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Measuring Chronic Multidimensional Poverty: A Counting Approach

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Abstract

How can indices of multidimensional poverty be adapted to produce measures that quantify both the joint incidence of multiple deprivations and their chronicity? This paper adopts a new approach to the measurement of chronic multidimensional poverty. It relies on the counting approach of Alkire and Foster (2011) for the measurement of multidimensional poverty in each time period and then on the duration approach of Foster (2009) for the measurement of multidimensional poverty persistence across time. The proposed indices are sensitive both to (i) the share of dimensions in which people are deprived and (ii) the duration of their multidimensional poverty experience. A related set of indices is also proposed to measure transient poverty. The behaviour of the proposed two families is analysed using a relevant set of axioms. An empirical illustration is provided with a Chilean panel dataset spanning the period from 1996 to 2006.

Keywords: Chronic poverty, Multidimensional poverty.

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The Oxford Poverty and Human Development Initiative (OPHI) is a research centre within the Oxford Department of International Development, Queen Elizabeth House, at the University of Oxford. Led by Sabina Alkire, OPHI aspires to build and advance a more systematic methodological and economic framework for reducing multidimensional poverty, grounded in people's experiences and values.

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1. Introduction

Sen (1976) argued that an index of poverty should identify persons who live in poverty and measure the extent of individual poverty. His seminal contribution inspired numerous proposals of unidimensional indices of poverty based on cross-sections of income or consumption data.

However, the duration of poverty at the individual or household level is a crucial issue for understanding how people experience poverty. Persistent conditions of insufficiency might precipitate detrimental effects on well-being. For instance, an increase in the duration of poverty increases the likelihood of impairment and illness. A person stricken by long-lasting poverty can become socially excluded and/or lose allegiance to the wider community (Walker, 1995). This, in turn, may lead to social unrest. Furthermore, it is important to know who among the poor are chronically poor and to understand their condition in order to improve policy predictions and responses (Lybbert et al., 2004; Carter and Barrett, 2006). Therefore it often becomes desirable to measure individual poverty dynamically using panel data.

An important recent development in poverty measurement research has been the definition of a robust multidimensional framework. The reason for its emergence is that the well-being depends on both monetary and non-monetary dimensions of life (see Kolm, 1977; Streeten, 1981; Sen, 1985, 1987; Anand and Sen, 1997; and Foster and Sen, 1997). Examples of non-income dimensions are housing, schooling, nutrition, etc. A person with a sufficiently high income may not always be well-off with respect to some non-monetary dimensions of life. For example, she may have an insufficient quantity of a non-club public good. Likewise, a pavement dweller with good nutritional status may have a low income. It may not be possible to trade off income and some non-income dimensions. It also may be necessary to develop policies to address specific deprivations or combinations of deprivations. If so, then the construction of a multidimensional poverty index and its analysis may be worthwhile.

It is extremely important to combine these two approaches for the study of chronic multidimensional poverty. Hulme et al. (2001) and Hulme and McKay (2005) argued explicitly that the measurement of chronic poverty should focus on multidimensional situations. 'Chronically poor are commonly multi-dimensionally deprived' (CPRC, 2004-5, p. 6). Furthermore, interesting analyses can be carried out when chronic and transient poverty measures are broken down by dimension. For example, one can perform an analysis to see whether chronic poverty has distinctive components that may comprise 'poverty traps'.

This paper extends the Alkire-Foster multidimensional approach to chronic poverty and, in a related manner, to transient poverty, using the Foster (2009) duration approach. The latter is chosen because it is parsimonious and easy to understand and it is based on the same axiomatic foundations as the Alkire-Foster family of multidimensional poverty indices. Moreover, unlike other inter-temporal poverty approaches, Foster's identification criteria explicitly identify the chronically poor; but can easily be adjusted to identify the transiently poor. The Alkire-Foster (2011) approach has the practical advantage that it can be computed with ordinal or ratio-scale data and is widely applied.

The next section briefly discusses the most recent literature on inter-temporal poverty measurement including existing proposals to measure chronic multidimensional poverty. This section's purpose is to clarify the concept of chronic poverty and its distinctiveness within the inter-temporal poverty literature. Section 3 presents some notation and definitions. Section 4 introduces our class of chronic multidimensional poverty measures. We also introduce a family of transient multidimensional poverty measures. Section 5 presents axioms for a general chronic multidimensional poverty index and investigates axiom fulfilment by the families introduced in the previous section. Section 6 offers two empirical illustrations that use ratio scale and, separately, ordinal variables, using the CASEN panel datasets in Chile with observations for 1996, 2001, and 2006. Section 7 concludes. In the Appendix, we compare our proposal and those put forward by Nicholas and Ray (2011) and Nicholas, Ray, and Sinha (2013) highlighting their main differences.

2. A conceptual clarification on the current state of the literature

The recent literature on poverty measurement that accounts for time, also known as *inter-temporal poverty*, provides normative evaluations that are sensitive to different aspects of people's lifetime poverty experience. This literature does not explicitly distinguish people who are chronically poor from those who are only transiently poor. Instead attention is generally focused on features like the number of consecutive spells in poverty, the number of consecutive spells outside poverty, or the timing of the poverty experience (e.g. whether it is concentrated at the beginning or at the end of a lifetime). For instance, in the individual poverty measures of Bossert et al. (2012), deprivation gaps belonging to longer spells are assigned greater weight.[1] Other interesting examples of inter-temporal poverty measures include the contributions of Hoy and Zheng (2011), Dutta *et al.* (2011) and Hojman and Kast (2009). For instance, Dutta *et al.* (2011) considered a variant of the approach by Bossert et al. (2012) by discounting the impact of a period in poverty using the number of periods outside poverty directly preceding it. Hojman and Kast (2009) described an inter-temporal poverty measure that trades off poverty levels and changes (gains and losses) over time. Hence this index is an increasing function of absolute levels of poverty and changes in poverty. Bossert *et al.* (2014) followed a similar approach.

[1] Gradin et al. (2012), in turn, generalized the proposal of Bossert *et al.* (2012).

Finally, Nicholas and Ray (2011) presented a generalization of the Chakravarty-D'Ambrosio (2006) class of multidimensional deprivation measures by explicitly taking into account the duration and persistence of deprivation. Essential to this generalization is the number of dimensions in which a person becomes deprived at different time periods. Nicholas, Ray and Sinha (2013) have generalized the proposal of Nicholas and Ray (2011) by including a more flexible poverty identification function and rendering the poverty experience sensitive to both the breadth of deprivation in any given time period and the duration of each deprivation. As these proposals combine a multidimensional framework with time, just like ours, we discuss them more thoroughly in the Appendix below.

None of the recent proposals mentioned above seek to identify the chronically poor (distinguishing them from the transiently poor), a purpose whose ongoing interest dates from an earlier literature. Several approaches to the measurement of chronic poverty have been suggested. Jalan and Ravallion (1998) proposed using a person's permanent income in order to identify him or her as chronically poor.[2] According to this approach, a person is regarded as chronically poor if the individual's permanent income falls below a certain poverty line. Because aggregation of incomes over the periods under consideration ignores income variations across periods, Foster and Santos (2014) followed the permanent income approach by explicitly allowing for an imperfect degree of substitutability across periods. They then used a decomposable Clark, Hemming and Ulph (1981) measure in order to compute chronic poverty. Porter and Quinn (2014) suggested a class of chronic poverty indices that incorporates the view that the poorer the individual is, the higher the negative impact of fluctuations in well-being.

Foster (2009) proposed a class of chronic poverty indices that rely on aggregation across time. He defined an individual as chronically poor if his income falls below an exogenously given poverty line for a minimum percentage of time periods. This approach to the measurement of chronic poverty is known as the spell, or duration, approach (see Yaqub 2000a, 2000b; McKay and Lawson 2002; Hoy, Thompson and Zheng 2012). The Foster indices, which are an extension of the Foster-Greer-Thorbecke (1984) family of indices to address chronic income poverty, fulfils a time anonymity condition under which the reordering of incomes in the individuals' trajectories does not change chronic poverty. Foster (2009) also suggested an associated index of transient poverty to evaluate shorter-duration poverty.[3]

The chronic multidimensional poverty measure presented here applies three sets of cut-offs: dimension-specific deprivation cut-offs, a multidimensional poverty cut-off, and a duration cut-off. We apply deprivation cut-offs to each person's achievement vector to determine the indicators in which they are deprived. Using the poverty cut-off we identify each person as multidimensionally poor or non-poor in

2 See also Rodgers and Rodgers (1993), Calvo and Dercon (2007), Calvo (2008) and Foster (2009).
3 Chakravarty (2009) investigated properties of subgroup-decomposable chronic poverty indices in this framework.

each period based on their weighted deprivation score. We then count the periods in which each person experienced multidimensional poverty. We identify those persons as chronically multidimensionally poor who have experienced multidimensional poverty in at least the number of periods specified by the duration cut-off. The final class of measures are the mean of a set of doubly censored deprivation gaps. Our methodology generates a range of intuitive and consistent partial indices. These include the incidence and intensity of chronic multidimensional poverty and the censored headcounts from the Alkire-Foster method. New indicators include the average duration of poverty and the average duration of deprivation in each indicator, as well as period-specific indicators of incidence and intensity. Thus our methodology proposes a way to identify and evaluate the experience of the chronically poor in a multidimensional sense. Like some previous contributions, our proposal is guided by a set of relevant axioms.

3. Preliminaries

We have observations on d dimensions or attributes of well-being for a set of N individuals at T different time points. Let x_{ij}^t stand for the quantity of attribute j possessed by person i in period t. Let $\mu(v)$ stand for the arithmetic mean of v. It is assumed that $x_{ij}^t \geq 0 \; \forall i, j, t$. Let X^t denote the matrix whose i^{th} row is the row vector $x_{i.}^t = (x_{i1}^t, x_{i2}^t, \dots, x_{id}^t)$. X^t is the $N \times d$ achievement matrix in period t. The distribution of attribute j in period t is represented by the column vector $x_{.j}^t$.

In this multidimensional set-up, a deprivation cut-off z_j is defined for each attribute; these are fixed across periods. These deprivation cut-offs give the minimal quantities of the d attributes necessary to be non-deprived in each attribute. Let $z = (z_1, \dots, z_d)$ be the vector of deprivation cut-offs in every period and $z_j > 0 \; \forall j$. z is an element of the set $Z \subset \mathbb{R}_{++}^d$, the strictly positive part of the *d*-dimensional Euclidean space. Person i is regarded as deprived with respect to dimension j in period t if $x_{ij}^t < z_j$. Person i is non-deprived in dimension j in period t if $x_{ij}^t \geq z_j$. Note that deprivation cut-offs can be applied to ordinal or cardinal data.

When some data are ordinal or binary – a common situation in multidimensional poverty measurement – we create an $N \times d$ deprivation matrix for period t; $G^t(0)$, whose typical element, $g_{ij}^t(0)$, takes the value of 1 if $x_{ij}^t < z_j$, and 0 if $x_{ij}^t \geq z_j$. If all data are cardinal, we create an $N \times d$ powered deprivation gap matrix for period ; $G^t(\alpha)$, whose typical element, $g_{ij}^t(\alpha)$, is constructed as follows. For any triplet(i, j, t), let $\widehat{x_{ij}^t} \equiv \min\{x_{ij}^t, z_j\}$. The powered deprivation shortfall of person i in dimension j at period t is: $g_{ij}^{t}(\alpha) \equiv \left(1 - \frac{\widehat{x_{ij}^{t}}}{z_j}\right)^{\alpha}$, where $\alpha \geq 0$. Clearly, individuals deprived in j at t have a positive deprivation gap, whereas otherwise $g_{ij}^{t}(\alpha) = 0$. Since we are using the Alkire-Foster method of identification and aggregation, we use $g_{ij}^{t}(\alpha)$. An alternative to this is $1 - \left(\frac{\widehat{x_{ij}^{t}}}{z_j}\right)^{\epsilon}$, where ϵ is a constant. This deprivation function was characterized by Chakravarty (1983).

