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AN ESTIMATION OF THE GRANT ELEMENT IN
FOREIGN AID

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An Estimation of the Grant Element in Foreign Aid

by

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Introduction

The purpose of aid is to effect a real transfer of resources from the developed countries to the developing ones. Aid, however, is given under certain conditions. Each donor country specifies the terms under which it is ready to grant assistance, and the three main variables that influence the terms of a loan are the interest rate, the grace period and the repayment period. By varying any one of these while keeping the other two constant, the terms of the loan change. For example, a loan with a 3 percent interest rate, a 5-year grace period, and a 10-year repayment period is different from one with a one percent interest rate even though the grace and repayment periods remain the same. The former is a "harder" loan compared with the latter.

The concept of hard and soft loans requires further elaboration. The distinction between a hard loan and a soft one is that the former has a lower grant element than the latter. The grant element, which is a measure of foreign aid, may be defined as the difference between the amount of the loaned principal and the future charges, such as interest payments, on a loan which are discounted to the present¹. The resulting figure, when multiplied

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¹ The grant element of a loan is zero when the discount rate used to calculate the present value of the future charges on a loan is equal to the rate of interest charged on the loan.

by 100, can be expressed as a percentage. The hardness or softness of a loan can also be influenced by the following consideration which may be stipulated in a repayment agreement. This is the possibility that a loan may be repayable in local instead of convertible currency; if this is so, then the grant element of the loan increases.

The aim of this paper is to show the grant element in the loans received by Pakistan over a ten-year period, 1965 to 1975. The question asked is whether the grant element has declined, remained constant or has increased over this time span. A secondary objective is to determine what affects each of the three variables, namely, the interest rate, and the grace and repayment periods, have on the grant element, so that appropriate policy recommendations may be made. A table is prepared showing the grant element in each country's loan to Pakistan. From this table, the degree of concession in each country's loan is also evident.

Before proceeding to the next section a brief overview of the aid flows to Pakistan is informative, (see Table 1). It can be seen that the relative shares of grants to loans in total aid has decreased from 13 percent in 1965-1970 to 4 percent in 1974-1975.

Pakistan's total indebtedness in June 1975 was \$6.649 billion. Aid commitments to the country total \$2 billion, which includes both capital aid as well as export credits.²

² Economic Affairs Division, Government of Pakistan, and 1 2 7.

Table 1

Aid Flows, (\$ million) 1965-1975

	Grants	Loans
1965 - 1970	356.274	2,280.750
1970 - 1971	57.113	810.779
1971 - 1972	18.735	65.142
1972 - 1973	28.944	500.995
1973 - 1974	26.251	1,212.538
1974 - 1975	48.468	1,066.440
1970 - 1975	179.511	3,655.894
1965 - 1975	535.785	5,936.644

Source: Economic Affairs Division, Government of Pakistan
and [2]

Methodology

As mentioned in the previous section aid is given on concessional terms.³ The question, then is, how concessional are these terms? In this section a methodology is outlined for calculating the degree of concession in the loans received by the country.

The process of calculating the grant element can be divided into two stages. The first deals with the calculation of the instalment repayable on every loan, and, the second involves the calculation of the present value of the loan. Once a loan has been contracted it can be repaid in one of three ways. The first is in equal annual instalments where the repayment rate or total debt servicing remains constant over time. The second method involves repaying of the loan at an increasing rate, i. e., in each successive year the payable instalment increases. The third way, by contrast, involves repayment of the loan at a decreasing rate over time. To simplify the analysis we assume that total debt servicing remains constant over time.

A formula is derived for the calculation of the grant element and is presented below:

It is

$$Q (1+P)^B (1+r)^t + \frac{A}{r} [1 - (1+r)^t] = 0 \dots\dots\dots (1)$$

where Q is the amount of the loaned principal

³ A capital transfer is not called "aid" or "official assistance" unless it is given on concessional terms.

P is the rate of interest during the grace period

B is the grace period in years

r is the rate of interest

A is the equal instalments of the loan

t is the repayment period of the loan and

$X^1 X^2 X^3 \dots$ is the outstanding principal plus interest less repayment in each year.

In the formula above it should be noted that interest is charged during the grace period but is paid out when the grace period ends.

Having obtained the value of A, the next step is to derive a formula for the calculation of the present value of the costs incurred on a loan.

