t - Y_t \qquad (2)$$

In our formulation, (i) Y_t is allowed to become negative — when αX_t the share of the tenant is not sufficient even to repay his outstanding debt with interest; (ii) Y_t is also allowed to be large enough to make D_t negative — when the output is high enough in a given year to generate a surplus for the tenant over his debt repayment and consumption needs; and also as in Bhaduri, (iii) αX_t may be large enough for repayment of last year's debt but the balance Y_t may be smaller than C_t.

The form of consumption function we have assumed in (1.1) and (1.2) implies that there is minimum subsistence level λ which conforms to a negative balance Y_t; in this case since the tenant is not even able to repay his outstanding debt equation (2) implies debt accumulation — a feature absent in Bhaduri's analysis.

Now, it is obvious that

$$Y_t = \alpha X_t - (1 + i)D_{t-1} \qquad \text{if } D_{t-1} > 0 \qquad (3.1)$$

$$= \alpha X_t - D_{t-1} \qquad \text{if } D_{t-1} \leq 0 \qquad (3.2)$$

The last equation merely says that if there was a surplus last year it will augment the current year's balance. We also see from (3.1) that

$$Y_t < 0 \quad \text{if and only if} \quad D_{t-1} > \frac{\alpha X_t}{(1+i)} \qquad (4)$$

From (1.1), (1.2) and (2) we get

$$D_t = \lambda \cdot (1-\beta)Y_t \qquad \text{if } Y_t > 0 \qquad (5.1)$$

$$= \lambda \cdot Y_t \qquad \text{if } Y_t \leq 0 \qquad (5.2)$$

in which we insert (3.1) and (3.2) and use (4) to derive

$$D_t = \lambda - \alpha(1-\beta)X_t + (1-\beta)D_{t-1} \qquad \text{if } D_{t-1} \leq 0 \qquad (6.1)$$

$$= \lambda - \alpha(1-\beta)X_t + (1-\beta)(1+i)D_{t-1} \quad \text{if } 0 < D_{t-1} \leq \frac{\alpha X_t}{1+i} \qquad (6.2)$$

$$= \lambda - \alpha X_t + (1+i)D_{t-1} \qquad \text{if } D_{t-1} > \frac{\alpha X_t}{1+i} \qquad (6.3)$$

Hence for a given D_0 (the initial status of the tenant) the parameters λ, α, β and i and the sequence of outputs $\{X_t\}$ determine the sequence $\{D_t\}$ which is the object of our analysis. We may note here that Bhaduri's analysis is restricted to the equilibrium solutions of (6.2).

Before characterising the relationship between $\{X_t\}$ and $\{D_t\}$ we must introduce the concept of perpetual indebtedness. For this purpose let us first consider sequences

$$\{D_t : \text{ for some n, } D_t > 0 \text{ for all } t \geq n\} \qquad (7)$$

The set of all sequences satisfying (7) can be partitioned into two sets, one for which

$$\{D_t : \text{ for some m, } 0 < D_t < \lambda \text{ for all } t \geq m\} \qquad (8)$$

and the other for which (7) holds along with

$$D_t > \lambda \text{ for infinitely many t.} \qquad (9)$$

Now it is evident from (6.2) and (6.3) that

$$D_t > \lambda \quad \text{if and only if} \quad D_{t-1} > \frac{\alpha X_t}{1+i} \qquad (10)$$

which together with (4) and (1.2) implies that

$$D_t > \lambda \qquad \text{if and only if} \qquad C_t = \lambda \qquad (11)$$

We now see that sequences satisfying (9) imply that the tenant's consumption is forced to the subsistence level infinitely often; on the other hand the condition in (8) implies that beyond a certain point of time the tenant, while never being free of indebtedness, remains above the subsistence level. The condition in (8) does not, however, rule out the possibility of the tenant being in a state of insolvency represented by $D_t > \lambda$ (or $C_t = \lambda$) for a finite number of years, nor does it rule out the possibility of the tenant enjoying a surplus in a few years: both these possibilities may occur through exogenous factors responsible for variations in output. Accordingly we have the following conditions:

<u>$\{D_t\}$ represents perpetual indebtedness
if and only if (8) is satisfied*</u> (12)

In contrast we may say that sequences D_t satisfying (7) and (9) imply the <u>ruin</u> of the tenant for obvious reasons. However, we are not attempting a full analysis of the ruin problem in this note.