3.1 The Alkire-Foster approach to the identification of the multidimensionally poor

Two well-known methods of identification of the multidimensionally poor have been analysed, among others, by Tsui (2002), Atkinson (2003) and Bourguignon and Chakravarty (2003). According to the *union method*, if a person is deprived in any dimension, then he is regarded as poor. On the other extreme, the *intersection method* demands that only persons who are deprived in all dimensions are identified as poor. As Alkire and Foster (2011) argued, a more general alternative to these two criteria is an identification approach which requires a person to be poor if she is deprived in at least k dimensions, where: $0 < k \leq d$. Equivalently, k can be defined as the share of total dimensions in which a person must be deprived in order to be identified as poor, $0 < k \leq 1$. Thus k is a poverty cut-off that identifies who is poor. When each dimension is assigned equal importance, then if $k = \frac{1}{d}$ the union method is obtained, whereas the intersection method requires $k = 1$.

However, different dimensions can be assigned different positive weights in order of importance, where $\sum_{j=1}^{d} w_j = 1$, where w_j is the non-negative weight assigned to dimension j. In such a case, if $0 < k \leq \min\{w_1, w_2, \ldots, w_d\}$, we obtain the union method. As before, $k = 1$ yields the intersection method.[4] In this paper we adopt the Alkire-Foster method for the identification of the multidimensionally poor in each period.

Identification of the multidimensionally poor in period t proceeds according to the following steps. Having defined a d-dimensional column vector of weights: $W = (w_1, w_2, \ldots, w_d)$, we generate an N-dimensional counting vector, $C^t = G^t(0)W'$. A typical element of C^t, e.g. c_i^t, gives the weighted sum of deprivations for person i in period t. Formally, $c_i^t = \sum_{j=1}^{d} w_j g_{ij}^{t}(0)$.[5] Second, we generate an N-dimensional identification (column) vector for period t, $I^t(k)$, such that a typical element, $\rho_i^t(k)$, is

4 See Alkire and Foster (2011) for further discussion.
5 Recall that $g_{ij}^{t}(0) = 1$ when individual i is deprived in dimension j.

defined by: $\rho_i^t(k) = \mathbb{I}(c_i^t \geq k)$.[6] The identification vector elements take two values: 0 and 1. The entry $\rho_i^t(k) = 1$ if and only if individual i is multidimensionally poor, according to deprivation cut-offs z, weights W and poverty cut-off k; and $\rho_i^t(k) = 0$ otherwise.

3.2 The duration approach

Having identified the poor in every period, the next step is to identify the chronically poor. As mentioned above, we assume that the attribute quantities have been appropriately transformed to take into account variations across time periods (e.g. due to discount factors) and hence for each dimension a common threshold can be used. Let $z = (z_1, z_2, \dots, z_d)$ be the vector of common deprivation cut-offs.

Given the Alkire-Foster method of identification of the multidimensionally poor, Foster's (2009) duration approach says that a person is chronically poor if she remains in poverty for at least a certain proportion τ of the total number of time periods, T (that is, $0 < \tau \leq 1$). We refer to τ as the duration cut-off. Thus, this duration-based approach involves a third identification step in addition to the two steps implemented above. In the previous subsection, we identified dimensional deprivation in every period (and for every individual) using the deprivation cut-offs (z). Then we identified the multidimensionally poor, in each period, using the Alkire-Foster dual cut-off approach and poverty cut-off k. The third step identifies the chronically poor among these multidimensionally poor persons in different periods using the duration cut-off τ.

We apply the deprivation cut-off across the number of periods in which each individual is multidimensionally poor. First, we count the periods of poverty by constructing a $N \times T$ matrix, $I(k)$, in which each of the *t* column vectors is the identification vector for the t[th] period, $I^t(k)$. Then we generate the N-dimensional chronic counting vector, L, whose typical element, $l_i = \frac{1}{T}\sum_{t=1}^{T} \rho_i^t(k)$, gives the proportion of periods in which person i is multidimensionally poor for a given k. Finally, we apply the cut-off τ to the chronic counting vector, to identify the chronically poor. We generate an N-dimensional column vector, $P^c(k;\tau)$, for the identification of the chronically poor, such that a typical element, $\rho_i(k;\tau)$, is defined by: $\rho_i(k;\tau) = \mathbb{I}(l_i \geq \tau)$. $\rho_i(k;\tau) = 1$ if and only if individual i is chronically multidimensionally poor, according to deprivation cut-offs z, weights W, poverty k and duration cut-off τ.[7]

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6 $\mathbb{I}(a)$ is an indicator function whose value is 1 if and only if a is true. Otherwise, it is equal to 0.

7 The measures presented subsequently could also use different identification strategies, such as the average deprivation level across years $\rho_i(k;\tau) = \mathbb{I}\left(\frac{1}{T}\sum_{t=1}^{T} c_i^t \geq k\right)$ or the inclusion of a functional form (or weights) to allow for different valuation

Finally, let X denote the $(N \times d) \times T$ achievement matrix for all periods. For a given $T > 1$ and > 1 , we denote the set of all inter-temporal achievement matrices of the form X by M^N.

4. A class of chronic multidimensional poverty measures

Closely following the functional forms proposed by Alkire and Foster (2011) and Foster (2009), we propose the following normalized population average of powered deprivation gaps, in which only the deprivation gaps of the chronically poor are considered. In essence, this measure is the mean *across people and time* of the weighted sum of deprivation gaps, $\sum_{j=1}^{d} w_j g_{ij}^t(\alpha)$, which are censored for individual i if $\rho_i(k;\tau) = 0$:

$$M_C^\alpha(X; z, W, k, \tau) = \frac{1}{NT} P^{c\prime} \sum_{t=1}^{T} G^t(\alpha) W' \tag{1}$$

Where, W' is the transpose of W, $G^t(\alpha)W'$ is a N-dimensional column vector whose typical element is $\sum_{j=1}^{d} w_j g_{ij}^t(\alpha)$, and $P^{c\prime}$ is the transpose of P^c, i.e. a N-dimensional row vector whose typical element is $\rho_i(k;\tau) = I(l_i \geq \tau)$ as defined in section 3.b. An alternative way of writing M_C^α is:

$$M_C^\alpha(X; z, W, k, \tau) = \frac{1}{N} \sum_{i=1}^{N} \rho_i(k;\tau) \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{d} w_j g_{ij}^t(\alpha) \tag{2}$$

M_C^α is the population sum of powered censored normalized deprivation gaps divided by the maximum possible value, NT; which arises if and only if $x_{ij}^t = 0 \; \forall (i,j,t) \in [1,N] \times [1,d] \times [1,T]$, for $\alpha > 0$.[8] If $\alpha = 0$ then the maximum is attained if and only if $x_{ij}^t < z_j \; \forall (i,j,t) \in [1,N] \times [1,d] \times [1,T]$.

M_C^α is an extension of the Alkire-Foster multidimensional poverty index to chronic poverty and is an extension of the Foster index to the multidimensional space. M_c^α can be expressed in terms of intuitive partial indices that convey meaningful information on different features of a society's experience of chronic multidimensional poverty. We focus particularly on the first measure in our class, the adjusted headcount ratio of chronic multidimensional poverty, M_C^0, because it can be constructed using ordinal data. The multiplicative decomposition is the following:

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across years; however, the axioms satisfied by such an approach would change; also, the resulting measures would not be associated with the set of intuitive partial indices of H^C, A^C, D^C presented below.

8 The intervals $[1,N]$, $[1,d]$ and $[1,T]$ are all subsets of the set of natural numbers.

$$M_C^0(X;z) = \frac{1}{N}\sum_{i=1}^{N}\rho_i(k;\tau)\frac{1}{T}\sum_{t=1}^{T}c_i^t = H^C \times A^C \times D^C$$

where:

- $\boldsymbol{H^C}$ is the headcount ratio of chronic multidimensional poverty, the percentage of the population that are chronically multidimensionally poor according to k and τ:

$$H^C = \frac{1}{N}\sum_{i=1}^{N}\rho_i(k;\tau)$$

- $\boldsymbol{A^C}$ is the average intensity of poverty among the chronically multidimenisonally poor, or the share of weighted deprivations that chronically poor people experience in the periods in which they are multidimensionally poor:

$$A^C = \frac{P^{c\prime}\sum_{t=1}^{T}C^t}{T \times P^{c\prime}(k;\tau)L} = \frac{\sum_{i=1}^{N}\rho_i(k;\tau)\sum_{t=1}^{T}c_i^t}{\sum_{i=1}^{N}\rho_i(k;\tau)\sum_{t=1}^{T}\rho_i^t(k)}$$

- $\boldsymbol{D^C}$ reflects the average duration of poverty among the chronically poor (i.e. $N \times H^C$) – the average share of T periods in which they experience multidimensional poverty:

$$D^C = \frac{P^{c\prime}(k;\tau)L}{N \times H^C} = \frac{\sum_{i=1}^{N}\rho_i(k;\tau)\sum_{t=1}^{T}\rho_i^t(k)}{N \times H^C \times T}$$

It may also prove useful to assess the duration of dimensional deprivations among the chronically poor. Construct an $N \times d$ censored deprivation duration matrix Q, whose typical entry q_{ij} reflects the share of periods in which person i was chronically poor (by k and τ) *and* was deprived in dimension j. For the chronic poor, $0 \leq q_{ij} \leq 1$ in each dimension, whereas $q_{ij} = 0$ for non-poor persons in all dimensions. Thus the matrix has at least one positive entry for $H^C N$ rows, while the rest of the rows, corresponding to people who are not chronically poor, only have zeroes.

Then the dimensional duration index for dimension j is:

$$D_j = \frac{1}{N \times H^C}\sum_{i=1}^{N}q_{ij}$$

The value of D_j provides the average percentage of periods in which chronically poor people are deprived in dimension j. The relationship between the weighted mean across all D_j and the adjusted headcount ratio of chronic multidimensional poverty is elementary:

$$M_C^0 = H^C\sum_{j=1}^{d}w_jD_j$$

And:

$$\sum_{j=1}^{d}w_jD_j = A^C \times D^C$$

Another interesting relationship between the adjusted headcount ratio of chronic poverty and partial indices pertains to censored headcounts. These represent the proportion of people who are chronically poor *and* deprived in dimension j in period t:

$$CH_j^t = \frac{1}{N}\sum_{i=1}^{N} \rho_i(k;\tau)\, g_{ij}^t(0)$$

Across time, the inter-temporal or longitudinal censored headcount can be defined as:

$$CH_j = \frac{1}{T}\sum_{t=1}^{T} CH_j^t = H_{ch} \times D_{ch} = \frac{1}{N}\sum_{i=1}^{n} I[q_{ij} > 0] \times \frac{1}{N \times H_{ch}}\sum_{i=1}^{N} q_{ij}$$

where H_{ch} is the percentage of individuals who are chronically poor and deprived in at least one period in dimension j over the total population. D_{ch} is the average duration of that deprivation among chronically poor individuals. Weights can be applied to portray the contribution of each dimension to overall chronic poverty in period t. Our chronic multidimensional poverty adjusted headcount ratio across all periods is simply the mean of the weighted average censored headcount ratios across all periods:

$$M_C^0 = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{d} w_j CH_j^t$$

When data are ratio scale and $\alpha = 1$, we compute the adjusted poverty gap, M_1^C, which can also be expressed as follows in an analogous way:

$$M_C^1(X; z, W, k, \tau) = H^C \times A^C \times D^C \times G^C$$

Where:

$$G^C = \frac{1}{N \times T \times M_0^C} P^{c\prime} \sum_{t=1}^{T} G^t(1) W\,'$$

That is, G^C is the average normalized gap that chronically poor people experience in those dimensions in which they are deprived. Likewise, when data are ratio scale and $\alpha = 2$, the adjusted squared gap measure of chronic poverty, M_2^C, is expressed as the product of the following partial indices:

$$M_C^2(X; z, W, k, \tau) = H^C \times A^C \times D^C \times S^C$$

Where:

$$S^C = \frac{1}{N \times T \times M_0^C} P^{c\prime} \sum_{t=1}^{T} G^t(2) W^{\prime}$$

That is, S^C is the average severity, or squared gap, that chronically poor people experience in those dimensions in which they are deprived.