The formula for deriving the present value of a loan is

$$\frac{A}{i(1+i)^B} \left[1 - \frac{1}{(1+i)^t} \right] \dots \dots \dots (2)$$

In the above formula

A is the equal annual instalment of the loan

B is the grace period

i is the discount rate, and

t is the repayment period.

Choice of Discount Rate

To obtain an accurate measure of the grant element it is necessary to discount the future charges on a loan by using an appropriate discount rate. Ideally, the social discount rate should reflect the real productivity of capital in an economy.

However, as this rate is difficult to calculate three rates of discount are arbitrarily assumed, namely 8, 10, and 12 percent. Hopefully, one of these is a reasonable approximation.

Having calculated the present value of a loan from formula 2 it is simple to derive the grant element for the loan. This is given below.

$$\frac{Q - PV}{Q} \times 100.$$

or

$$\left(1 - \frac{PV}{Q}\right) \times 100 \dots\dots\dots (3)$$

where Q is the amount of the principal loaned and PV is its present value.

The next step involves the calculation of the weighted grant elements, interest rates, and grace and repayment periods. This is achieved simply by multiplying the grant element for each loan by the value of the loan, and dividing the sum of the products by the total quantity of the loans received in that particular year. In mathematical form, this can be represented as

$$\frac{\sum G. E \times Q}{\sum Q}$$

where $G. E.$ is the grant element and Q is the quantity of the loan. Similarly, weighted values can be calculated for each year for the other variables. The rationale for calculating these weighted values for each of the variables is to discover their trend, if any, over time.

Tied Aid

The issue of tied aid or "mark-up" has been ignored in the methodology presented above. However, it can be easily incorporated by employing the formula presented below. The symbol Q refers to the face value of the loan, x is the mark-up due to the tying of aid and the formula is

$$\frac{Q}{Qx + Q} = n \dots\dots\dots (4)$$

The resulting figure n is the value of the loan with the tied element taken into account. This value, n , is inserted into formula 3 above in the following manner.

$$\frac{Qn - PV}{Q} \times 100 \dots\dots\dots (5)$$

Solving, we get a figure for the grant element adjusted for the mark-up inherent in tied aid. For the purposes of illustration three rates of mark-up are chosen, namely, 10, 20 and 30 percent.

Ranking of Countries

An important consideration in ranking the countries is to separate the various supplier's credits from official development assistance. The reason for making this distinction is that the former is extended on more or less commercial terms, with a low grant element, while the latter is more concessionary. Thus, if the two are combined, a distorted figure of the grant element emerges.

Data for the project were obtained from the following two sources:

- (1) The Pakistan Economic Survey 1974-1975, and
- (2) Unpublished Statistics from the Economics Affairs Division, Ministry of Finance, Government of Pakistan.

Results

The grant element fluctuated over the period 1965-1975 and the other variables exhibited a similar behaviour. The weighted annual values of these variables are shown in Table 2. The relationship between the grant element and the other three variables, the rate of interest (r) and the grace and repayment periods (B and t respectively) is also brought out in Table 2. The rate of interest is negatively related to the grant element, as the latter increases the former decreases. The grace period (B) and the repayment period (t) are positively related to the grant element, as the latter increases so do B and t .

To check the sensitivity of the grant element with respect to the other variables, namely the rate of interest and the grace, and repayment periods, we have estimated their transformation elasticities. These may be defined as

$$\frac{\frac{d \text{ G. E.}}{\text{G. E.}}}{\frac{d X_i}{X_i}}$$

where G. E. is the grant element and X_i is one of the three variables.⁴ These transformation elasticities may be estimated by regressing the natural logarithm (\log) of the G. E. against $\log X_i$.

⁴ See [3] for further information on this process.

Table 2

Weighted Annual values of the Grant Element, the Rate of Interest, the Grace Period and the Repayment Period

Year	Grant Element*	r	Grace Period	Repayment Period
1965-66	59.558	.030	4.207	21.845
1966-67	63.163	.029	4.340	24.255
1967-68	65.938	.028	4.580	25.737
1968-69	54.055	.040	4.253	19.737
1969-70	64.546	.028	4.549	25.322
1970-71	60.204	.026	9.582	14.697
1971-72	64.630	.045	4.584	25.163
1972-73	71.350	.025	5.691	28.030
1973-74	56.286	.041	5.139	20.994
1974-75	61.603	.029	4.838	22.730

* The discount rate used to calculate the grant element was 12 percent.