Remark 1: The condition (8) guarantees an ultimate perpetual stream of net incomes from usury to the landlord even if the tenant initially starts with a surplus. This is because firstly, the condition implies an ultimate positive debt sequence and secondly, since $D_t > \lambda$ entails debt accumulation the condition (8) ensures that defaulting of consumption loans does not take place except possibly in a few years. Thus the trap of indebtedness operates in such a way that usury is ultimately economically 'paying' to the landlord and not just an instrument of political control.

The land under tenancy is assumed to be fixed and hence it is reasonable to assume that X_t is bounded. Let us write

$$\delta = \sup_{t \geq 1} X_t, \quad 0 < \delta < \infty \qquad (13)$$

We may think of δ as the highest potential output corresponding to a given technology: δ can increase only with a

* We may say that a tenant <u>is</u> perpetually indebted only in the sense that the sequence $\{X_t\}$ and the parameters of the system imply condition (8).

technological improvement.

We now present some results that follow from our structure. We state these (with proofs following) as propositions P1 to P7.

(P1) The condition $\delta < \frac{\lambda}{\alpha(1-\beta)}$ implies condition (7) irrespective of the values of D_0 and i ($i = 0$ not excluded)

Put $\epsilon = \lambda - \delta\alpha (1-\beta)$. Since $X_t \leq \delta$ for all t, the hypothesis then implies that

$$\lambda - \alpha(1-\beta)X_t \geq \epsilon > 0 \quad \text{for all t} \qquad \qquad (14)$$

Now if $D_0 > 0$, either (6.2) or (6.3) applies and in either case $D_1 > 0$, because of (14). Induction establishes the validity of the assertion in this case.

But the case $D_0 < 0$ is more interesting. Suppose $D_0 < 0$, then choose a positive integer m such that

$$-(1-\beta)^m D_0 < \epsilon \qquad \qquad (15)$$

and consider the set $(D_1, D_2 \ldots, D_m)$

Now, if any one member D_j of this set is positive or zero the inequality (14) and equations (6.2) and (6.3) ensure the positivity of D_t for $t \geq j$. On the other hand if all of them are negative then equation (6.1) applies for $t = 1, \ldots, m$ and so we have

$$D_m = \lambda - \alpha(1-\beta)X_m + (1-\beta)D_{m-1}$$
$$> (1-\beta)D_{m-1} \ldots \ldots > (1-\beta)^m D_0 > -\epsilon \qquad (16)$$

the last inequality following from (15). And since D_m is also negative

$$D_{m+1} = \lambda - \alpha(1-\beta)X_m + (1-\beta)D_m$$
$$> \epsilon - (1-\beta)\epsilon = \beta\epsilon \quad 0$$

and $D_t > 0$ for $t \geq m+1$

Thus if the highest potential output corresponding to a given technology is less than $\frac{\lambda}{\alpha(1-\beta)}$ condition (7) is satisfied which guarantees that the tenant will be either perpetually indebted or ruined irrespective of the values of D_0 and i.

It is easy to show that the condition $\delta < \frac{\lambda}{\alpha(1-\beta)}$ is not necessary for perpetuating indebtedness. This apparent stringency in the condition arises out of our unwillingness to assume anything about D_0 and i. However the proposition does show that the consumption loan emerges ultimately as a consequence of the output restriction. Ruling out the possibility that $D_t < 0$ is thus not necessary for perpetual indebtedness. Similarly it can be shown that if the interest rate and the initial debt are high enough both perpetual indebtedness and ruin are possible with output levels higher than $\frac{\lambda}{\alpha(1-\beta)}$