4.1 A class of transient multidimensional poverty measures

Using the same framework we also propose a family of indices of *transient* (multidimensional) poverty, M_{tr}^{α}. The main difference between the two families is in the identification of the poor. We identify a person as transiently poor if $0 < l_i < \tau$. Hence we use a different N-dimensional vector, $P^{tr}(k;\tau)$, for the identification of the transiently poor, such that a typical element, $\omega_i(k;\tau)$, is defined by $\omega_i(k;\tau) = \mathbb{I}(0 < l_i < \tau)$. $\omega_i(k;\tau) = 1$, if and only if, individual i is transiently multidimensionally poor, according to deprivation cut-offs z, weights W, multidimensional cut-off k and duration cut-off τ. The family is:

$$M_{tr}^{\alpha}(X; z, W, k, \tau) = \frac{1}{NT} P^{tr\prime} \sum_{t=1}^{T} G^t(\alpha) W^{\prime} \tag{3}$$

An alternative way of expressing M_{tr}^{α} is:

$$M_{tr}^{\alpha}(X; z, W, k, \tau) = \frac{1}{NT} \sum_{i=1}^{N} \omega_i(k;\tau) \sum_{t=1}^{T} \sum_{j=1}^{d} w_d g_{ij}^t(\alpha) \tag{4}$$

5. Desirable properties

We now define a chronic multidimensional poverty index as a real-valued non-negative function, $\Psi(X; z, W, k, \tau)$, such that $\Psi\colon \bigcup_{N=1}^{\infty} M^N \times \mathbb{R}_{++}^d \times [0,1]^{d+2} \rightarrow [0,1]$, where $X = (X^1, X^2, \ldots, X^T)$. We assume at the outset that the poverty index is normalized between 0 and 1, and that it is scale invariant, i.e. positive scale transformations of the attribute quantities in all the periods and cut-offs do not change the level of poverty. This property shows that the attributes are measurable on ratio scales.

The next axiom ensures that chronic poverty remains unchanged if individuals trade their places:

A1 Anonymity (ANY): Suppose X is obtained from Y as follows: $X^t = BY^t$, where B is an $N \times N$ permutation matrix, and $X^l = Y^l \; \forall l \neq t$. Then $\Psi(X; z, W, k, \tau) = \Psi(Y; z, W, k, \tau)$.[9]

This axiom says that, in the measurement of chronic poverty, only people's achievements, in different periods and in different dimensions, matter.

A2 Time Anonymity (TAN): If the sequence $(Y^1, Y^2, \dots, Y^T)$ in the achievement matrix Y is obtained by a reordering of the sequence $(X^1, X^2, \dots, X^T)$ in the matrix Y, then $\Psi(X; z, W, k, \tau) = \Psi(Y; z, W, k, \tau)$.

This postulate requires that the time-sequencing of the attributes' distributions does not affect the value of the chronic poverty index. It rules out the possibility that longer poverty spells get higher weights in the aggregation. The following axiom enables poverty comparisons among societies with different populations, by measuring it in per capita terms:

A3 Population Replication Invariance (PRI): Let Y be the matrix obtained from a q-fold replication of the achievement matrix X, where $q \geq 2$ is a positive integer; that is, in Y the matrix X appears q times. Then $\Psi(X; z, W, k, \tau) = \Psi(Y; z, W, k, \tau)$.

The following axioms are multidimensional counterparts to Foster's (2009) single dimensional chronic poverty axioms and/or chronic counterparts to multidimensional poverty axioms:

A4 Chronic Poverty Focus (CHF): Suppose person i is not chronically poor in the achievement matrix $X$and the matrix Y is obtained from X as follows: $y_{ij}^t = x_{ij}^t + \delta$ for a triplet (i, j, t), where $\delta > 0$, and $y_{sq}^l = x_{sq}^l \forall (s, q, l) \neq (i, j, t)$. Then $\Psi(X; z, W, k, \tau) = \Psi(Y; z, W, k, \tau)$.

This axiom says that if a person is not chronically poor, then an increase in the quantity of any of his attributes, in any period, does not affect the value of the poverty index. That is, the poverty index is independent of the achievement levels of non-chronically poor people.

A5 Chronic Deprivation Focus (CDF): Suppose person i is chronically poor in the achievement matrix X and the matrix Y is obtained from X as follows: $y_{ij}^t = x_{ij}^t + \delta$ for a triplet (i, j, t), where $x_{ij}^t \geq z_j$, $\delta > 0$ and $y_{sq}^l = x_{sq}^l \forall (s, q, l) \neq (i, j, t)$. Then $\Psi(X; z, W, k, \tau) = \Psi(Y; z, W, k, \tau)$.

This property says that for a chronically poor person who is non-deprived in an attribute in a period, an increase in the quantity of that attribute in the same period leaves poverty unchanged. Thus, if a person

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9 A non-negative $N \times N$ matrix $B = (b_{ij})$ is called a bi-stochastic matrix of order N if all its cells are non-negative, and each of its rows and columns sums to one. A bi-stochastic matrix is called a permutation matrix if there is exactly one positive entry in each row and column.

is not deprived in an attribute, then giving her more of the attribute does not change the extent of chronic poverty, even if she is deprived in one or more of the other dimensions in that period. A trade-off between two attributes is not possible for a person who is deprived in one but not in the other. This does not exclude the possibility of a trade-off if the person is deprived in both attributes.

A6 Chronic Normalization (CHN): $\Psi(X; z, W, k, \tau) = 0$ if and only if $\rho_i(k; \tau) = 0\ \forall i \in (1,2,\ldots,N)$.

According to this axiom, if there are no chronically poor people in society then the poverty index takes the value zero, and vice versa.

Now the analysis of multidimensional poverty across time requires the definition of new assumptions on how a poverty measure should behave. The following axioms, which have not been suggested earlier in the literature, also seem appropriate for a duration-based index:

B1 Chronic Dimensional Monotonicity (CDM): Suppose the achievement matrices Y and X are related as follows: for some period t', some attribute j' and a person i' who is chronically poor in X, $x_{ij}^t \geq z_j, y_{ij}^t = x_{ij}^t - \delta < z_j, \delta > 0$ for $(i,j,t) = (i',j',t')$, and $y_{ij}^t = x_{ij}^t\ \forall (i,j,t) \neq (i',j',t')$. Then $\Psi(X; z, W, k, \tau) < \Psi(Y; z, W, k, \tau)$.

According to this axiom, if a chronically poor person who is non-deprived in a dimension but poor in a period becomes deprived in the dimension in that period, then chronic poverty increases.

B2 Chronic Weak Monotonicity (CHM): Suppose person i is chronically poor in the achievement matrix X and the matrix Y is obtained from X as follows: $y_{ij}^t = x_{ij}^t - \delta$ for a triplet (i,j,t), where $x_{ij}^t < z_j, \delta > 0$ and $y_{sq}^l = x_{sq}^l \forall (s,q,l) \neq (i,j,t)$. Then$\Psi(X; z, W, k, \tau) \leq \Psi(Y; z, W, k, \tau)$.

This property says that if a person who is chronically poor becomes more deprived in an attribute, then poverty does not decrease. A strong version of CHM, requiring the poverty measure to increase strictly when a chronically poor person becomes more deprived in an attribute is also worth stating, since it is relevant for poverty measures based on cardinal variables:

B2a Chronic Strong Monotonicity (CHMS): Suppose person i is chronically poor in the achievement matrix X and the matrix Y is obtained from X as follows: $y_{ij}^t = x_{ij}^t - \delta$ for a triplet (i,j,t), where $x_{ij}^t < z_j, \delta > 0$ and $y_{sq}^l = x_{sq}^l \forall (s,q,l) \neq (i,j,t)$. Then$\Psi(X; z, W, k, \tau) < \Psi(Y; z, W, k, \tau)$.

B3 Time Monotonicity (TIM): Suppose the achievement matrices Y and X are related as follows: for some period t', some attribute j' and a person i' who is chronically poor in Y, $y_{ij}^t < z_j \leq x_{ij}^t$, $(i,j,t) = (i',j',t')$ and $y_{ij}^t = x_{ij}^t \forall (i,j,t) \neq (i',j',t')$. Then $\Psi(X; z, W, k, \tau) < \Psi(Y; z, W, k, \tau)$.

This postulate says that, for a chronically poor person, an increase in the duration of poverty experienced in a dimension leads to an increase in poverty.

B4 Chronic Monotonicity in Thresholds (CMT): Let the vector of cut-off points z be transformed into the vector z^*, where $z_j^* = z_j + \beta$ for some $j \in (1,2,\dots,d)$, $z_q = z_q^*\ \forall q \neq j$ and $\beta > 0$ is a constant. Then given the achievement matrix X: $\Psi(X; z^*, W, k, \tau) \geq \Psi(X; z, W, k, \tau)$.

This axiom says that an increase in the deprivation threshold of a dimension does not decrease the chronic poverty associated with a given achievement matrix X.

B5 Monotonicity in Multidimensional Poverty Identifier (MMI). Given the achievement matrix X and (z, W, τ), $\delta > 0$, then $\Psi(X; z, W, k, \tau) \geq \Psi(Y; z, W, k + \delta, \tau)$.

Since an increase in the value of *k* may reduce the number of poor people, although the intensity of their poverty may rise, the poverty index does not increase.

B6 Chronic Duration Monotonicity (CDUM): Given the achievement matrix X and (z, W, τ), $\gamma > 0$, then $\Psi(X; z, W, k, \tau) \geq \Psi(X; z, W, k, \tau + \gamma)$.

Resembling the rationale of MMI, a higher duration cut-off cannot increase the number of people identified as chronically poor.

None of the axioms stated so far deals with the inequality among the chronically poor. In the case of cross-sectional income poverty, if there is a (progressive) transfer of income from a richer poor to a poorer poor that does not change their relative positions, then we say that the post-transfer income distribution of the poor is obtained from the pre-transfer one by a 'smoothing of incomes'. This reduces inequality in the income distribution of the poor (Sen, 1976). In multidimensional measurement, smoothing requires that poverty should not increase under (progressive) transfers of attribute quantities from richer poor to poorer poor persons, given the relative positions of the donors and the recipients. This is achieved if the post-transfer achievement matrix of the chronically poor in any period can be expressed as the product of a bi-stochastic matrix and the pre-transfer achievement matrix in the period (Kolm, 1977).

For any $t \in (1,2,\dots,T)$, we say that X^t is obtained from Y^t by an averaging or smoothing of achievements among the chronically poor if $X^t = BY^t$ for some non-permutation bi-stochastic matrix B of order N such that $b_{ii} = 1$ for every non-chronically poor person i in Y^t. The condition $b_{ii} = 1$ ensures that the distributions of the attributes among the non-chronically poor remain unaffected and that smoothing occurs only among the chronically poor (Alkire and Foster, 2011). Hence inequality of the chronically poor in X^t is not higher than that in Y^t. If B is a permutation matrix, then the rows of X^t are a rearrangement of the rows of Y^t.

We can now formally state the following:

B7 Chronic Weak Transfer (CHT): If the achievement matrix Y is transformed into the matrix X as follows: For any arbitrary $t \in (1,2,\dots,T)$, X^t is obtained from Y^t by an averaging among the chronically poor and $X^l = Y^l \ \forall\ l \neq t$, then $\Psi(X;z,W,k,\tau) \leq \Psi(Y;z,W,k,\tau)$.

A transfer from a less chronically poor individual to a more chronically poor individual in a defined period should not increase the poverty index. A strong version of CHT, requiring the poverty measure to increase strictly when a chronically poor person becomes more deprived in an attribute, is also worth stating, since it is relevant for poverty measures based on cardinal variables:

B7a Chronic Strong Transfer (CHTS): If the achievement matrix Y is transformed into the matrix X as follows: For any arbitrary $t \in (1,2,\dots,T)$, X^t is obtained from Y^t by an averaging among the chronically poor and $X^l = Y^l \ \forall\ l \neq t$, then $\Psi(X;z,W,k,\tau) < \Psi(Y;z,W,k,\tau)$.

Following Tsui (2002), Bourguignon and Chakravarty (2003), and Alkire and Foster (2011), we also propose an inequality axiom related to transfers between pairs of chronically poor people that reduce the degree of association between the dimensions. We say that X^t is obtained from Y^t by an association-decreasing switch among the poor; if for a pair of chronically poor people, i and i', it is the case that: 1) $y_{ij}^t \geq y_{i'j}^t \forall j \in \{1,2,\dots,d\}$, 2) $\exists j | x_{ij}^t \leq x_{i'j}^t$; and 3) $y_{qj}^t = x_{qj}^t \forall q \neq i,i'$. That is, the vector dominance of i over i' is broken by the association-decreasing switch. The following property describes one way in which a chronic multidimensional poverty measure should react to association-decreasing switches (see Bourguignon and Chakravarty, 2003):

B8 Non-increasing Chronic Poverty under Association-decreasing Switch (NIPA): Suppose X^t is obtained from Y^t by an association-decreasing switch among the poor and $Y^l = X^l \ \forall l \neq t$.Then $\Psi(X;z,W,k,\tau) \leq \Psi(Y;z,W,k,\tau)$.