The time series data for 1965-1975 showed good results. The regression coefficients for the rate of interest, the grace period and the repayment period are -0.08484, 0.16344 and 0.39659 respectively. Of the three variables the repayment period was the most important one followed by the grace period and the rate of interest. The policy implication of the result is that loans with longer amortization periods should be preferred over those with shorter repayment periods.

For policy makers one can draw up a list of tables showing how the grant element varies with different discount rates, maturity periods, grace periods, and interest rates. For a listing of such tables see [1].

Ranking of Countries

Over the ten year period, the country that regularly gave Pakistan the most concession in its loans was Canada, (See Table 3). A close second was the IDA, the International Development Agency. Both Canada and the IDA have a grant element of over 90 percent if a discount rate of 12 percent is used. In 1970-1971, China, broke the pattern when the grant element of her loan was 98 percent. It is notable that the smaller countries of Western Europe, such as Belgium, Denmark, and ^{the} Netherlands gave concessionary loans with the grant element varying between 70 and 80 percent. Swedish assistance, after 1967-1968, was in the form of outright grants.

II
Table 3

and
The Grant Element by Country, Discount Rate, 1965 - 1975

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Belgium Cap. aid	8	-	43.430	34.006	47.869	59.598	66.097	-	66.097	66.097	66.097
	10	-	54.138	42.775	58.935	69.321	75.530	-	75.530	75.530	75.530
	12	-	62.471	49.793	67.287	76.426	82.071	-	82.071	82.071	82.071
" Export credit	8	-	10.978	8.831	8.831	-	-	-	-	6.659	3.868
	10	-	18.481	16.515	16.515	-	-	-	-	14.526	4.886
	12	-	25.039	23.231	23.231	-	-	-	-	21.403	12.538
Canada CIDA	8	86.191	86.191	86.191	86.191	86.191	86.191	-	86.191	86.191	86.191
	10	90.574	90.574	90.574	90.574	90.574	90.574	-	90.574	90.574	90.574
	12	93.364	93.364	93.364	93.364	93.364	93.364	-	93.364	93.364	93.364
" EDC	8	18.218	-	-	-	-	15.303	-	-	-	-
	10	32.470	-	-	-	-	27.743	-	-	-	-
	12	43.735	-	-	-	-	37.934	-	-	-	-
France Export credit	8	12.043	12.043	12.043	25.682	20.812	18.503	-	23.774	25.025	26.572
	10	19.456	19.456	19.456	33.960	30.399	33.152	-	33.267	34.696	36.417
	12	25.936	25.936	25.936	40.283	38.283	44.829	-	40.929	42.440	44.215
Germany Jar. aid	8	30.574	38.681	46.005	53.364	48.541	53.751	-	66.097	59.258	63.099
	10	42.343	50.234	58.160	64.760	58.472	65.052	-	75.530	69.626	72.969
	12	51.448	58.959	67.167	73.031	65.832	73.214	-	82.071	77.007	79.918

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Germany Export credit	8	8.831	8.831	8.831	8.831	8.831	8.831	2.243	-	-	-
	10	16.515	16.515	16.515	16.515	16.515	16.515	10.482	-	-	-
	12	23.231	23.231	23.231	23.231	23.231	23.231	17.684	-	-	-
Italy	8	8.831	8.831	8.831	8.831	8.831	8.831	8.831	8.831	8.831	8.831
	10	16.515	16.515	16.515	16.515	16.515	16.515	16.515	16.515	16.515	16.515
	12	23.231	23.231	23.231	23.231	23.231	23.231	23.231	23.231	23.231	23.231
" Cap. aid	8	-	-	-	-	-	16.966	-	-	-	-
	10	-	-	-	-	-	30.629	-	-	-	-
	12	-	-	-	-	-	41.700	-	-	-	-
Japan Cap. aid	8	18.815	18.815	20.723	22.596	22.596	-	-	35.988	-	43.156
	10	31.998	31.998	33.596	35.165	35.165	-	-	50.109	-	56.251
	12	42.632	42.632	43.980	45.304	45.304	-	-	60.709	-	65.911
" Export credit	8	-	-	9.908	-	8.831	-	-	-	-	-
	10	-	-	17.501	-	16.515	-	-	-	-	-
	12	-	-	24.138	-	23.231	-	-	-	-	-
Netherlands Cap aid	8	29.266	51.099	51.099	51.099	61.988	58.222	-	60.611	60.611	60.611
	10	45.561	62.365	62.365	62.365	72.289	69.544	-	70.868	70.868	70.868
	12	57.581	70.675	70.675	70.675	79.465	77.430	-	78.086	78.086	78.086