<u>Remark 2</u>: The condition $\delta < \frac{\lambda}{\alpha(1-\beta)}$ ensures that an initial surplus to the tenant cannot be sustained. This is implicit in the proof of (P1) but is also obvious from (6.1): when $D_0 < 0$ even if output in the period 1 reaches its highest level δ the surplus $-D_1$ cannot exceed $-(1-\beta)D_0 < -D_0$. We are designating this condition as the 'output restriction' but it is equally instructive to write it as $\alpha < \frac{\lambda}{\delta(1-\beta)}$ and regard it as the 'share restriction'. Both low shares and low output levels may be valid historical reasons for the conditions of perpetual indebtedness and ruin. The output restriction may itself come about as a consequence of restricting the land given for share-cropping.

(P2) <u>If $\delta < \frac{\lambda}{\alpha(1-\beta)}$ and $i > \frac{\beta}{1-\beta}$ then $D_t \to \infty$ as $t \to \infty$</u>.
<u>irrespective of the value of D_0</u>

Putting $\epsilon = \lambda - \delta\alpha(1-\beta)$ we see that (14) holds. First suppose that there is n such that $D_n > \lambda$. Then

$$\frac{X_{n+1}}{1+i} \leq \frac{\delta\alpha}{1+i} \leq \frac{\lambda}{(1-\beta)(1+i)} < \lambda < D_n$$

since $i > \frac{\beta}{1-\beta}$ implies that $(1-\beta)(1+i) > 1$.

This shows that equation (6.3) applies for determining D_{n+1} and hence $D_{n+1} > \lambda$. Thus once D_t crosses λ it stays above that level implying ruin of the tenant. We now show that the conditions in (P2) imply that $D_n > \lambda$ for some n; for this we need to

remember that (P1) guarantees that $D_t > 0$ for some $t = m$. Now consider $(D_m, D_{m+1} \ldots D_{m+p})$. If none of these is greater than λ then (6.2) determines D_t for $t = m+1, \ldots m+p+1$ and so we have, (in view of (14))

$$D_{m+p+1} > (1-\beta)(1+i)D_{m+p}$$

$$> \left[(1-\beta)(1+i)\right]^{p+1} D_m$$

which is greater than λ for sufficiently large p. This proves that there is a n such that $D_t > \lambda$ for all $t \geq n$. Using the equivalence (10) we therefore have

$$D_t = \lambda - \alpha X_t + (1+i)D_{t-1} \qquad \qquad .. \qquad (17.1)$$

and
$$D_{t-1} > \frac{\alpha X_t}{1+i} \qquad \qquad .. \qquad (17.2)$$

valid for all $t \geq n$. Now (17.1) can be rewritten as

$$D_t = \lambda - \alpha(1-\beta)X_t + (1+i)(1-\beta)D_{t-1}$$

$$+ \beta \left\{(1+i)D_{t-1} - \alpha X_t\right\}.$$

(14) and (17.2) now ensure that

$$D_t > \epsilon + (1+i)(1-\beta)D_{t-1} \qquad \text{for } t \geq n$$

That $D_t \to \infty$ is obvious now since $(1+i)(1-\beta) > 1$. The conditions in (P2) guarantee ruin of the tenant (and much worse) whatever his initial status might be; however, they are not necessary for the ruin of the tenant (or even for $D_t \to \infty$).

(P2) immediately yields the following:

(P3) If $\delta < \dfrac{\lambda}{\alpha(1-\beta)}$ then perpetual indebtedness implies that $i \leq \dfrac{\beta}{1-\beta}$.

For if $i > \dfrac{\beta}{1-\beta}$, (P2) implies that $D_t \to \infty$ which implies ruin and not perpetual indebtedness (see (12)).

(P4) If $i > \dfrac{\beta}{1-\beta}$ then perpetual indebtedness implies that*

$\limsup D_t \geq \dfrac{\lambda}{\alpha(1-\beta)}$.