Alternatively, we could also consider a property of non-decreasing chronic poverty under association decreasing switches (NDPA), as well as a property of poverty insensitivity to association-decreasing switches.

Finally, the next postulate ensures the coherence between local and global assessments of chronic poverty:

C1 Additive Subgroup Decomposability (ASD): For an arbitrary subgroup division of the achievement matrix X into m matrices X_1 through X_m, each with respective subgroup populations of N_1 through N_m: $\Psi(X;z,W,k,\tau)=\sum_{q=1}^{m}\frac{N_q}{N}\Psi(X_q;z,W,k,\tau)$.

This axiom says that for any partitioning of the population into m ($\in \mathbb{N}$) subgroups, overall chronic poverty is given by the population-share weighted average of the subgroup chronic poverty levels. Thus, if chronic poverty in one subgroup decreases (increases), while remaining unchanged in other subgroups, then global poverty falls (rises). The latter property, known as subgroup consistency, is fulfilled by any measure satisfying ASD.

The following theorem describes the behaviour of M_C^{α} in terms of its fulfilment of the axioms introduced above:

Theorem 1: $M_C^{\alpha}(X;z,W,k,\tau)$ satisfies ANY, TAN, PRI, CHF, CDF, CHN, CDM, CHM, TIM, CMT, MMI, CDUM, CHT, NIPA, and ASD for all $\alpha \geq 0$. $M_C^{\alpha}(X;z,W,k,\tau)$ also satisfies CHMS for $\alpha > 0$, and CHTS when $\alpha \geq 1$. Proof: Available upon request.

The following theorem describes the behaviour of M_{tr}^{α} in terms of its fulfilment of the axioms introduced in the previous section:

Theorem 2: $M_{tr}^{\alpha}(X;z,W,k,\tau)$ satisfies ANY, TAN, PRI, CDF, TIM, CMT, MMI, and ASD for all $\alpha \geq 0$.

Note that several axioms are not fulfilled by M_{tr}^{α}. In many cases the reason is that the axioms are stated for chronic poverty. For example, CHF states that improvements in an attribute of a non-chronically poor person should not affect the poverty measure. However, that person could still be transiently poor, in which case a transient measure, sensitive to the poverty status and intensity of that person, may be affected. However it is straightforward to show that the following axioms can also be fulfilled by M_{tr}^{α} if they are rephrased in terms of transiently poor people: CHF, CHM, CHT, CHN, CDM and NIPA. In the case of CDUM, an increase in the duration cut-off does not decrease transient poverty (as opposed to not increasing chronic poverty). Proof: available upon request.

6. Empirical illustration

In this section we study chronic multidimensional poverty in Chile with a panel dataset whose data points are 1996, 2001 and 2006. These years relate to three identifiable GDP growth experiences. First, in 1996 Chile began one of its most successful decades of GDP growth and income poverty reduction (Contreras, 2003; Contreras et al., 2001). In 2001 the country suffered from the negative impact of the

Asian crises (Corbo and Schmidt-Hebbel, 2010), and in 2006 a public policy response to lower growth rates was implemented (Galasso, 2011; Glick and Menon, 2009). We provide one empirical illustration with ordinal variables and another one with cardinal variables. The next subsection discusses the data and the choice of well-being indicators. Then the application with ordinal variables is described, followed by the application with cardinal variables. We also provide estimates of dimensional and period contributions to overall chronic multidimensional poverty.

6.1 Data and indicators

The CASEN (National Survey of Economic Characterization) panel follows households in three regions (covering 60% of Chile's population) in three rounds: 1996, 2001 and 2006. The panel survey began in 2001 when the Chilean government, together with the University of Chile, selected a representative subsample of 5,209 households (20,942 individuals) based on the cross-sectional survey of 1996.[10] This is an illustration; consequently, extremely disaggregated results might not fully represent the respective population.

We provide two illustrations of the chronic poverty indices. First we compute a measure providing a broad understanding of multidimensional poverty by including ordinal variables. When ordinal, categorical or binary variables are included, we calculate only M_c^0 and M_t^0. Second, three continuous variables are used to construct a chronic multidimensional poverty measure sensitive to the deprivation gap of each indicator. With continuous indicators we can compute M_c^α and M_t^α for any level of α, thus generating information on the breadth and severity of chronic poverty.

The survey's breadth allows for the computation of multiple well-being indicators. As with most household surveys, the questions elicit information on command over resources and functionings, rather than capabilities. There are examples of several choices of well-being dimensions and respective indicators made in the literature. Asselin (2009) presents a summary of commonly used dimensions. Our choices were constrained by the need to guarantee longitudinal comparability (e.g. changes in questionnaires preclude using certain indicators).

We select three dimensions: education, housing and employment/income. For the ordinal illustration three indicators are selected in each dimension; for the cardinal illustration one indicator is selected in

10 The survey is deemed one of the longest panel datasets for a developing country with longitudinal and cross-sectional representativeness (Dercon and Shapiro, 2007). By design, it tends to overestimate income poverty levels vis-à-vis national ones by approximately 5%. Inflation factors were produced in order to adjust for attrition among young (20–29 years) and elderly people (over 60) in large households, and in rented dwellings (Bendezu et al., 2007). To correct for attrition, sample weights for longitudinal consistency were implemented; consequently, results are not comparable with cross-sectional data from 2006.

each dimension. Table 1 presents our chosen set of dimensions and indicators for both illustrations; additionally, deprivation ratios per indicator (raw headcounts) are included.

6.2 Ordianl illustration

The ordinal illustration provides a comprehensive picture of multidimensional poverty whose dimensions and indicators are shown in Table 1.

Table 1: Dimensions, Indicators, Weights and Uncensored Headcounts

Dimension	Indicator	Deprivation Cut-off: *An individual is deprived if he/she lives in a household with…*	Weights: Cardinal illustrat.	Weights: Ordinal illustrat.	Raw Headcounts[11]: 1996	Raw Headcounts[11]: 2001	Raw Headcounts[11]: 2006
Education	Educational Achievement	no household member fulfilling the legal number of compulsory years of education relevant to their birth cohort[12]	1/3	1/9	8% (7% -10%)	6% (5% -7%)	5% (4% -6%)
	School Attendance	at least one individual of school age (6 to 17 years) not attending school, or evidencing a gap greater than 3 years between his/her highest achieved school year and the appropriate school year by the individual's age.		1/9	9% (7% -10%)	7% (5% -9%)	5% (4% -7%)
	Illiteracy	at least one member older than 17 not able to read or write[13]		1/9	8% (7% -10%)	7% (5% -8%)	5% (4% -6%)
Housing	Overcrowding	more than 2.5 persons per bedroom as defined by the Chilean Ministry of Social Development[14]	1/3	1/9	17% (14% -20%)	12% (10% -14%)	8% (7% -10%)
	Shelter	insufficient housing materials as defined by the Chilean Ministry of Social Development[15] (one or more deprived indicators for walls, floor or roof)		1/9	44% (39% -48%)	37% (33% -42%)	38% (34% -42%)
	Toilet	at least 1 toilet in the household[16]		1/9	19% (15% -23%)	12% (10% -15%)	6% (4% -7%)
Income - Employment	Income	a per capita income lower than the relevant national poverty line defined by the Social Planning Ministry	1/3	1/9	24% (20% -27%)	21% (17% -24%)	11% (9% -12%)
	Unemployment	no member older than 17 is employed[17]		1/9	6% (5% -7%)	10% (8% -12%)	8% (7% -10%)
	Quality of Employment	no member older than 17 has access to the pension system or has signed contract – excluding rentiers, pensioners and entrepreneurs as defined by the Chilean Law		1/9	22% (19% -26%)	23% (19% -26%)	22% (19% -25%)

11 In parentheses: lower and upper 95% confidence intervals.

12 In 1920, the Law 3.654 defines primary education as compulsory. In 1929, the Decree 5.291 extends this regulation to 6 years. Then, in 1965, Government Decree 27.953 increases the levels of compulsory education to 8 years. Finally, in 2003, the Constitutional Law 19.876 sets the minimum compulsory schooling to 12 years.

13 The Chilean Government defined a set of policies to promote literacy regardless the age of the individuals (Contigo Aprendo). This indicator differs from schooling because it tries to capture the skill of literacy of each individual in the household. Consequently, if one individual is deprived the entire household is deprived. Conversely, in the schooling indicator if one individual has enough school the household is immediately non-deprived.

14 Available at http://www.ministeriodesarrollosocial.gob.cl/casen/definiciones/vivienda.html

15 Deprived walls: adobe, wall without interior protection, mud, thatch, artisanal construction, rubbish, cardboard, tin or rubber. Deprived roof: clinkstone, straw, bulrush, rubbish or cane. Deprived floor: no protected cement foundation.

16 There is no additional qualitative information regarding the type of toilet.

17 In Narayan (2000), individuals remark about the relevance of employment not only for the pecuniary benefits but also due to social and other outcomes.

Table 2 shows cross-sectional and longitudinal results with $k = \frac{3}{9}$. Full results for several other cut-off combinations are available in Table 3. Similar to the cardinal illustration results below, cross-sectional multidimensional poverty falls from 0.071 to 0.028 between 1996 and 2006. Most of the improvement is due to a lower headcount ratio, whereas the intensity level declines marginally. Clearly, the largest contributors to multidimensional poverty are housing, toilet, overcrowding, quality of employment and income.

Table 2: Cross-sectional and Longitudinal Poverty Measures with $k = \frac{3}{9}$

	Cross-sectional Results			*Longitudinal Results*		
Main Statistics	***1996***	***2001***	***2006***	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$
Headcount Ratio (H/Hc)	13.93%	9.98%	5.66%	18.32%	8.49%	2.77%
Duration (/Dc)	-	-	-	53.82%	77.53%	100%
Intensity (A/Ac)	51.07%	49.85%	49.96%	50.44%	51.77%	52.45%
Adjusted Headcount Ratio (M0/M0c)	0.071	0.050	0.028	0.050	0.034	0.015
Censored Headcount	***Cross-sectional Censored Headcount***			***Longitudinal Censored Headcount***[18]		
Overcrowding	6.52%	4.86%	2.25%	4.54%	2.92%	1.06%
Housing	13.07%	9.10%	5.14%	9.10%	6.01%	2.42%
Toilet	10.34%	6.59%	2.60%	6.51%	4.43%	1.62%
Attendance	4.28%	2.51%	1.94%	2.91%	1.96%	0.97%
Schooling	5.42%	3.28%	2.17%	3.63%	2.83%	1.45%
Illiteracy	4.22%	2.68%	1.46%	2.79%	2.18%	1.22%
Employment	1.87%	2.33%	1.92%	2.04%	1.43%	0.75%
Employment Quality	8.02%	6.15%	4.04%	6.07%	4.27%	1.75%
Income	10.28%	7.26%	3.96%	7.17%	4.64%	1.82%
Percentage Contribution	***Percentage Contribution to M0***			***Percentage Contribution M0c***		
Overcrowding	10.13%	10.40%	8.21%	10.15%	9.52%	8.15%
Housing	20.32%	19.47%	18.76%	20.34%	19.60%	18.55%
Toilet	16.08%	14.09%	9.49%	14.55%	14.43%	12.40%
Attendance	6.65%	5.37%	7.08%	6.50%	6.41%	7.42%
Schooling	8.43%	7.03%	7.94%	8.10%	9.24%	11.11%
Illiteracy	6.57%	5.73%	5.34%	6.23%	7.12%	9.31%
Employment	2.91%	4.99%	7.01%	4.56%	4.66%	5.75%
Employment Quality	12.46%	13.16%	14.74%	13.56%	13.91%	13.37%
Income	15.98%	15.52%	14.45%	16.01%	15.12%	13.94%

The longitudinal results show that under the time union approach ($\tau = \frac{1}{3}$), 18.32% of the population is poor, experiencing poverty spells during 53.82% of the periods in 50.44% of the possible dimensions.[19] The chronic adjusted headcount ratio in this case is 0.05. When $\tau = 1$, only 2.77% of the population is

18 Average censored headcount among those chronically poor or $\frac{1}{T}\sum_{t=1}^{T} CH_j^t$.

19 We note that the concept of chronic poverty would only be meaningful when $\tau > \frac{1}{T}$.

chronically poor in 52.45% of their dimensions. The chronic adjusted headcount ratio for intersection approach is 0.015.

