Oxford Poverty & Human Development Initiative (OPHI)
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OPHI WORKING PAPER NO. 68

Measuring and Decomposing Inequality among the Multidimensionally Poor Using Ordinal Data: A Counting Approach

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January 2014

Abstract

Poverty has many dimensions, which, in practice, are often binary or ordinal in nature. A number of multidimensional measures of poverty have recently been proposed that respect this ordinal nature. These measures agree that the consideration of inequality across the poor is important, which is typically captured by adjusting the poverty measure to be sensitive to inequality. This, however, comes at the cost of sacrificing certain policy-relevant properties, such as not being able to break down the measure across dimensions to understand their contributions to overall poverty. In addition, compounding inequality into a poverty measure does not necessarily create an appropriate framework for capturing disparity in poverty across population subgroups, which is crucial for effective policy. In this paper, we propose using a separate decomposable inequality measure – a positive multiple of variance – to capture inequality in deprivation counts among the poor and decompose across population subgroups. We provide two illustrations using Demographic Health Survey datasets to demonstrate how this inequality measure adds important information to the adjusted headcount ratio poverty measure in the Alkire-Foster class of measures.

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This study has been prepared within the OPHI theme on multidimensional poverty measurement.

OPHI gratefully acknowledges support from the UK Economic and Social Research Council (ESRC)/(DFID) Joint Scheme, Robertson Foundation, Praus, UNICEF N'Djamena Chad Country Office, German Federal Ministry for Economic Cooperation and Development (GIZ), Georg-August-Universität Göttingen, International Food Policy Research Institute (IFPRI), John Fell Oxford University Press (OUP) Research Fund, United Nations Development Programme (UNDP) Human Development Report Office, national UNDP and UNICEF offices, and private benefactors. International Development Research Council (IDRC) of Canada, Canadian International Development Agency (CIDA), UK Department of International Development (DFID), and AusAID are also recognised for their past support.

ISSN 2040-8188 ISBN 978-19-0719-455-9

Keywords: inequality decomposition, inequality among the poor, multidimensional poverty, counting approach, variance decomposition.

JEL classification: D6, O1

Acknowledgements

We are grateful to our colleagues at OPHI including James Foster, to the participants of the Centre for the Study of African Economies annual conference 2013 at Oxford, the Development Studies Association conference 2013 at Birmingham, the Southern Economic Association annual meetings 2013 at Tampa as well as to participants in the March 2013 HDRO workshop for helpful initial responses and to Milorad Kovacevic in particular for comments.

The Oxford Poverty and Human Development Initiative (OPHI) is a research centre within the Oxford Department of International Development, Queen Elizabeth House, at the University of Oxford. Led by Sabina Alkire, OPHI aspires to build and advance a more systematic methodological and economic framework for reducing multidimensional poverty, grounded in people's experiences and values.

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1. Introduction

The progress of a society remains incomplete without improving the conditions of those stricken with poverty. It is commonly agreed that there are three important aspects of poverty – incidence, intensity and inequality – that should receive practical consideration.[1] Any policy design for alleviating poverty and the relevant outcome may be influenced by the mechanism used for assessing the progress of the poor, which is normally a poverty measure. Whether a poverty measure captures incidence, intensity or inequality has large consequences for the incentives of a policy maker. A measure that only captures the incidence of poverty but neither the intensity of poverty nor the inequality across the poor (e.g., a headcount ratio measure like the 2005 UNICEF child poverty index or the $1.25/day poverty rate in the single-dimensional context) would create incentives for a policy maker, who is keen on showing a large reduction in overall poverty, to improve the lives of those who are least poor, but it would implicitly create incentives to deliberately ignore those who experience the severest poverty. On the other hand, a measure such as the global Multidimensional Poverty Index (MPI) published in the UNDP's *Human Development Reports*, which captures the incidence and also the intensity of poverty, provides an incentive to policy makers to address the poorest as well as the least poor. But it does not give over-riding incentives to the policy maker to prioritize the conditions of the poorest. Even with similar levels of overall poverty alleviation, one may seek to ascertain whether the fruit of poverty alleviation has been shared uniformly or equally across different population subgroups within a society. This is important in order to avoid aggravating horizontal inequality (Stewart 2008). Again, existing methodologies like the MPI, which are decomposable by subgroup, may be used to check the changes in performance of different subgroups separately but may not provide any conclusive response regarding the disparity across subgroups.