First note that it is immediately obvious from (P2) that $\delta \geq \dfrac{\lambda}{\alpha(1-\beta)}$ for otherwise $D_t \to \infty$. Now (p.i) implies that for some n, $0 < D_t < \lambda$ for $t \geq n$ and because of the equivalence (10), equation (6.2) is valid for all $t \geq n+1$ from which we get

$$\limsup D_t \geq \liminf \left[\lambda - \alpha(1-\beta)X_t\right] + (1-\beta)(1+i)\limsup D_t \quad (18)$$

Note that the validity of (18) does not depend on the values of D_1, $D_2 \ldots D_{n-1}$. From (18) it follows that

$$\alpha(1-\beta)\limsup X_t \geq \lambda + \left[(1-\beta)(1+i) - 1\right] \limsup D_t$$

which is $\geq \lambda$ since $(1-\beta)(1+i) > 1$ and $\limsup D_t \geq 0$.

We can now see that the own rate of interest i and the output restriction play a dual role: If the output is technologically restricted to $\lambda/\alpha(1-\beta)$ it is necessary to have a relatively low interest rate for perpetuating indebtedness; similarly if i is very high perpetual indebtedness can be maintained only if outputs higher than $\lambda/\alpha(1-\beta)$ are realised infinitely often. The absence

* Throughout the note all limits are taken as $t \to \infty$.

of these counteracting forces may imply ruin of the tenant.

(P5) <u>If $\liminf X_t > \dfrac{\lambda}{\alpha(1-\beta)}$ then perpetual indebtedness implies</u>
<u>that $i > \dfrac{\beta}{1-\beta}$.</u>

Again equation (6.2) applies for all $t \geq n$, for some n and
we have

$$\limsup D_t \leq \limsup \left[\lambda - \alpha(1-\beta)X_t\right] + (1-\beta)(1+i)\limsup D_t$$

and hence

$$\left[1 - (1-\beta)(1+i)\right] \limsup D_t \leq \lambda - \alpha(1-\beta)\liminf X_t < 0$$

The conclusion follows because $\limsup D_t$ is finite and
positive.

(P6) <u>If $i < \dfrac{\beta}{1-\beta}$ then perpetual indebtedness implies that</u>

<u>$\liminf X_t \leq \dfrac{\lambda}{\alpha(1-\beta)}$</u>

Equation (6.2) applies beyond some n and it follows that

$$\liminf D_t \leq \limsup \left[\lambda - \alpha(1-\beta)X_t\right] + (1-\beta)(1+i)\liminf D_t$$

and hence

$$\lambda - \alpha(1-\beta)\liminf X_t \geq \left[1 - (1-\beta)(1+i)\right]\liminf D_t$$

The conclusion follows from the facts $i < \dfrac{\beta}{1-\beta}$

and $\liminf D_t \geq 0$.

(P5) and (P6) thus throw further light on the operation of the
output restriction in combination with usury.

If technologically outputs higher than $\dfrac{\lambda}{\alpha(1-\beta)}$ are possible
infinitely often than the interest rate has to be higher than $\dfrac{\beta}{1-\beta}$
for perpetuating indebtedness; similarly with a relatively low

interest rate perpetual indebtedness is possible only if the output

restriction becomes operative infinitely often. The absence of these

combinations may yield surpluses to the tenant infinitely often.

Remark 3: We must add that the conditions lim inf $X_t < \frac{\lambda}{\alpha(1-\beta)}$ and i $>$ are not sufficient for ensuring perpetual indebtedness. This is because the output can be high enough infinitely often (without violating the restriction on lim inf X_t) to yield surpluses.

Remark 4: As already remarked the output restriction can also be written as a share restriction if δ is assumed to be given. This will enable us to see the duality between the two modes of exploitation, usury and extraction of surplus through right of property represented by the instruments i and α respectively. For the trap of perpetual indebtedness to operate i and α have both to be either low or high simultaneously.

(P7) If $0 < D_0 < \lambda$ then the condition

$$\frac{\lambda(1+i)}{\alpha} < X_t < \frac{\lambda}{\alpha(1-\beta)} \qquad\qquad .. \qquad (19)$$

implies perpetual indebtedness.