Following Yalonetzky (2011) and accounting for the complex survey design, standards errors and confidence intervals for the longitudinal adjusted headcount ratio can also be calculated for each poverty and temporal cut-off. Figure 1 shows the results.

Figure 1: Longitudinal Multidimensional Poverty by Poverty (k) and Temporal (t) Cut-off

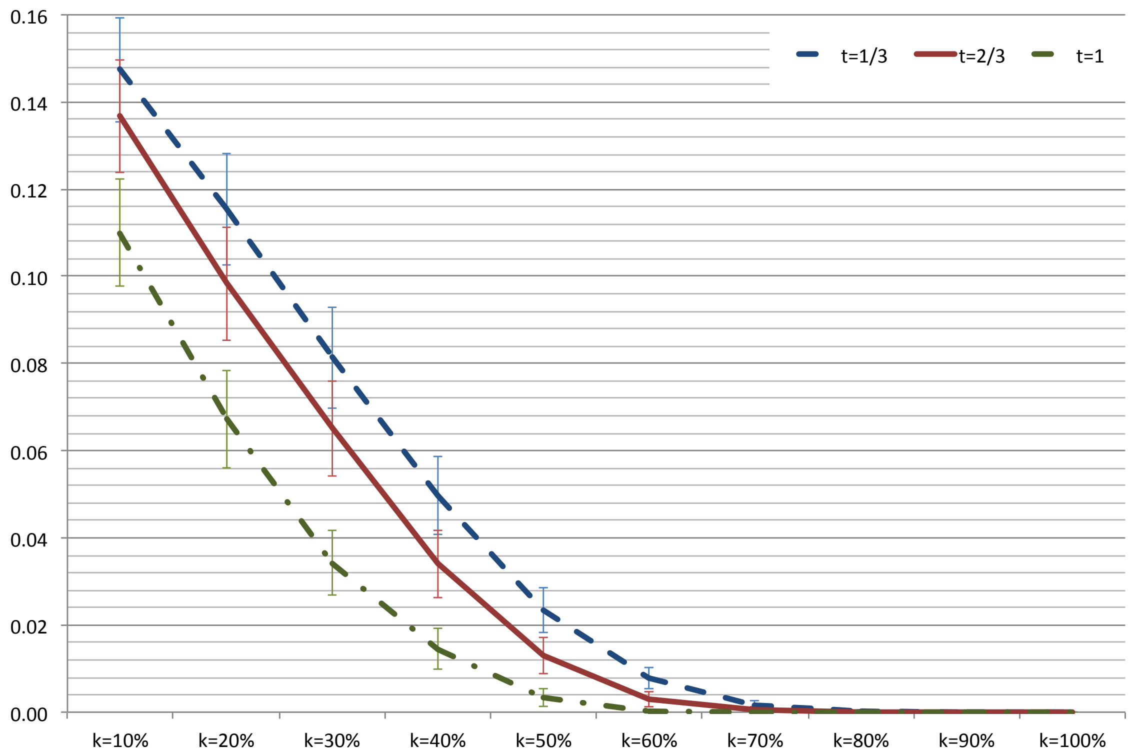
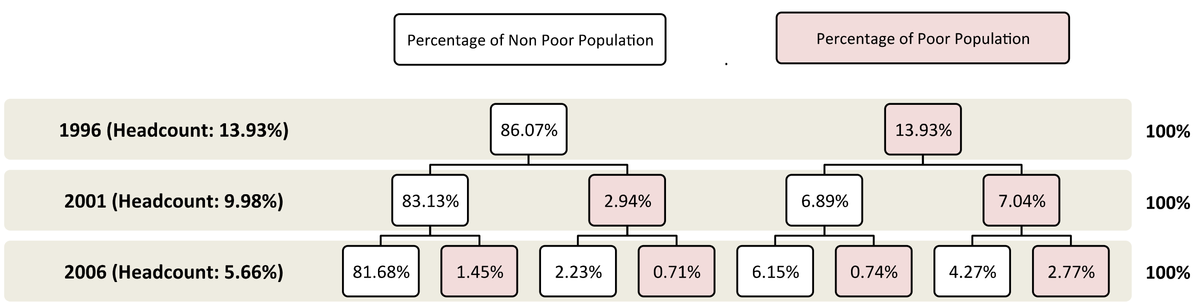


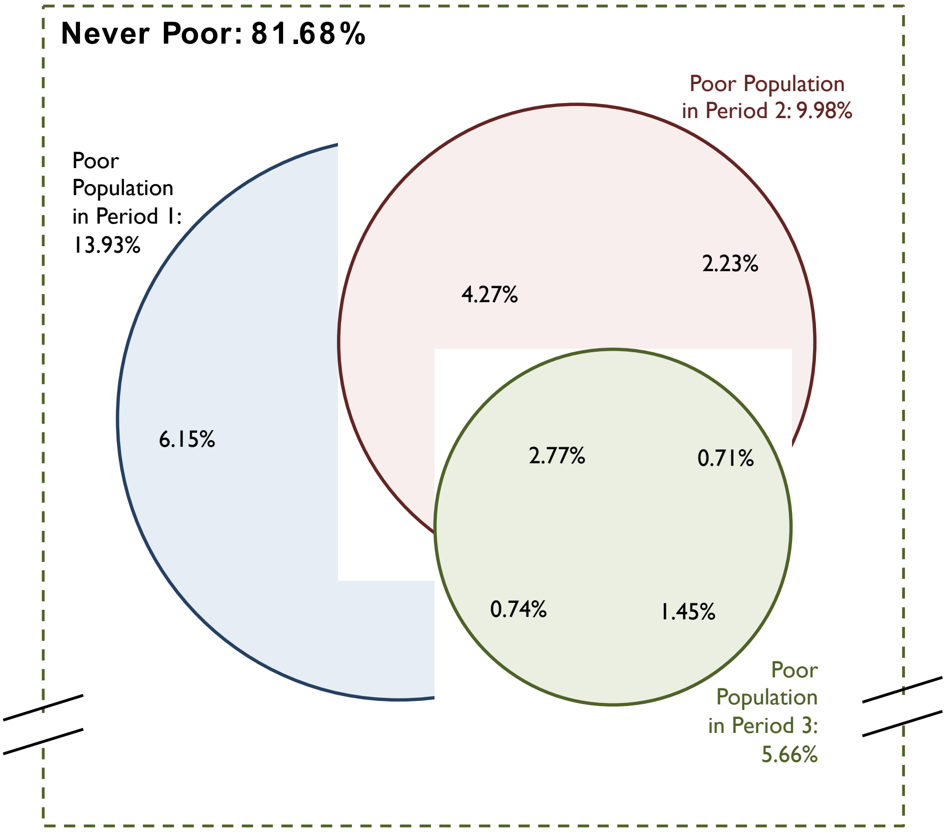
Figure 2 displays the transitions into and out of poverty spells in a way that highlights the connection between the year-specific poverty headcounts and their chronic counterparts for different choices of τ, similar to the Venn diagram in Figure 3.

Figure 2: Transitions Entry and Exit from Multidimensional Poverty ($k = \frac{3}{9}$)



For instance, with $\tau = \frac{1}{3}$, the chronic poverty headcount of 18.32% is equal to the headcount of 1996 (13.93%) plus the new poor in 2001 (2.94%) and the new poor in 2006 (1.45%). With $\tau = 1$, the chronic poverty headcount is compounded by those who were always poor (2.77%). The longitudinal intersection approach suggests that with $\tau = \frac{2}{3}$, the chronic poverty headcount of 8.49% is equal to the percentage of individuals who are always poor (2.77%) plus those who were poor in the first and last period (0.74%) and those who became poor in the second period and remained in that condition until the last period (0.71).

Figure 3: Longitudinal Multidimensional Poverty $k = \frac{3}{9}$



Following Figures 2 and 3, we can identify and compute different headcounts of chronic and transient poverty using different time cut-offs. For each of these groups of poor people we can also compute complementary measures of incidence, duration and intensity using the methods described in Section 4.

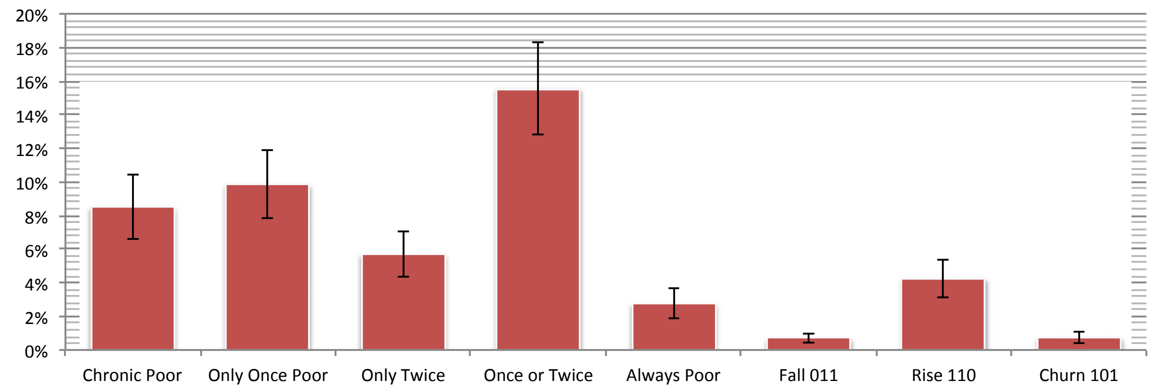
Table 3: Chronic and Transient Poverty for Selected Groups with $k = \frac{3}{9}$

	Chronic Poor (τ=2/3)	*Only Once Poor*	*Only Twice poor*	*Once or Twice poor*	*Always Poor*	*Fall 011*	*Rise 110*	*Churn 101*
Headcount Ratio (Hc)	8.49%	9.83%	5.72%	15.55%	2.77%	0.71%	4.27%	0.74%
Duration (Dc)	77.53%	33.33%	66.67%	45.60%	100.00%	66.67%	66.67%	66.67%
Adj Av Dep Share (Ac)	51.77%	47.77%	51.28%	49.66%	52.45%	49.36%	52.12%	48.29%
Adj Headcount ratio (M0c)	0.03	0.02	0.02	0.04	0.01	0.00	0.01	0.00

**Fall 011: Non-poor in 1996, then poor in the subsequent periods. **Rise 110: Poor in 1996 and 2001, then non-poor in 2006. ***Churn 101: Poor in 1996, non-poor in 2001, poor in 2006.*

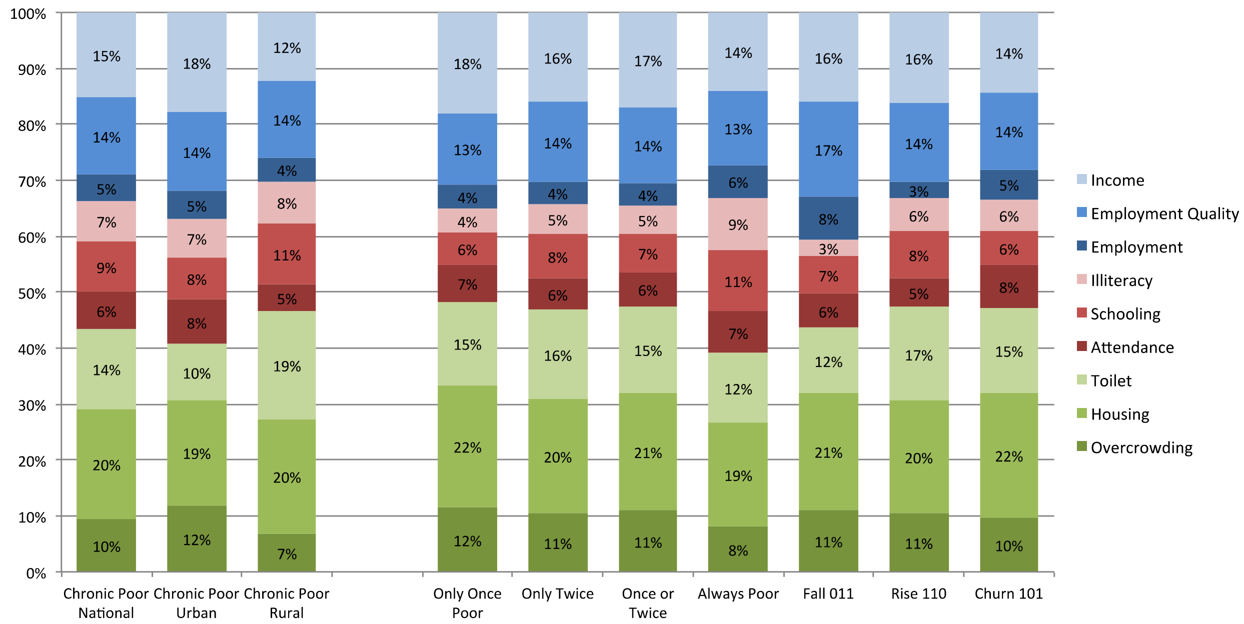
Table 3 and Figure 4 show the adjusted headcount ratio and its components for different groups of poor people identified by different criteria of chronicity and transiency. Clearly, transient poverty is more prevalent than chronic poverty, although the average intensity of poverty (second-to-last row in Table 3) is lower among the chronically poor in the Chilean case.