Since the seminal work of Sen (1976), a number of poverty measures reflecting all three aspects have been proposed in the context of unidimensional income poverty measurement.[2] However, there is a growing consensus that the measurement of poverty should not be confined to any single dimension, such as income, because poverty has multiple facets or dimensions. This understanding of the multidimensional nature of poverty has caused interest in the analysis of multidimensional poverty to grow significantly and a number of poverty measures have been proposed with the

[1] Jenkins and Lambert (1997) refer these three aspects as 'three I's of poverty'.
[2] See Thon (1979); Clark, Hemming, and Ulph (1981); Chakravarty (1983); Foster, Greer and Thorbecke (1984), and Shorrocks (1995).

objective of improving its understanding and assessment. One branch of measures has been constructed under the assumption that dimensions are cardinal in nature (Chakravarty, Mukherjee and Ranade 1998; Tsui 2002; Bourguignon and Chakravarty 2003, Massoumi and Lugo 2008). The wider application of these measures may be limited by the fact that many dimensions are ordinal in practice or binary in nature. This has led to the development of another branch of measures that takes into consideration this practical nature – ordinal and dichotomous – of dimensions (Alkire and Foster 2007, 2011; Bossert, Chakravarty and D'Ambrosio 2009; Jayaraj and Subramanian 2009; Rippin 2012). We refer to this second branch of measures as counting measures of multidimensional poverty following the extensive literature (Atkinson 2003). These measures concentrate on counting the number of dimensions to identify the poor. The counting approach to the measuring of multidimensional poverty is our focus in this paper.

It may be quite straightforward to capture inequality among the poor within each dimension for measures using cardinal data, but such liberty is greatly circumscribed when dimensions are ordinal and dichotomous. Yet inequality among the poor can still be captured through the extent of the simultaneous deprivations that they suffer. This is a natural starting point, when each person's deprivation profile is summarized in a cardinally meaningful score that shows the weighted sum of their deprivations, and can be used to assess inequality, as we shall show.

A common approach to capture inequality among the poor in both unidimensional and multidimensional frameworks has been to adjust a poverty measure so that it is sensitive to inequality. This approach has also been adopted while developing many counting-based measures (see Bossert, Chakravarty and D'Ambrosio 2009, Jayaraj and Subramanian 2009, Rippin 2012). This approach, however, may suffer from certain limitations in practice. First, inequality-adjusted poverty indices necessarily compromise a crucial policy-relevant property that allows overall poverty to be expressed as a weighted average of dimensional deprivations (Alkire and Foster 2013).[3] Second, because of this, the inequality-adjusted poverty indices are useful for poverty comparisons across space and time, but the overall index may become rather intricate to interpret and the underlying policy message may become difficult to convey. Third, some of the inequality-adjusted poverty indices are broken down into different partial indices – each separately capturing the incidence, intensity, and inequality across the poor – in order to study how they contribute to overall poverty.

[3] This property is referred to as factor decomposability by Chakravarty, Mukherjee and Ranade (1998) and dimensional breakdown by Alkire and Foster (2011).

One limitation of this approach is that it does not provide the appropriate framework to capture disparity in poverty across population subgroups. This is because an overall improvement in poverty may come with a reduction in inequality among the poor and with a more uniform reduction in intensities across the poor, but, simultaneously, come with a large non-uniform improvement in poverty across different population subgroups.

In this paper, we propose using a separate inequality measure, rather than an inequality-adjusted poverty measure, in order to assess inequality across the poor and also to capture disparity in poverty across and within population subgroups. In particular, we propose a separate inequality measure that is consistent with and can accompany the widely used counting poverty measure, the adjusted headcount ratio proposed by Alkire and Foster (2011 – henceforth AF). The adjusted headcount ratio is an intuitive index for measuring multidimensional poverty, which can be expressed as a product of the incidence and intensity of poverty among the poor.[4] Our proposed measure of inequality among the poor, as we will subsequently show, can create a policy-amiable poverty assessment mechanism that builds on the AF method and provides more direct policy information than can be gained by incorporating inequality into a poverty measure itself.

Which inequality measure would serve our purpose best depends on an important methodological consideration and the properties that the measure should satisfy. We now turn to describe these properties that undergird our selection of a variance-based inequality measure.