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Netherlands Suppliers	8	9.908	9.908	-	-	8.831	-	-	-	-	-
	10	17.501	17.501	-	-	16.515	-	-	-	-	-
	12	24.138	24.138	-	-	23.231	-	-	-	-	-
" Export credit	8	-	-	-	-	-	-	14.097	-	-	-
	10	-	-	-	-	-	-	17.105	-	-	-
	12	-	-	-	-	-	-	19.939	-	-	-
Sweden Cap. aid	8	62.387	62.387	82.721	-	-	-	-	-	-	-
	10	72.179	72.179	88.205	-	-	-	-	-	-	-
	12	79.196	79.196	91.697	-	-	-	-	-	-	-
" Export credit	8	6.659	-	-	-	-	-	-	-	-	-
	10	14.526	-	-	-	-	-	-	-	-	-
	12	21.402	-	-	-	-	-	-	-	-	-
UK Cap. aid	8	69.620	69.620	69.620	69.620	69.620	69.620	69.620	69.620	69.620	69.620
	10	76.619	76.619	76.619	76.619	76.619	76.619	76.619	76.619	76.619	76.619
	12	81.781	81.781	81.781	81.781	81.781	81.781	81.781	81.781	81.781	81.781
" Suppliers	8	-	8.831	8.831	10.978	10.978	8.931	-	-	-	-
	10	-	16.515	16.515	18.481	18.481	16.515	-	-	-	-
	12	-	23.231	23.231	25.039	25.039	23.231	-	-	-	-
US Aid	8	61.945	61.945	58.915	57.090	57.090	57.090	82.618	57.090	57.090	57.090
	10	69.701	69.701	67.588	66.061	66.061	66.061	87.885	66.061	66.061	66.061
	12	75.206	75.206	73.698	72.393	72.393	72.393	91.355	72.393	72.393	72.393

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974 - 75
PL 480	8	-	61.945	58.915	57.090	57.090	57.090	57.090	57.090	57.090	57.090
	10	-	69.701	67.588	66.061	66.061	66.061	66.061	66.061	66.061	66.061
	12	-	75.206	73.698	72.393	72.393	72.393	72.393	72.393	72.393	72.393
Eximbank	8	18.564	8.311	-	13.622	11.821	10.75	-	-	15.172	-
	10	31.303	15.057	-	24.991	21.961	20.116	-	-	27.755	-
	12	41.424	21.078	-	34.535	30.694	28.298	-	-	38.234	-
US Suppliers	8	-	7.749	15.551	8.831	-	-	-	-	-	-
	10	-	15.524	22.631	16.515	-	-	-	-	-	-
	12	-	22.320	28.856	23.231	-	-	-	-	-	-
IBRD	8	19.250	19.604	14.557	15.000	9.816	-	-	-	6.620	5.774
	10	31.273	34.478	27.520	31.201	25.988	-	-	-	21.779	15.137
	12	41.039	45.930	37.529	44.698	38.576	-	-	-	33.961	31.168
IDA	8	82.721	82.721	82.721	82.721	82.721	82.721	-	82.721	82.721	82.721
	10	88.205	88.205	88.205	88.205	88.205	88.205	-	88.205	88.205	88.205
	12	91.697	91.697	91.697	91.697	91.697	91.697	-	91.697	91.697	91.697
ADB	8	-	-	-	8.050	-	4.077	-	33.804	46.184	46.184
	10	-	-	-	22.502	-	18.409	-	47.632	60.122	60.122
	12	-	-	-	34.202	-	30.010	-	58.013	69.899	69.899
Nonconsortium Denmark	8	61.164	-	-	-	-	70.069	-	-	-	80.222
	10	68.515	-	-	-	-	77.018	-	-	-	86.002
	12	74.236	-	-	-	-	82.128	-	-	-	89.899

Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Czechoslovakia Export credit	8	23.331	-	-	23.331	23.331	-	-	-	-	-
	10	29.792	-	-	29.792	29.792	-	-	-	-	-
	12	35.441	-	-	35.441	35.441	-	-	-	-	-
Poland Export credit	8	23.331	-	23.331	-	-	-	-	-	-	-
	10	29.792	-	29.792	-	-	-	-	-	-	-
	12	35.441	-	35.441	-	-	-	-	-	-	-
Switzerland	8	5.564	-	-	5.564	5.564	5.564	-	-	-	-
	10	13.523	-	-	13.523	13.523	13.523	-	-	-	-
	12	20.480	-	-	20.480	20.480	20.480	-	-	-	-
USSR Capital aid	8	23.331	23.331	23.331	24.964	23.331	23.331	26.532	26.532	-	26.532
	10	29.792	29.792	29.792	31.732	29.792	29.792	33.575	33.575	-	33.575
	12	35.441	35.441	35.441	37.591	35.441	35.441	39.612	39.612	-	39.612
Yugoslavia Export credit	8	21.337	21.337	-	21.337	-	21.337	-	-	-	-
	10	27.967	27.967	-	27.967	-	27.967	-	-	-	-
	12	33.762	33.762	-	33.762	-	33.762	-	-	-	-
Turkey	8	-	-	-	17.271	-	-	-	-	-	-
	10	-	-	-	24.243	-	-	-	-	-	-
	12	-	-	-	30.338	-	-	-	-	-	-
Austria	8	-	-	-	-	10.978	-	-	-	17.861	-
	10	-	-	-	-	18.481	-	-	-	27.494	-
	12	-	-	-	-	25.039	-	-	-	35.687	-
China	8	-	-	-	-	-	93.332	-	-	-	-
	10	-	-	-	-	-	96.479	-	-	-	-
	12	-	-	-	-	-	98.114	-	-	-	-

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Country/Agency	Discount Rate	1965-66	1966-67	1967-68	1968-69	1969-70	1970-71	1971-72	1972-73	1973-74	1974-75
Bulgaria Export credit	8	-	-	-	-	-	-	-	-	28.785	-
	10	-	-	-	-	-	-	-	-	36.387	-
	12	-	-	-	-	-	-	-	-	42.884	-
Rumania Export credit	8	-	-	-	-	-	-	-	-	22.678	-
	10	-	-	-	-	-	-	-	-	30.483	-
	12	-	-	-	-	-	-	-	-	37.217	-
GDR Export credit	8	-	-	-	-	-	-	-	-	-	2.343
	10	-	-	-	-	-	-	-	-	-	10.989
	12	-	-	-	-	-	-	-	-	-	18.596
Nontraditional											
Libya	8	-	-	-	-	-	-	-	-	23.600	68.600
	10	-	-	-	-	-	-	-	-	35.100	76.100
	12	-	-	-	-	-	-	-	-	43.800	81.300
Qatar	8	-	-	-	-	-	-	-	-	-	26.800
	10	-	-	-	-	-	-	-	-	-	35.770
	12	-	-	-	-	-	-	-	-	-	42.770
Saudi Arabia	8	-	-	-	-	-	-	-	-	-	54.100
	10	-	-	-	-	-	-	-	-	-	61.800
	12	-	-	-	-	-	-	-	-	-	67.800
Abu Dhabi	8	-	-	-	-	-	-	-	-	-	67.700
	10	-	-	-	-	-	-	-	-	-	77.600
	12	-	-	-	-	-	-	-	-	-	84.000
Iran	8	-	-	-	-	-	-	-	-	25.900	0
	10	-	-	-	-	-	-	-	-	34.000	11.000
	12	-	-	-	-	-	-	-	-	40.300	21.400

Some countries, for example, Germany and Japan, softened their terms of lending for the period 1965-1975. The grant element of German loans rose from 51 percent in 1965 to 82 percent in 1972-1973. Japanese loans, too, increased the content of their grant element from 42 percent in 1965 to 65 percent in 1975. The Asian Development Bank, ADB, improved its terms of lending as the grant element of its loans increased from 34 percent in 1968-1969 to 69 percent in 1974-1975. The degree of concession in the official development assistance from the United Kingdom remained constant, at 82 percent for the period. However, the terms and conditions of US loans became marginally stiffer, their grant element declined from 75 percent in 1965 to 72 percent in 1974-1975. The International Bank for Reconstruction and Development, IBRD, did poorly; the degree of concession varied in the first three years of the period and then showed a steady decline towards the end of the period; the grant element having declined to 31 percent in 1974-1975 from a high of 45 percent in 1968-1969.⁵

The Socialist countries of Eastern Europe and the USSR are near the lower end of the ranking scale. Their average grant element varied slightly over this time span, ranging between 35 percent and 39 percent. Switzerland, with a grant element of 20 percent in her loans to Pakistan, did poorly, too.