Figure 4: Headcount Ratios of Chronic and Transient Poverty for Selected Groups with $k = \frac{3}{9}$



Additionally, we can assess the contribution of each deprivation to the adjusted headcount ratio of each one of the above poverty groups. The contributions are based on the censored headcounts, i.e. the proportions of people who are poor (e.g. chronically or transiently) *and* deprived in a specific variable.

Figure 5: Dimensional Breakdown of Longitudinal Poverty in Selected Poverty Groups ($k = \frac{3}{9}$)



In Figure 5 the first three leftmost bars decompose poverty by dimensions for the chronically poor at the national level and by urban and rural areas. The contributions of toilet, housing and schooling are most significant in rural locations, while income and housing are prominent in cities. Similar analyses can be performed for the other poverty groups (see six rightmost bars in Figure 5). More censored headcounts and relative contributions, for different choices of k and τ are available in the tables of Section b in the Appendix.

The contribution results are based on the longitudinal censored headcount of each indicator, and they can be calculated as the average of censored headcounts across time for those individuals living in each condition of chronic or transient poverty. However, it does not capture explicitly the duration of the deprivation.

Figure 6: Duration of Deprivation (D_j) in Chronic Multidimensional Poverty by Zone ($k = \frac{3}{9}, \tau = \frac{2}{3}$)

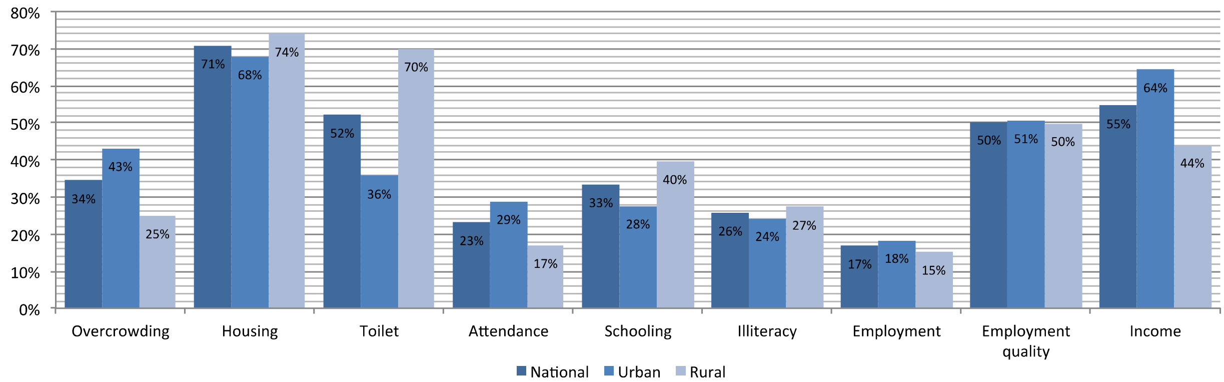
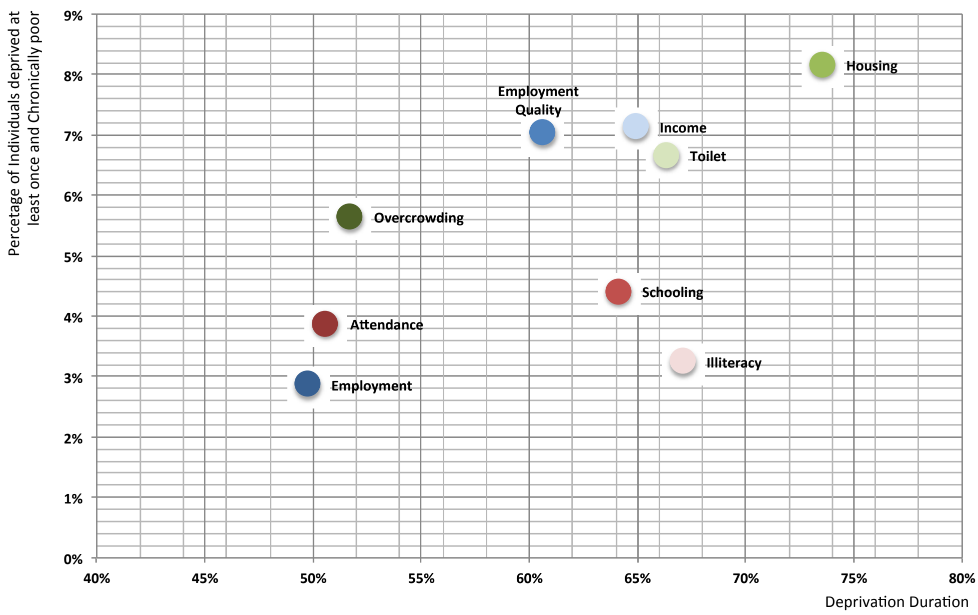


Figure 6 shows the duration of the deprivation in each dimension (D_j) at the national, urban and rural level. The figure shows the persistence of each deprivation among those individuals who are identified as chronic multidimensional poor. On an average, an individual in chronic multidimensional poverty is deprived in overcrowding 34% of the time and 43% in urban areas and 25% in rural areas. Housing shows the highest duration at the national level, indicating reduced improvements in the dimension over time. The duration of deprivation is higher in urban areas for overcrowding, school attendance, employment and income.

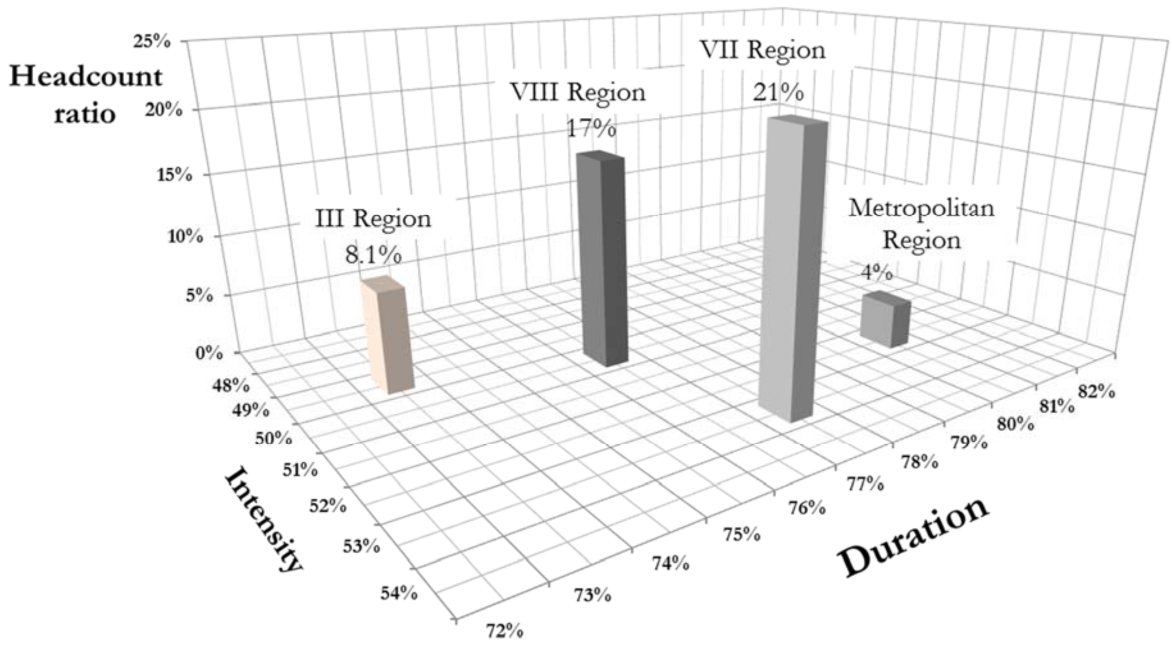
Figure 7: Decomposition of Censored Headcounts in H_{ch} and D_{ch} for Chronic Poverty Population ($k = \frac{3}{9}, \tau = \frac{2}{3}$)



The longitudinal censored headcount can also be decomposed by the percentage of people who are chronically (or transiently) poor and deprived in dimension j for at least one period (H_{ch}) and the average duration of the deprivation in this subgroup (D_{ch}). In Figure 7, the area under the point represents the longitudinal censored headcount. For instance, more than the 8% of the population have experienced housing deprivation and chronic poverty; on an average, they have been deprived for 74% of the time. It is important to note that the percentage of individuals deprived in employment and illiteracy are similar (around 3%). However, illiteracy is a more persistent deprivation.

Finally, Figure 8 compares the situation of four regions based on the average deprivation share (A^c) – or intensity, the duration of poverty, and the chronic poverty headcount for $k = \frac{3}{9}$ for those individuals who are chronically poor. The metropolitan region presents the lowest level of poverty with the lowest headcount but the highest duration. Compared to the metropolitan region, the III region has twice the percentage of poverty but lower duration and intensity. The VII region has the highest proportion of chronically poor people (nearly 21%), although its duration is below that of the III region. The VIII region exhibits a similar level of duration to the VII region but with a lower headcount and intensity. In each case, the volume represented by the headcount times the duration times the intensity represents the level of multidimensional poverty.

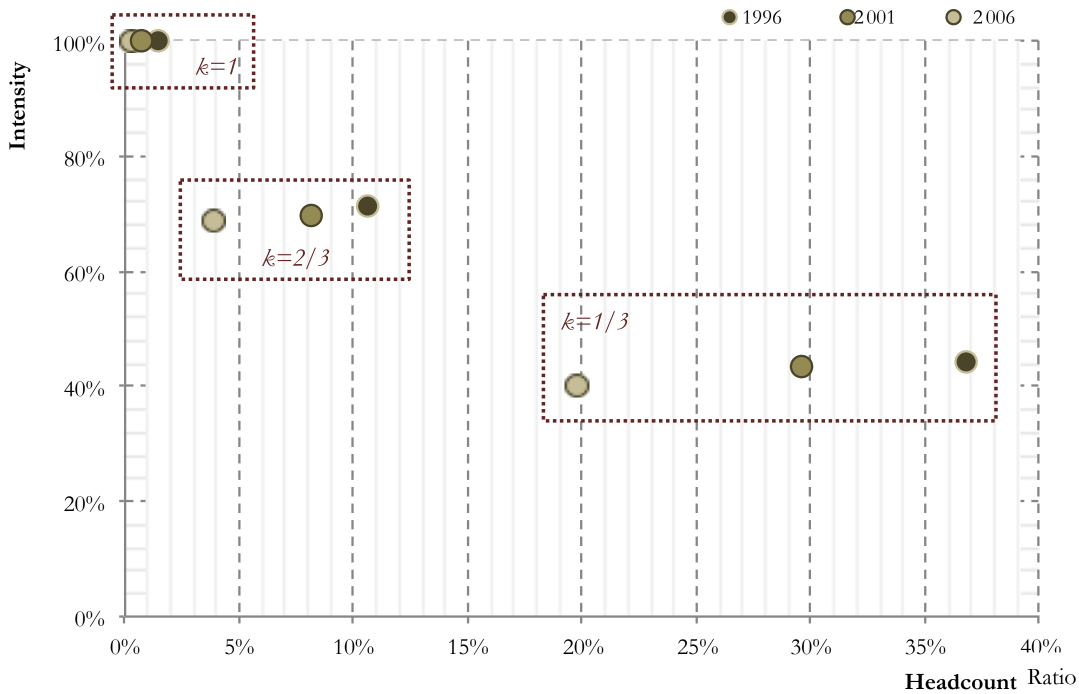
Figure 8: Chronic Multidimensional Poverty by Region ($k = \frac{3}{9}$, $\tau = \frac{2}{3}$)



6.3 Cardinal illustration

The evolution of the three indicators used in the cardinal illustration shows similar patterns. The deprivation headcounts of schooling, overcrowding and income fell between 1996 and 2006. Figure 9 shows that multidimensional poverty reduction is apparent across years and robust to choices of the poverty cut-off and the measure (adjusted headcount ratio, adjusted gap and adjusted squared gap).