The important methodological consideration is whether inequality across deprivation counts should be measured in relative or absolute terms. When the inequality measure is relative, then multiplying all deprivation counts by the same factor leaves overall inequality unchanged. On the other hand, when the inequality measure is absolute, then adding a deprivation to everybody's deprivation count leaves overall inequality unaltered. Most income inequality measures have been relative, but this trend has been the subject of debate (Kolm 1976). In the counting approach framework, we argue that inequality across deprivation counts should be measured in absolute terms, largely because each deprivation may arguably have a direct or even intrinsic importance. In fact, we show that any

[4] The adjusted headcount ratio, a counting poverty measure proposed by Alkire and Foster (2011), has already seen a number of applications by international organizations and country governments. The United Nations Development Programme has used it to introduce the Global Multidimensional Poverty Index (MPI) into their annual *Human Development Reports* (Alkire and Santos 2010); the Colombian, Mexican and Bhutanese governments have used this measure to create their official poverty measures (Foster 2007; CONEVAL 2011; Angulo, Diaz and Pardo 2011); the Government of Bhutan also adapted the measure to create the Gross National Happiness Index (Alkire, Ura, Wangdi and Zangmo 2012).

relative inequality measure may result in contradictory conclusions in a counting approach framework depending whether the attainments of the poor are counted or their deprivations.

In order to fit our goal of studying both inequality among the poor within population subgroups and disparity between population subgroups, we require the inequality measure to be additively decomposable. This means that the inequality measure can be broken down into a within-group and a between-group component. Using certain additional but relevant properties, we show that the only reasonable absolute inequality measure that fits our purpose is a positive multiple of "variance". The additive decomposability property allows overall inequality to be decomposed into a total within-group and a between-group component. The total within-group component can be expressed as a population-share weighted average of the within-group inequalities. As a result, total within-group inequality does not change if there is no change in inequality within any of the population subgroups. An additional feature of the inequality measure is that it reflects the same level of inequality whether poverty is assessed by counting deprivations or counting attainments.

We support our methodological development with two empirical illustrations: a cross-country illustration using 23 Demographic Health Surveys (DHS) and an inter-temporal illustration in the Indian context by classifying the population into various mutually exclusive and collectively exhaustive population subgroups. For both illustrations, we use the global MPI and its parametric specifications. In the cross-country illustration, despite a positive association between the MPI and the level of inequality among the poor across countries, we found several instances in which low levels of MPI was not accompanied by lower inequality. In one instance, we found that the MPI of Colombia is less than one-seventh of Lesotho's MPI but the level of inequality across the poor is almost the same. In another instance, we find that Kenya's MPI is much lower than Bangladesh's MPI and both have similar level of inequality across the poor, but the disparity in sub-national MPIs is much larger in Kenya than in Bangladesh. This shows why we also need to study the regional disparities in poverty in addition to studying inequality across the poor. In the inter-temporal illustration, we find that the MPI reductions across different population subgroups were not necessarily accompanied by corresponding reductions in inequality across subgroups. For instance, when the Indian population was classified into four major caste categories, both the incidence and intensity of poverty went down in each category, but inequality across the poor did not.

The rest of the paper proceeds as follows. In Section 2, we introduce the notation that we use in the rest of the paper. We then review various existing ways of reflecting inequality in poverty measurement based on counting approaches in Section 3. In Section 4, we obtain the inequality measure that is suitable for capturing inequality across the multidimensionally poor in a counting approach framework and is decomposable across population subgroups. We discuss various decomposition formulations of the inequality measure to study inequality among the poor and disparity across population subgroups in Section 5. The empirical illustrations are given in Section 6. Concluding remarks are provided in Section 7.

2. Notation

In this section, we introduce the notation that we will be using in the rest of our paper. We assume that our hypothetical society contains $n \geq 2$ persons and their wellbeing is assessed by a fixed set of $d \geq 2$ indicators.[5] We use subscript i to denote persons and subscript j to denote dimensions. The achievement of person i in dimension j is denoted by $x_{ij} \in \mathbb{R}_+$ and the achievement matrix is denoted by $X \in \mathbb{R}_+^{n \times d}$. The set of all achievement matrices of size n is denoted by $\mathcal{X}_n$ and the set of all possible matrices of any size is denoted by $\mathcal{X} = \bigcup_n \mathcal{X}_n$. We denote the deprivation cutoff of dimension j by $z_j \in \mathbb{R}_{++}$ such that person i is considered deprived in dimension j whenever $x_{ij} < z_j$ and non-deprived if $x_{ij} \geq z_j$. The d deprivation cutoffs are summarized in vector z and the set of all possible deprivation cutoff vectors are summarized by $\mathcal{Z}$.