First, note that (19) implies that $(1+i) < \frac{1}{(1-\beta)}$ or that $i < \frac{\beta}{1}$. Since $D_0 > 0$ and $X_t < \frac{\lambda}{\alpha(1-\beta)}$ equations (6.2) and (6.3) imply that D_t can never be negative. So all we need to show is that

D_t can never cross λ .

Now we know from (10) that $D_t > \lambda$ implies that $D_{t-1} > \frac{\alpha X_t}{1+i}$ which

is $> \lambda$ by hypothesis. Thus if $D_t > \lambda$ for any t, the above argument

can be repeated to show that $D_0 > \lambda$ which contradicts the hypothesis.

This proves the proposition.

We conclude this section with the observation that a fuller

characterisation of the necessary and sufficient conditions for

perpetual indebtedness as well as ruin of the tenant is possible

but not attempted here. The propositions (P1) to (P7) are sufficient

to illustrate the range of possibilities.

III   TECHNOLOGICAL CHANGE

In Bhaduri's analysis technological change brings about a rise in productivity and the resulting increase in output is a once-for-all phenomenon because output variations arising out of uncertainties are assumed away. Obviously this type of analysis may be misleading.

To be more realistic we assume that $\{X_t\}$ is a sequence of random variables (r.v); for a given technology it is quite reasonable to assume that the r.v.s are independent and identically distributed although the realised values of the output may differ depending on exogenous factors like rainfall etc. If the underlying cumulative distribution function (c.d.f) is $F(.)$ we may now regard δ as the right extreme of the support of $F(.)$, i.e.

$$\delta = \inf\{x : F(x) = 1\} \qquad (20)$$

so that output is below δ almost surely (a.s.) i.e. with probability one.

Technological change gives rise to a new sequence of r.v.s $\{X_t'\}$ independent and identically distributed with a c.d.f., say $G(.)$ which has a corresponding right extreme point $\delta' > \delta$. We may also write

$$X_t' = X_t + \theta \qquad \qquad .. \quad (21)$$

and assume that
$$E(\theta) > 0 \qquad \qquad .. \quad (22)$$

where E is the expectation operator. For our purposes there is no need to assume anything more than (22) about the distribution of θ: for example it is tempting to assume that $\mathrm{Prob}(\theta \geq 0) = P(\theta \geq 0) = 1$ implying that the new technology yields outputs higher than what

would have been obtained under the old one almost surely irrespecti
of all exogenous factors.

Finally we may assume that D_0 is also a r.v with a given
distribution. Then it is possible to derive the distributions of
$\{D_t\}$ using equations (6), (6.2) and (6.5) provided we assume that
λ, α, β and i are constants

In this framework the implications of technical change can be
analysed in terms of the flows of expected incomes of the tenant
and the landlord. In this note we shall concentrate on the
analysed by Bhaduri in his "equilibrium - no uncertainty" framework
But first it is necessary to review the validity of the proposition
P1 to P7 when $\{X_t\}$ are treated as r.v.s.

It is not hard to establish the almost sure validity of the
propositions but it is necessary to make some modifications. We
illustrate this for (P7) which we need for analysing the Bhaduri case

(P8)   Let D_0 be distributed independently of $\{X_t\}$ such that
$P(0 < D_0 < \lambda) = 1$ and suppose that the c.d.f. $F(.)$ of X_t
(for all t) be such that $F(b) - F(a) = 1$ where $a = \dfrac{\lambda(1+i)}{\alpha}$
and $b = \dfrac{\lambda}{\alpha(1-\beta)}$. Then the tenant is perpetually indebted
almost surely.

This is a trivial consequence of (P7). For we immediately have

$$P(0 < D_0 < \lambda \; ; \; \frac{\lambda(1+i)}{\alpha} \leq X_t \leq \frac{\lambda}{\alpha(1-\beta)})$$

$\leq$ Probability that the tenant is perpetually indebted.

since the event on the left hand side implies by (P7) the event on
the right. We need only note that the left hand side equals one
by hypothesis.