Figure 9: Headcount Ratio by Poverty Cut-off and Year



In terms of decomposition, income is the most important dimension, followed by housing and education. In this example, changes in multidimensional poverty are mainly explained by a reduction in the headcount ratio (Table 2b in the Appendix, section b).

Figure 10: Headcount Ratio with All Possible Poverty (k) and Time (τ) Cut-offs

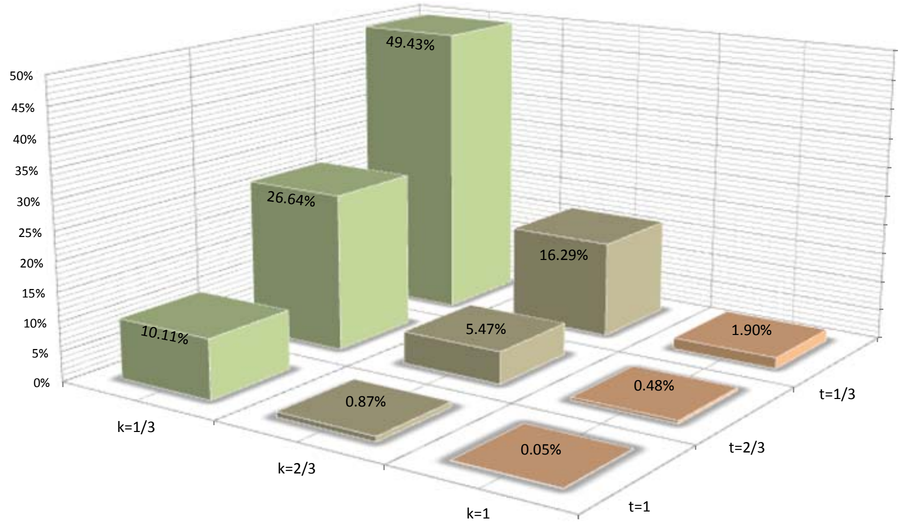


Table 4: Headcount Ratio for relevant k and τ cutoffs

	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$
$k = \frac{1}{3}$	49.43%	26.64%	10.11%
$k = \frac{2}{3}$	16.29%	5.47%	0.87%
$k = 1$	1.90%	0.48%	0.05%

Figure 10 shows the headcount ratio for all possible combinations of poverty (k) and time (τ) cut-offs. A double union approach ($k = \frac{1}{3}$ and $\tau = \frac{1}{3}$) identifies 49.43% of the population as chronically poor with an average duration (D^c) of 58.12% periods and an intensity (A^c) of 43.02%. On the other extreme, a double intersection approach ($k = 1$ and $\tau = 1$) identifies only 0.05% of the population as chronically poor, with an average duration and intensity equal to 1. With an intermediate approach of $k = \frac{2}{3}$ and $\tau = \frac{2}{3}$, 5.47% of the population would be identified as chronically poor with an intensity of 72.1% and a duration of 71.95%. Detailed information can be found in Table 4 above.

With a poverty cut-off of $k = \frac{1}{3}$, the intersections among poor populations in every year are represented in the Venn diagram of Figure 11. In 1996, 2001 and 2006, 10.11% of the population was poor. They are the chronically poor for $k = \frac{1}{3}$ and $\tau = 1$ (as in Figure 10). We can further deduce that the proportion of transiently poor people was 39.31%, given the proportions of chronically poor people (10.11%) and never-poor people (50.57%, top left of Figure 11). Likewise, other headcounts of chronic and transient poverty can be computed from the diagram, which also provides information about the transition of entry into and exit from poverty.

Figure 11: Multidimensional Transitions 1996-2001-2006 ($k = \frac{1}{3}$)

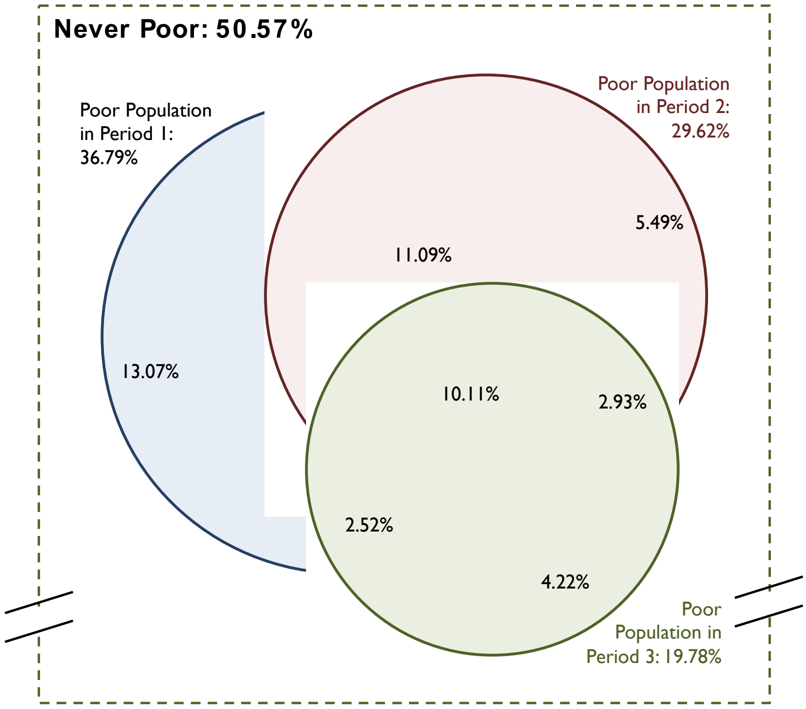
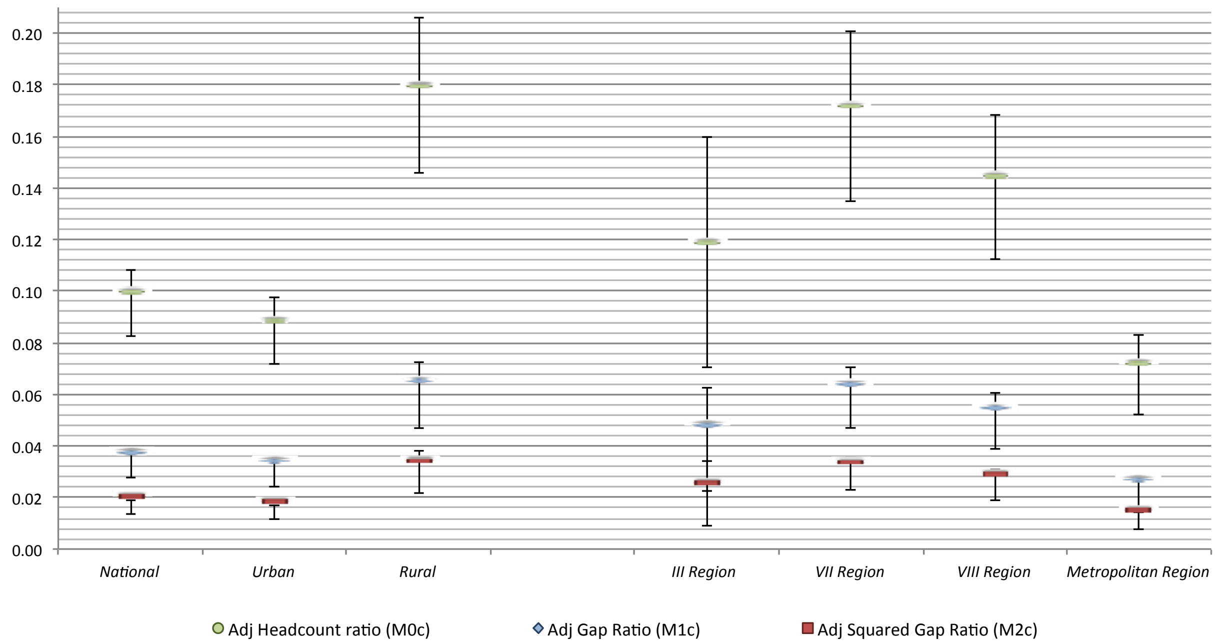


Figure 12 presents the regional breakdown of chronic poverty for $\alpha = 0,1,2$, using $k = \frac{1}{3}$ and $\tau = \frac{2}{3}$. This is an interesting case showing that higher breadth of deprivations does not necessarily mean higher intensity. Results for the three measures and all natural cut-offs appear in Table 3b (Appendix, section b).

Figure 12: Chronic Multidimensional Poverty by Region with $k = \frac{1}{3}$ and $\tau = \frac{2}{3}$, for different α



7. Conclusions

It has been argued explicitly in the literature that poverty should be measured multidimensionally in terms of shortfalls of well-being attributes from minimally acceptable levels defined for different individuals in a society. Since, for many people worldwide, poverty is a situation from which it is difficult to escape over time, often it becomes important to track it over multiple periods. This, of course, requires panel data on different dimensions of well-being. Following Foster's (2009) income-based analysis, we have considered the spell, or duration, approach to chronic multidimensional poverty. We have defined multidimensional poverty following Alkire and Foster (2011). In this context two notions of identification are present: the identification of the multidimensionally poor in each period and the minimum number of periods a person has to spend in poverty in order to be identified as chronically poor.

The indices of chronic and transient poverty proposed in this paper represent the most straightforward merger between the snapshot multidimensional poverty and the duration approaches to chronic poverty. Being both counting approaches to poverty measurement, they blend naturally. As illustrated by the comparison of this paper's proposal with that of Nicholas and Ray (2011), there is scope for further developments on suitable indices of inter-temporal, multidimensional poverty, but these have costs in terms of policy relevance if they do not allow dimensional breakdown (Chakravarty et al. 1998; Alkire and Foster, 2013). Future research should study the theoretical, empirical and policy implications of combining different approaches to the identification and measurement of multidimensional poverty with different ways of understanding, indentifying and measuring chronic and transient poverty.

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Appendix

a. Comparison with other approaches

The measures of Nicholas and Ray (2011)

Nicholas and Ray (2011) proposed the first inter-temporal extension of a multidimensional poverty index. Their measures combine the multidimensional approach of Chakravarty and D'Ambrosio (2006) with the inter-temporal poverty approach of Bossert et al. (2012) and Gradin et al. (2012). Expressed in our notation, their family of indices is the following:

$$\Omega_\beta(X;z) = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{dT}\sum_{j=1}^{d}\sum_{t=1}^{T} g_{ij}^{t}(0)s_{ijt}\right]^{\beta} \quad \text{(A1)}$$

Where $\beta \geq 0$, and s_{ijt} is a weight that depends on the length of the deprivation spell to which the deprivation experience of individual i in dimension j and period t belongs.[20] A first fundamental difference between (A1) and (1) is that (A1) neither identifies the chronically poor, nor explicitly distinguishes them from the transiently poor. That is, (A1) is an index of inter-temporal poverty, but not of chronic poverty. Implicitly, it adopts a union approach to both chronic and multidimensional poverty. By contrast, our indices can adopt several approaches for the identification of the chronically poor, ranging from union to intersection. For this reason $\Omega_\beta = 0 \leftrightarrow g_{ij}^{t}(0) = 0 \forall (i,j,t)$, whereas this is not the case for M_C^{α}, unless the union approach is considered for both multidimensional and chronic identification steps. Both (A1) and (1) are counting measures, but only (1) uses a counting approach explicitly for chronic multidimensional poverty identification.