For any $X \in \mathcal{X}$, person i is assigned a *deprivation status value* of $g_{ij} = 1$ in dimension j if $x_{ij} < z_j$ and $g_{ij} = 0$ otherwise for all $j = 1, \ldots, d$ and $i = 1, \ldots, n$. A relative weight of w_j is assigned to each dimension j based on its value relative to other indicators, such that $w_j > 0$ and $\sum_j w_j = 1$. The weights are summarized by vector w. The *deprivation score* of person i is obtained by the weighted average of the deprivation status values and is denoted by $\pi_i = \sum_j w_j g_{ij}$. Thus, $\pi_i \in [0,1]$ for all i. The deprivation scores of all n persons in the society is summarized by vector $\pi = (\pi_1, \ldots, \pi_n)$.[6] An alternative approach may be to attach an *attainment status value* of $\tilde{g}_{ij} = 1$ to person i in dimension j

[5] In many studies, the terms 'dimensions' and 'indicators' are used differently, where dimensions are assumed to be the pillars of wellbeing and each dimension is measured using one or more indicators.

[6] In this paper, we use a slightly different notation than Alkire and Foster (2011) to denote the deprivation vector and censored deprivation vector because of simplicity of presentation. Alkire and Foster (2011) denote the deprivation score vector by c and the censored deprivation score vector by $c(k)$. In this paper, we use notation c to denote the censored deprivation vector instead. Therefore, we recommend caution when interpreting the results.

if $x_{ij} \geq z_j$ and $\tilde{g}_{ij} = 0$ otherwise for all $j = 1, \dots, d$ and $i = 1, \dots, n$ (see Alkire and Foster 2013). Then, the *attainment score* of person i is $\tilde{\pi}_i = \sum_j w_j \tilde{g}_{ij}$ and we summarize the attainment scores of all persons by vector $\tilde{\pi}$. Note that for the same achievement matrix $X \in \mathcal{X}$, deprivation cutoff vector $z \in \mathcal{z}$ and weight vector w, by construction $\tilde{\pi}_i = 1 - \pi_i$ for all i.

In a counting approach framework, any person i is identified as poor if $\pi_i \geq k$ for any poverty cutoff $k \in (0,1]$.[7] Let us define the identification function for any person i as $\rho_i(k) = 1$ if $\pi_i \geq k$ and $\rho_i(k) = 0$, otherwise. We denote the post-identification *censored deprivation score* of person i by $c_i = \pi_i \rho_i(k)$ and the corresponding vector by $c = (c_1, \dots, c_n)$. Thus, $c_i = \pi_i$ if $\pi_i \geq k$ and $c_i = 0$, otherwise. Alternatively, a person can also be identified as poor through counting attainments, setting an equivalent poverty cutoff $\tilde{k} \in [0,1)$, and an identification function for any person i such that $\tilde{\rho}_i(\tilde{k}) = 1$ if $\tilde{\pi}_i \leq \tilde{k}$ and $\tilde{\rho}_i(\tilde{k}) = 0$, otherwise. A corresponding *censored attainment-score* $\bar{c}_i$ can also be obtained such that $\tilde{c}_i = \tilde{\pi}_i \tilde{\rho}_i(\tilde{k})$ and the corresponding vector is denoted by $\tilde{c}$. For the same achievement matrix $X \in \mathcal{X}$, deprivation cutoff vector $z \in \mathcal{z}$ and weight vector w, by construction $\tilde{c}_i = 1 - c_i$ for all i whenever $\tilde{k} = 1 - k$.

Note that $c = \pi$ for the union approach for identifying the poor. We denote the number of poor after identification by q and the proportion of poor is denoted by $H = q/n$. Without loss of generality, we assume that $c_1 \geq \cdots \geq c_n$. Thus, if at least one person is identified as poor, then $c_i > 0$ for all $i \leq q$ and $c_i = 0$ for all $i > q$. We summarize the deprivation scores of the poor persons by a having q elements such that $a_i = c_i$ for all $i = 1, \dots, q$. Similarly an attainment score vector $\tilde{a}$ may be constructed with q elements such that $\tilde{a}_i = 1 - a_i$ for all i. In this paper, we present the results using the deprivation counts rather than the attainment counts, but the results hold for any analysis using attainment counts as well.

We also introduce the analogous subgroup notations in order to facilitate the decomposition analysis. We assume that there are $m \geq 2$ mutually exclusive and collectively exhaustive population subgroups within the society. The population subgroups may be geographic regions, castes or religious groups. The number of all persons and the number of poor persons in subgroup ℓ are denoted by n^ℓ and $q^\ell \geq 1$, respectively, for all $\ell = 1, \dots, m$ such that $\sum_{\ell=1}^m n^\ell = n$ and $\sum_{\ell=1}^m q^\ell = q$.

[7] If $k = 1$, then it is the *intersection approach*. If $k \in (0, \min_j \{w_j\}]$, it is the *union approach*. If $\min_j \{w_j\} < k < 1$, it is the *intermediate approach* (Alkire and Foster 2011).