⁵ The reason IBRD did poorly is that this agency borrows funds on the international money market and relends it at a rate of interest marginally below the one it borrowed them at.

Export credits and loans extended by the British, American and Dutch financial institutions were made at almost commercial terms and thus the grant element was low, ranging between 24 and 12 percent.

Tied Aid

Countries that tie their aid include: Canada, France, Italy, Netherlands, the UK and USA and the Socialist Countries of Eastern Europe as well as the USSR. Countries that partially tie their aid are Belgium and Japan. International agencies, such as the World Bank, the IDA and ADB give aid that is untied. German aid, as that of the Islamic bloc of countries is fully untied.

Disaggregated data required to quantify the mark-up present in tied aid is difficult to obtain. However, a simple table can be constructed showing the effect of tied aid on the grant element under different assumptions, (See Table 4).

Table 4

The Effect of Tied Aid on the Grant
Element

	G. E. adjusted for tied aid		
	$X^* = 10$	20	30
G. E. unadjusted	$n = 0.91$	0.83	0.77
20	11	3	negative
30	21	13	7
40	31	23	17
etc.			
90	81	73	67

* The notations X refers to the rate of mark up, and n to the grant element adjusted for the mark up.

Conclusion

In conclusion, several results of this project may be recapitulated. Firstly, the grant element varied for the period under review, 1965-1975, it showed no definite trend. Secondly, the variable t , the repayment period, was the most significant. Thirdly, the use of different discount rates produced noticeable variations in the grant element, (See Table 3). It should also be noted that several countries, for example, UK, Japan, Belgium, and Germany have improved their terms of granting assistance. UK aid from 1975 is completely in grant form, Japan has lowered the rate of interest on her loans, and Belgium has raised the amortization period. German aid is on the same terms as that of the IDA. Also, all Norwegian and Swedish assistance is in grant form. A new development in the past three years has been the increased aid inflow from nontraditional sources, that is from the Middle Eastern Arab states, namely, Saudi Arabia, Libya, Qatar and Abu Dhabi. Total assistance from these nontraditional sources amounted to \$ 948.930 million upto March 1976

[2].

Appendix

The formula for deriving the calculation of the grant element is given below. Suppose a country has received a loan which has to be repaid in 10 years after the end of the first year after the grace period would be

$$Q \left[(1+p)^B (1+r) \right] - A = X^1$$

at the end of the second year

$$Q \left[(1+p)^B (1+r)^2 \right] - A(1+r) - A = X^2$$

at the end of the third year

$$Q \left[(1+p)^B (1+r)^3 \right] - A(1+r)^2 - A(1+r) - A = X^3$$

and so on until the tenth year when the position would be

$$Q \left[(1+p)^B (1+r)^{10} \right] - A(1+r)^9 - A(1+r)^8 \\ \dots - A = X^{10} = 0$$

This can be written as

$$Q \left[(1+p)^B (1+r)^{10} \right] - \left[A + A(1+r)^2 \right. \\ \left. \dots + A(1+r)^9 \right] = 0$$

simplyfying

$$Q \left[(1+p)^B (1+r)^{10} \right] - \left[1 + (1+r) + (1+r)^2 \right. \\ \left. \dots + (1+r)^9 \right] = 0$$

which can then be rewritten as

$$Q \left[(1+p)^B (1+r)^{10} \right] + \frac{A}{r} \left[1 - (1+r)^{10} \right] = 0.$$

In the case of t years the formula can be expressed as

$$Q \left[(1+p)^B (1+r)^t \right] + \frac{A}{r} \left[1 - (1+r)^t \right] = 0$$

Q is the amount of the loaned principal

P is the rate of interest during the grace period

B is the grace period in years.

r is the rate of interest

A is the equal annual instalments of the loan

t is the repayment period of the loan and

$X^1 X^2 X^3$ is the outstanding principal plus interest less repayment in each year.

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