Second, (1) fulfils Time Anonymity (TAN), which is inconsistent with a property of Durational Persistence Monotonicity (TPM), fulfilled by (A1). According to (TPM), a poverty measure should increase with increases in s_{ijt}. Hence the timing of deprivation experiences matters; particularly, an index satisfying (TPM) shows higher poverty when poverty experiences are consecutive rather than scattered. (1) does not fulfil (TPM) but could be extended to do so. Note that (A1) uses s_{ijt}, but not W.

Among minor differences, note that (A1) cannot be broken down by dimensional contributions, unless $\beta = 1$, whereas (1) can be broken down by dimensional contributions, although in a censored way when non-union approaches are used. Also Nicholas and Ray (2011) focus on $g_{ij}^{t}(0)$, although extensions for $g_{ij}^{t}(\alpha)$ should be straightforward.

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20 For different formulations of s_{ijt} see Bossert et al. (2012) and Gradin et al. (2012).

Finally, it is worth noting that:

$$M_c^\alpha = \Omega_\beta \leftrightarrow \beta = 1, \alpha = 0, s_{ijt} = 1, W = (1, \dots, 1), 0 < k \leq 1, 0 < \tau \leq 1 \quad \text{(A2)}$$

That is, (1) and (A1) are equivalent, if and only if, a union approach is adopted for both chronic and multidimensional poverty identification, dimensions and spells are weighted equally, only deprivation counts are considered ($\alpha = 0$), and $\beta = 1$.

<u>The measures of Nicholas, Ray and Sinha (2013)</u>

Nicholas, Ray and Sinha extended the original family of Nicholas and Ray (2011): first, by allowing for more poverty identification approaches and, second, by rendering the individual poverty function sensitive to both the breadth of deprivations in any given time period and the duration of each deprivation experience. In our notation, their family of indices is the following:

$$\Omega_{\alpha\beta\gamma\delta}(X; z) = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{dT}\sum_{j=1}^{d}\sum_{t=1}^{T} s_{ij}^t(\alpha, \beta, \gamma)\right]^\delta C_i \quad \text{(A3)}$$

Where $\alpha, \beta, \delta \geq 0;\ 0 \leq \gamma \leq 1$ and :

$$C_i = \mathbb{I}(\textstyle\sum_{j=1}^{d}\sum_{t=1}^{T} g_{ij}^t(0) \geq z) \quad \text{(A4)}$$

$$s_{ij}^t(\alpha, \beta, \gamma) = \gamma\left[\frac{\sum_{j=1}^{d} g_{ij}^t(\alpha)}{d}\right]^\beta g_{ij}^t(\alpha) + (1-\gamma)\left[\frac{\sum_{t=1}^{T} g_{ij}^t(\alpha)}{T}\right]^\beta g_{ij}^t(\alpha)$$

(A5)

Nicholas, Ray and Sinha (2013) elucidate the resilience of (A3) in terms of its fulfilment of several desirable properties for specific combined choices of the four parameters determining the index's functional form. However, the authors acknowledge that this flexibility comes at the cost of having to make choices on a four-dimensional parameter space (p. 24). By contrast, our measures of chronic and transient poverty only require choosing one parameter (α) for the functional form of the individual poverty index. In other words, while our family of poverty indices leans toward the parsimony side of the trade-off between axiomatic resiliency and parametric choice complexity, the family of Nicholas, Ray and Sinha favours a richer flexibility in terms of fulfilment of desirable properties.

Now at first sight it would seem that our index of chronic poverty can be regarded as a special case of (A3) for a special choice of β, γ and δ. However there is a fundamental difference in the way the two index families identify the chronically poor (in addition to the fact that we distinguish between the chronically and the transiently poor). While Nicholas, Ray and Sinha identify the chronically poor using C_i (A4), we use the following criteria for chronic and transient poverty respectively:

$\rho_i(k;\tau) = \mathbb{I}(l_i \geq \tau)$ (A6)

$\omega_i(k;\tau) = \mathbb{I}(0 < l_i \leq \tau)$ (A7)

Where $l_i = \frac{1}{T}\sum_{t=1}^{T}\mathbb{I}(c_i^t \geq k)$. Note the key differences between (A4) and (A6)–(A7). In our identification approach we first identify the poor with a counting approach to multidimensional poverty *in each period independently*. Then using the duration approach of Foster (2009) we differentiate between those who are chronically poor (in the counting sense), transiently poor, and neither. By contrast the criterion in (A4) compresses the time dimension and takes an implicit view of chronic poverty in which identification of the poor requires comparing the sum of all deprivations in a lifetime against one single cut-off (z), which can take a maximum value of dT. The notion of being poor in one time period, regardless of whether that experience is persistent or not, is lost in their approach (even though the breadth of poverty in each period is captured in s_{ij}^t when $\beta, \gamma > 0$).

Finally, it is worth noting that:

$$M_c^\alpha = \Omega_{\alpha\beta\gamma\delta} \leftrightarrow \beta = 0, \delta = 1, W = (1, \dots, 1), 0 < k \leq 1, 0 < \tau \leq 1, 0 < z \leq 1 \qquad \text{(A8)}$$

That is, (1) and (A3) are equivalent for every matrix of attainments if and only if: (i) a union approach is adopted for both chronic and multidimensional poverty identification as well as for lifetime poverty identification ($0 < z \leq 1$); (ii) each deprivation gap is neither weighted by poverty breadth in its respective period nor by the duration of its experience (i.e. $\beta = 0$); and (iii) the social poverty indices are linear on the individual poverty indices ($\delta = 1$), which renders the indices insensitive to some forms of deprivation transfers.

b. Additional tables

Table 1b: Ordinal Illustration with Different Values of k and τ

	Cut-off k=20%			*Cut-off k=40%*			*Cut-off k=60%*			*Cut-off k=80%*			*Cut-off k=100%*		
	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$
Headcount Ratio (Hc)	0.528	0.33	0.181	0.183	0.085	0.028	0.028	0.006	0.0	0.001	0.0	0.0	0.0	0.0	0.0
Duration (Dc)	0.656	0.85	1.00	0.538	0.775	1.00	0.41	0.674	1.00	0.333	.	.	.	.	.
Adj Av Dep Share (Ac)	0.333	0.351	0.37	0.504	0.518	0.524	0.691	0.699	0.75	0.889	.	.	.	.	.
Adj Headcount ratio (M0c)	0.115	0.098	0.067	0.050	0.034	0.015	0.008	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Censored Headcount															
Overcrowding	10.4%	8.3%	5.4%	4.5%	2.9%	1.1%	0.8%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Housing	27.2%	23.1%	15.1%	9.1%	6.0%	2.4%	1.1%	0.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Toilet	11.9%	10.8%	7.3%	6.5%	4.4%	1.6%	0.9%	0.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Attendance	6.2%	5.4%	3.9%	2.9%	2.0%	1.0%	0.7%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Schooling	6.0%	5.8%	4.6%	3.6%	2.8%	1.5%	0.9%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Illiteracy	5.5%	5.1%	4.0%	2.8%	2.2%	1.2%	0.5%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Employment	5.4%	4.2%	3.2%	2.0%	1.4%	0.8%	0.3%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Empl_qual	15.5%	12.8%	8.6%	6.1%	4.3%	1.7%	0.8%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Income	15.7%	13.0%	8.4%	7.2%	4.6%	1.8%	1.0%	0.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Relative Contribution M0															
Overcrowding	10.0%	9.3%	9.0%	10.2%	9.5%	8.1%	11.4%	12.6%	13.4%	12.5%	.	.	.	.	.
Housing	26.2%	26.1%	25.0%	20.3%	19.6%	18.5%	15.9%	15.9%	14.8%	12.5%	.	.	.	.	.
Toilet	11.5%	12.2%	12.1%	14.5%	14.4%	12.4%	13.4%	13.9%	10.0%	12.5%	.	.	.	.	.
Attendance	6.0%	6.1%	6.4%	6.5%	6.4%	7.4%	9.6%	7.7%	2.6%	11.9%	.	.	.	.	.
Schooling	5.8%	6.5%	7.6%	8.1%	9.2%	11.1%	12.3%	12.8%	14.8%	12.5%	.	.	.	.	.
Illiteracy	5.3%	5.8%	6.6%	6.2%	7.1%	9.3%	7.2%	7.2%	13.5%	10.4%	.	.	.	.	.
Employment	5.2%	4.7%	5.3%	4.6%	4.7%	5.7%	4.4%	6.1%	11.2%	4.7%	.	.	.	.	.
Empl_qual	14.9%	14.5%	14.3%	13.6%	13.9%	13.4%	11.7%	10.0%	8.6%	11.9%	.	.	.	.	.
Income	15.1%	14.7%	13.9%	16.0%	15.1%	13.9%	14.1%	13.7%	11.2%	11.1%	.	.	.	.	.

Table 2b: Cross-sectional Results for Cardinal Illustration and All Possible Cut-offs

	Poverty cut-off $k=\frac{1}{3}$			***Poverty cut-off*** $k=\frac{2}{3}$			***Poverty cut-off*** $k=1$		
	1996-	*2001-*	*2006-*	*1996-*	*2001-*	*2006-*	*1996-*	*2001-*	*2006-*
Headcount Ratio	0.37	0.30	0.20	0.11	0.08	0.04	0.01	0.01	0.00
Intensity	0.44	0.43	0.40	0.71	0.70	0.69	1.00	1.00	1.00
Average Gap	0.31	0.32	0.32	0.34	0.32	0.33	0.40	0.31	0.31
Squared Gap	0.15	0.16	0.15	0.16	0.15	0.15	0.21	0.13	0.14
Adjusted Headcount Ratio	0.16	0.13	0.08	0.08	0.06	0.03	0.01	0.01	0.00
Adjusted Gap Ratio	0.05	0.04	0.03	0.03	0.02	0.01	0.01	0.00	0.00
Adjusted Squared Gap Ratio	0.03	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.00
Censored Headcount									
Education	8.22%	5.98%	5.08%	4.25%	2.22%	1.36%	1.45%	0.72%	0.26%
Income	23.71%	20.57%	10.63%	9.91%	7.82%	3.62%	1.45%	0.72%	0.26%
Housing	16.92%	11.96%	8.19%	8.51%	7.01%	2.98%	1.45%	0.72%	0.26%
Relative Contribution M0									
Education	*16.8%*	*15.5%*	*21.2%*	*18.7%*	*13.0%*	*17.1%*	*33.3%*	*33.3%*	*33.3%*
Income	*48.5%*	*53.4%*	*44.5%*	*43.7%*	*45.9%*	*45.5%*	*33.3%*	*33.3%*	*33.3%*
Housing	*34.6%*	*31.0%*	*34.3%*	*37.5%*	*41.1%*	*37.4%*	*33.3%*	*33.3%*	*33.3%*

Table 3b: Cardinal Illustration with Relevant Values of k and τ

	Cut-off $k=\frac{1}{3}$			*Cut-off* $k=\frac{2}{3}$			*Cut-off* $k=1$		
	$\tau=\frac{1}{3}$	$\tau=\frac{2}{3}$	$\tau=1$	$\tau=\frac{1}{3}$	$\tau=\frac{2}{3}$	$\tau=1$	$\tau=\frac{1}{3}$	$\tau=\frac{2}{3}$	$\tau=1$
Headcount Ratio (Hc)	0.49	0.27	0.10	0.16	0.05	0.01	0.02%	0.005	0.0005
Duration (Dc)	0.58	0.79	1.00	0.46	0.72	1.00	0.43	70.01%	1.00

Adj Av Dep Share (Ac)	0.43	0.45	0.48	0.70	0.72	0.75	1.00	1.00	1.00
Adj Av Gap Share	0.138	0.153	0.183	0.233	0.265	0.288	0.363	0.367	0.368
Adj Av Squared Gap Share	0.067	0.077	0.095	0.110	0.132	0.144	0.181	0.187	0.159
Ratio M1c/M0c (S1c)	0.321	0.339	0.380	0.331	0.368	0.382	0.363	0.367	0.368
Ratio M2c/M0c (S2c)	0.155	0.170	0.198	0.156	0.183	0.191	0.181	0.187	0.159
Adj Headcount ratio (M0c)	0.124	0.095	0.049	0.053	0.028	0.007	0.008	0.003	0.000
Adj Gap Ratio (M1c)	0.040	0.032	0.018	0.018	0.010	0.002	0.003	0.001	0.000
Adj Squared Gap Ratio (M2c)	0.019	0.016	0.010	0.008	0.005	0.001	0.001	0.001	0.000