Vectors $\overline{n} = (n^1, \dots, n^m)$ and $\overline{q} = (q^1, \dots, q^m)$ summarize the subgroup population and subgroup poor population, respectively. The overall censored deprivation score vector and the censored deprivation score vector of the poor for subgroup ℓ are denoted by c^ℓ and a^ℓ, respectively. As earlier, without loss of generality, we assume that within each subgroup ℓ, $c_i^\ell > 0$ for all $i \leq q^\ell$ if there is at least one poor in the subgroup and $c_i^\ell = 0$ for all $i > q^\ell$.

We denote the mean of all elements in any vector x by $\mu(x)$. Then, $\mu(c)$ is the *adjusted headcount ratio* and $\mu(a)$ is the *average deprivation scores* among the poor following the terminology of Alkire and Foster (2011). In fact, $\mu(c) = H \times \mu(a)$. Note that $\mu(c) = \sum_\ell \nu^\ell \mu(c^\ell)$ and $\mu(q) = \sum_\ell \theta^\ell \mu(c^\ell)$, where $\nu^\ell = n^\ell / n$ is the population share of subgroup ℓ to the total population and $\theta^\ell = q^\ell / q$ is the share of poor of the subgroup ℓ to total poor population. Finally we define vectors $\overline{\mu}_c = (\mu(c^1), \dots, \mu(c^m))$ and $\overline{\mu}_a = (\mu(a^1), \dots, \mu(a^m))$.

3. Consideration on Inequality among the Poor with Ordinal Data

Poverty can be mitigated by reducing its incidence or by reducing its intensity, but neither ensures that the reduction would benefit those poor with the highest deprivation scores. How can the concern for inequality be incorporated in the counting approach framework? It is customary, following Sen (1976), to fine-tune a poverty measure to be sensitive to inequality across the poor, whether it is unidimensional or multidimensional.[8] In a multidimensional analysis of poverty, inequality across the poor can be captured within each dimension when dimensions are cardinal. However, in this paper, our primary focus is counting approaches where dimensions can be cardinal, binary, or ordinal. In these approaches, inequality across the poor can be captured across their deprivation scores. Let us first review the proposed approaches for capturing inequality among the poor in counting approaches.

Some of the methods integrate inequality into a poverty measure; whereas, others use a separate inequality measure. Bossert, Chakravarty and D'Ambrosio (2009) propose using the extended symmetric means across the censored deprivation scores:

[8] The inequality-adjusted unidimensional poverty indices are Sen (1976), Thon (1979), Clark, Hemming, and Ulph (1981), Chakravarty (1983), Foster, Greer and Thorbecke (1984), and Shorrocks (1995). The inequality-adjusted multidimensional indices are Chakravarty, Mukherjee and Ranade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Massoumi and Lugo (2008), Bossert, Chakravarty and D'Ambrosio (2009), Jayaraj and Subramanian (2009), Alkire and Foster (2011) and Rippin (2012).

$$P_{BCD}(\pi) = \left(\frac{1}{n}\sum_{i=1}^{n}\pi_i^{\beta}\right)^{1/\beta} \text{; with } \beta \geq 1. \tag{3.1}$$

For $\beta = 1$, $P_{BCD}(\pi) = \mu(\pi)$, but for $\beta \geq 2$, the measure assigns higher weights to larger deprivation scores and thus is sensitive to inequality across the poor.

Similarly, while proposing a measure of social exclusion, Chakravarty and D'Ambrosio (2006) examine a class of additively decomposable poverty indices:

$$P_{CD}(\pi) = \frac{1}{n}\sum_{i=1}^{n}\pi_i^{\beta} \text{; with } \beta \geq 1. \tag{3.2}$$

This class of indices is analogous to the Foster-Greer-Thorbecke (FGT) class of poverty indices in the unidimensional framework. Jayaraj and Subramanian (2009) apply the class of indices in Equation (3.2) to analyze multidimensional poverty in the Indian context. This measure can be expressed in various ways. Following Aristondo et al. (2010), it can be expressed as:

$$P_{CD}(\pi) = H \times \mu(a_U) \times [1 + \beta(1-\beta)GE(a_U;\beta)], \tag{3.3}$$

where a_U is the vector of deprivation scores of the poor identified by a union approach, $GE(a_U;\beta)$ is the generalized entropy measure of order β.[9]For $\beta = 2$, $GE(a_U;\beta)$ is related to the squared coefficient of variation. While, for $\beta = 2$, Chakravarty and D'Ambrosio (2006) express their measure as:

$$P_{CD}(\pi) = \sigma^2(\pi) + [\mu(\pi)]^2 = H \times [\sigma^2(a_U) + [\mu(a_U)]^2], \tag{3.4}$$

where, σ^2 stands for variance.