The Bhaduri type of equilibrium analysis can be done in the following way: suppose the conditions of (P8) hold. This would imply $i < \frac{\beta}{1-\beta}$ (or $\beta > \frac{i}{1+i}$). The existence of an equilibrium level of expected debt, say $E(D)$, has to be demonstrated but supposing it does, it would satisfy

$$E(D) = \lambda - \alpha(1-\beta)E(X) + (1-\beta)(1+i)E(D) \qquad (23)$$

where we have dropped the suffix of X_t since they have identical distributions.

Now let us consider a technical improvement that implies a sequence of outputs $\{X_t'\}$ satisfying (21) and (22) and independent and identically distributed with a c.d.f. $G(.)$. Let us further assume that $G(.)$ also satisfies $G(b) - G(a) \approx 1$ so that p.i holds almost surely even after the change. A new level of equilibrium debt, say $E(D')$, when it exists satisfies

$$E(D') = \lambda - \alpha(1-\beta)E(X') + (1-\beta)(1+i)E(D') \qquad (24)$$

From (21), (23) and (24) we then get after simplification

$$E(D) - E(D') = \frac{\alpha(1-\beta)}{[1-(1+i)(1-\beta)]} E(\theta) = L > 0 \qquad (25)$$

It is obvious that iL would then represent the loss in expected income from usury to the landlord - but only if the loss is defined in terms of the two positions of equilibria ignoring all the intermediate stages.

As against this, the landlord derives a gain in expected income through increase in productivity every year (i.e. irrespective of equilibrium considerations) by an amount $(1-\alpha)E(\theta) = M$, say. It is easy to check that $M < iL$ if and only if $\beta < \frac{i}{1+i-\alpha}$ which is

condition (26) in the Bhaduri paper. The conclusion that in this case the relations of production stand in the way of forces of production is a consequence of the comparison between the two equilibria.

A more realistic way of analysing the problem is to look at changes in $E(D_t)$ and not worry about equilibrium. fFor this purpose let us retain all the assumptions we have made for the foregoing equilibrium analysis and try to evaluate the impact of introducing the new technology in time period $t = 1$.

The expected debt sequence corresponding to the old technology is determined by

$$E(D_t) = \lambda - \alpha(1-\beta)E(X) + (1-\beta)(1+i)E(D_{t-1})$$

and that corresponding to the new technology by

$$E(D_t') = \lambda - \alpha(1-\beta)\left[E(X) + E(\theta)\right] + (1-\beta)(1+i)E(D'_{t-1})$$

and hence,

$$L_t = E(D_t) - E(D_t') = \alpha(1-\beta)E(\theta) + (1-\beta)(1+i)L_{t-1} \qquad (26)$$

Remembering that $D_0 = D_0'$ i.e. $L_0 = 0$, it is easy to see that $L_t = k + kh + kh^2 + \ldots + kh^{t-1}$ where $k = \alpha(1-\beta)E(\theta)$ and $h = (1-\beta)(1+i) < 1$ and $L_t \rightarrow \frac{\alpha(1-\beta)}{[1-(1-\beta)(1+i)]} E(\theta)$ which is precisely the same as L in (25). Now, expected gain from increased productivity is $(1-\alpha)E(\theta) = M$ every year. Therefore, as remarked before the condition $L > M$ (or $\beta < \frac{i}{1+i-\alpha}$) becomes a fetter on the productive forces only if the landlord makes long run calculations and Bhaduri's analysis following this conclusion is valid in our more general framework subject to this qualification.

But we cannot ignore the contrary case when technical improvement may be necessary to bring about perpetual indebtedness. This

happens when $i > \frac{\beta}{1-\beta}$ and $\delta < \frac{\lambda}{\alpha(1-\beta)}$ which guarantee the ultimate ruin of the tenant and from (P4) it follows that technological improvement is essential if we assume that the landlord has no stake in the ruin of the tenant but only wishes to keep him in bondage or equivalently to ensure for himself a perpetual stream of net incomes from usury. This is discussed in more detail in the following section.

IV. IMPLICATIONS OF THE MODEL

We can now study the interaction between the two modes of exploitation on the one hand and the possibility of technical progress on the other. It is clear from the discussion in the last two sections that the nature of the interaction would depend on the precise values of the parameters of the system. However, we can make an exhaustive list of the possibilities.