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[9] The generalized entropy measure of order β can be written as $GE(a,\beta) = \left[q^{-1}\sum_{i=1}^{q}\left(a_i/\mu(a)\right)^{\beta} - 1\right]/[\beta(1-\beta)]$. Rippin (2012) uses the same measure to develop the correlation-sensitive index and shows the decomposition. In the single-dimension context, the FGT index can be broken down into the headcount ratio, income gap ratio and Generalized Entropy measure of order two (Foster and Sen 1997, Aristondo et al. 2010). Similarly, the Sen-Shorrocks-Thon index can be broken down into the headcount ratio, income gap ratio and Gini coefficient (Xu and Osberg 2001).

Note that both Chakravarty and D'Ambrosio (2006) and Bossert, Chakravarty and D'Ambrosio (2009) use an index of poverty that is sensitive to inequality across the poor. There also exist empirical studies that use a separate inequality measure to capture inequality across the poor. For example, while studying child poverty using a counting approach, the standard deviation has been used by Delamonica and Minujin (2007) in order to study inequality (or severity) in deprivation counts among poor children.[10]

The approaches that integrate inequality into poverty measures are primarily useful for ranking regions in terms of overall poverty, discounting for inequality among the poor. For example, Jayaraj and Subramanian (2009) found that the ranking of Indian states altered when inequality-sensitive poverty indices were used instead of a poverty index insensitive to inequality. The ranking altered because of the levels of inequality across deprivation counts among the poor within different states were different. If the indices are, in addition, additively decomposable, then this means that overall poverty can be expressed as a population-weighted average of subgroups' poverty. This is helpful in understanding how different subgroups have contributed to overall poverty in a particular period and how they have contributed to the overall change in poverty.

These integrated approaches may, however, have certain limitations. First, the inequality-adjusted poverty indices may lack intuitive interpretation. For example, the adjusted headcount ratio (Alkire and Foster 2011) of a society can be intuitively interpreted as the share of deprivations experienced by the poor out of the maximum possible deprivations within the society. The index can be presented as a product of the incidence and intensity of poverty among the poor. The inequality-adjusted poverty indices may lack such intuitive appeal in general.

Second, it has been shown in Alkire and Foster (2013) that a counting measure of poverty cannot be sensitive to the breadth of the distribution of deprivation counts *and* be broken down by dimensions at the same time.[11] The property of dimensional breakdown allows one to express overall poverty as a weighted average of dimensional deprivations (among the poor). The dimensional breakdown property thus enables one to understand the contribution of each dimension to overall poverty, as well as how dimensional changes contribute to overall changes in poverty, and indeed has important policy relevance.

[10] The approach has been followed by Roche (2013) while studying child poverty in Bangladesh.
[11] The dimensional breakdown property of Alkire and Foster (2011) is similar in spirit to the factor decomposability property of Chakravarty, Mukherjee and Ranade (1998).

Third, the inequality-adjusted poverty measures often involve an inequality aversion parameter. The particular value that the parameter should take depends on how averse an evaluator is to inequality among the poor. The inequality aversion parameter discounts for larger inequality by increasing the overall poverty index. Thus, for the same distribution across the poor, a more inequality-averse evaluator would conclude more poverty in the distribution that what a less inequality-averse evaluator would. Depending on the particular value of the parameter chosen, one may have different ranking of regions. Hence, this involves additional parametric decision-making, which can be a subject of significant debate.

Fourth, a poverty measure that combines incidence, intensity and inequality into one may not make transparent the relative weight that the measure places on each of these aspects. And that is important. For example, two different expressions of the poverty measure proposed by Chakravarty and D'Ambrosio (2006) in (3.3) and (3.4) would attach quite different weights to incidence, intensity and inequality across the poor for $\beta = 2$, depending on whether the value judgment of inequality is absolute or relative. We discuss various implications of this value judgment in the next section.[12] While it is certainly better to have lower inequality among the poor than high inequality among the poor, even with low inequality across the poor it is far better to have this situation with a low than a high average intensity. Of course, the fundamental aim of poverty reduction is not to reduce inequality among the poor, nor the intensity of poverty. Rather, it is to eradicate poverty, bringing the incidence to zero. Combined measures, however, rarely make the relative importance of these policy goals transparent.

Fifth, the inequality-adjusted poverty measures do not provide an appropriate framework for studying disparity in poverty across different population subgroups, even when poverty measures are additively decomposable. The consideration of disparity in poverty between subgroups is no less important than inequality in deprivation counts among the poor, because a large disparity in poverty across subgroups may reflect large horizontal inequality and thus may create an environment for potential conflict across groups, which may have further adverse consequences on poverty (Stewart 2010). A similar level of poverty may be accompanied by a very different level of subgroup disparities or a large overall reduction in poverty may be accompanied by an increasing disparity across subgroups. For example, Alkire, Roche, and Seth (2011) found that Malawi and Senegal,

[12] In fact, Zheng (1994) shows in the single-dimensional context that the only poverty index that is both absolute and relative is related to the headcount ratio and that there can be no meaningful index of inequality that can be both relative and absolute.

which have similar population sizes, the same number of sub-national regions, and similar MPI values, had very different levels of sub-national disparity. Alkire and Seth (2013) found that a strong reduction in national MPI in India between 1999 and 2006 was accompanied by non-uniform reductions in multidimensional poverty across different social and regional subgroups.

The disparity in poverty across subgroups should not be misunderstood as between-group inequality among the poor or the disparity in subgroups' intensities. In welfare analysis, an additively decomposable inequality measure can be expressed as a sum of a total within-group inequality component and a between-group inequality component. But the between-group inequality among the poor is not the same as disparity in poverty across subgroups. In fact, a reduction in inequality among the poor may be accompanied by a reduction in between-group inequality among the poor but with an increase in disparity in poverty across subgroups.

Let us consider the following simple example with a ten-person hypothetical society containing two subgroups – Subgroup A and Subgroup B – having five persons each. To keep it simple, we assume that every dimension is equally weighted and a person is identified as poor by union criterion. Suppose the deprivation score vector of the society is (0, 0, 0, 0.6, 0.6, 0.6, 0.6, 0.6, 0.7, 0.7). The deprivation count vector of Subgroup A is (0, 0, 0.6, 0.6, 0.7) and that of Subgroup B is (0, 0.6, 0.6, 0.6, 0.7). Clearly, there exists inequality in deprivation scores among the poor within each subgroup and inequality in average deprivation scores among the poor between two subgroups. Besides, any poverty measure satisfying standard properties ensures that Subgroup B will have more poverty than Subgroup A.

Now, suppose that over time, the overall deprivation score vector is transformed to (0, 0, 0, 0.6, 0.6, 0.6, 0.6, 0.6, 0.6, 0.6), which implies a reduction in inequality across the poor. If the transformed deprivation count vectors of Subgroups A and B are (0, 0, 0, 0.6, 0.6) and (0.6, 0.6, 0.6, 0.6, 0.6), respectively, then certainly inequality in deprivation scores among the poor within each subgroup has gone down along with a reduction in inequality in average deprivation scores among the poor between the two subgroups. However, it is harder to argue that disparity in poverty between these two subgroups has gone down. This particular concern is not only applicable to measures using a counting approach but also applies to single-dimensional inequality-adjusted poverty measures.

In sum, the integrated poverty measures are certainly useful for ranking. For example, Jayaraj and Subramanian (2009) use the integrated poverty measures in (3.2) to the Indian context and show

how the rankings of states change when the aversion towards inequality (denoted by parameter β) vary. However, the integrated approach has certain limitations as discussed above. Some of the integrated measures are broken down into components related to incidence, intensity and inequality among the poor, but without making the relative weight attached to these components transparent. In fact, if there is a need to break down the final unintuitive figure of the integrated poverty measures into different components and the integrated poverty measures fail to satisfy certain policy amiable properties, such as dimensional breakdown, then why one ought not to use a separate inequality measure to study inequality among the poor besides using an intuitive but informative measure to assess poverty. Thus, in this paper, we propose using a separate inequality measure to study and decompose inequality across the poor. The next section justifies our choice of which inequality measure should be used.

4. Which Inequality Measure?

Using a counting approach to assess multidimensional poverty, inequality may be assessed at different levels: (i) inequality in deprivation scores across the poor, (ii) inequality in deprivation scores between the poor and the non-poor, (iii) inequality in the level of poverty (or an associated partial index) between population subgroups, and (iv) inequality in deprivation scores among the poor across subgroups. Thus, the overall inequality across deprivation scores within a society may have several components. Some take into account inequality within populations or population subgroups (referred as *within-group inequality*) and some take into account inequality between the subgroup means (referred as *between-group inequality*).