Case 1: $i \leq \frac{\beta(1-\alpha)}{1-\beta} < \frac{\beta}{1-\beta}$ and $\alpha\delta < \frac{\lambda}{1-\beta}$.

In this case the maximal output level (or the share α) is low enough to prevent the tenant from breaking loose of the trap and the interest rate is low enough to prevent his ruin. However the condition $i \leq \frac{\beta(1-\alpha)}{1-\beta}$ is equivalent to $\beta \geq \frac{i}{1+i-\alpha}$ which implies that i is so low that gains in income through increases in productivity offset losses in income from usury. There is thus apparently no contradiction between perpetual indebtedness and technical change in this case but it is obvious that the magnitude of the expected increment to output is restricted to $\frac{\lambda}{\alpha(1-\beta)} - \delta$ where δ corresponds to the maximal output under the old technology, for otherwise the output restriction becomes inoperative leading to the possibility of the freedom of the tenant.

Case 2: $\quad \frac{\beta(1-\alpha)}{1-\beta} < i < \frac{\beta}{1-\beta}$ and $\quad \alpha\delta < \frac{\lambda}{1-\beta}$

The difference between this case and the last one is that while the interest rate is low enough to prevent the ruin of the tenant it is sufficiently high to make the loss in income from usury outweigh the gains from increase in productivity to the landlord. The two modes of exploitation can continue in this case only with stagnant output levels. This is the case analysed by Bhaduri.

Case 3: $\quad i \geq \frac{\beta}{1-\beta}$ and $\quad \alpha\delta < \frac{\lambda}{1-\beta}$

Here the interest rate is high and the maximal output level (or the share α) is low. These conditions imply the ultimate ruin of the tenant. In practice a growing level of indebtedness merely means that the tenant is paid a subsistence wage λ every year; the rest of the output goes to the landlord and there is no income from usury. In such a situation, as already remarked in the last paragraph of Section III, technical improvement may become necessary; alternatively it may be necessary for the landlord either to lower i or raise α. Otherwise the landlord may not be able to keep the tenant in bondage.

Case 4: $\quad i < \frac{\beta}{1-\beta}$ and $\quad \alpha\delta \geq \frac{\lambda}{1-\beta}$

In this situation it is impossible to keep the tenant in a state of perpetual bondage.

Case 5: $\quad i \geq \frac{\beta}{1-\beta}$ and $\quad \alpha\delta \geq \frac{\lambda}{1-\beta}$

Perpetual indebtedness is possible in this case and as in Case 1 there is apparently no contradiction between the 'forces of production' and semi-feudal relations. But here again it is obvious that the level of i places an upper limit to the increase in productivity if

the state of perpetual indebtedness is to continue.[1]

However, Cases 3, 4 and 5 may be of no practical significance as indicated in the following section.

V. CONCLUSION.

The empirical relevance of the model lies in the fact that , the marginal propensity to consume, is likely to be high in semi-feudal systems and hence $\frac{\lambda}{1-\beta}$ (in grain units) may be so high that for a large number of tenants — who lease in only small bits of land — the condition $\alpha\delta < \frac{\lambda}{1-\beta}$ may be automatically satisfied even with low levels of subsistence consumption. Similarly, if we take, for example, $\beta = 0.75$ then $\frac{\beta}{1-\beta} = 3$ and interest rates upto 300%, covered by cases 1 and 2 discussed above, may coexist with the state of perpetual indebtedness of a large number of tenants; if we rule out higher rates of interest as unrealistic we need to concentrate only on these two cases for all practical purposes. Thus we can see, for example, that if α, the share of a typical tenant is set at 50% then $\frac{\beta}{1-\beta}(1-\alpha) = 1.5$ and interest rates less than 150% do not stand in the way of technical progress while those in the range 150-300% constitute a definite barrier to improvements in productivity.

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We omit the mathematical arguments; it depends on the fact that for any given level of i there is an upper limit to the level of output for ensuring perpetual indebtedness.