The inequality measure that should be used depends on the desirable properties that it should satisfy. Let us denote the inequality measure by I. We discuss the properties in terms of a general t-dimensional vector x, such that $x_i \in [0,1]$ for all $i = 1, \ldots, t$. Depending on situations, vector x may represent vectors π, $\tilde{\pi}$, c, $\tilde{c}$, a or $\tilde{a}$ and symbol t may represent n or q as required, introduced in Section 2. The corresponding subgroup notations and operators introduced in Section 2 equally applies to x and t.

Given that we are interested in within-group and between-group inequalities, it is meaningful for the inequality measure to be *additively decomposable* so that overall inequality can be expressed as a sum of total within-group inequality and between-group inequality. Let us denote the within-group term by

I_W and the between group term by I_B. The overall within-group inequality can be expressed as a weighted average of within-group inequalities of the population subgroups, i.e., $I_W(x) = \sum_{\ell=1}^{m} \omega^\ell(\bar{t},\bar{\mu}_x) I(x^\ell)$, where $\omega^\ell(\bar{t},\bar{\mu}_x)$ is the weight attached to inequality within subgroup ℓ.[13] The between-group term can be expressed as $I_B(x) = I(\bar{\mu}_x;\bar{t})$, where $I(\bar{\mu}_x;\bar{t}) = I(\mu(x^1)1^{t^1},\dots,\mu(x^m)1^{t^m})$ and 1^{t^ℓ} is the t^ℓ-dimensional vector of ones for all $\ell = 1,\dots,m$. The between-group inequality can be interpreted as the inequality across deprivation scores when everybody within each subgroup suffers the mean deprivation score of that subgroup.

Additive Decomposability: For deprivation score vector x,

$$I(x) = I_W(x) + I_B(x) = \sum_{\ell=1}^{m} \omega^\ell(\bar{t},\bar{\mu}_x) I(x^\ell) + I(\bar{\mu}_x;\bar{t}).$$ [14]

What does it imply when the weight attached to a within-group term depends on subgroup means? It implies that if the mean deprivations of the subgroups change disproportionately, but the level of inequality and the population shares within these subgroups do not change, the share of within-group inequality to overall inequality *may change* without any justifiable reason. In order to avoid such circumstances, we impose a restriction such that the overall within-group inequality should not change when the inequality level and population size of each group remains unchanged but subgroup means change disproportionately.

Within-group Mean Independence: For any two deprivation score vectors x and x', if $t^\ell = t'^\ell$ and $I(x^\ell) = I(x'^\ell)$ for all $\ell = 1,\dots,m$, then $I_W(x) = I_W(x')$.[15]

The third property, which decides whether the concept of inequality across deprivation scores among the poor should be judged in a relative or in an absolute sense, is crucial. If the normative assessment of inequality depends on absolute distance, then a change in every poor person's deprivation score by the same *amount* leaves the level of inequality unchanged. If, on the other hand, the assessment of inequality is relative, then a change in every poor person's deprivation score by the

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[13] Note that weight to subgroups denoted by $\omega^\ell(\bar{t},\bar{\mu}_x)$ for subgroup ℓ is different from the weights attached to each dimension and denoted by w_j for dimensions j.

[14] This is the usual definition of additive decomposability also used by Shorrocks (1980), Foster and Shneyerov (1999), and Chakravarty (2001).

[15] Note that the property is analogous to the *path independence* property of Foster and Shneyerov (2000) for relative inequality measures. The within-group mean independence property does not require an index to be absolute or relative *a priori*. The additive decomposability property along with the within-group mean independence implies path independence.

same *proportion* leaves the level of inequality unchanged. As stated earlier, relative inequality has frequently been used with income. Atkinson (1970) proposed considering inequality in a relative sense in order to make the measure of inequality independent of mean. The other appealing reason in favor of relative inequality measures is that they satisfy the property of unit consistency (Zheng 2007), which requires that if the variable under consideration is expressed in different units, the inequality ordering should not change. Kolm (1976), on the other hand, discussed the social disadvantages of considering inequality in a relative than an absolute sense.

The value judgment of relative inequality is difficult to justify across deprivation scores. Let us consider the following three examples. For simplicity, we assume there are ten dimensions that are equally weighted and a union approach is used for identification. In the first example, suppose there are two poor persons in the society with deprivation score vector $x = (0.2, 0.5)$. Suppose, over time, the deprivation score vector becomes $x' = (0.4, 1)$, which means that the first poor person become deprived in two additional dimensions, whereas, the poorer person become deprived in five additional dimensions. It is hard to argue that inequality between the two poor persons has not changed. In fact, if the second poor person had become deprived in four instead of five additional dimensions, it would have even been harder to argue that inequality between the two poor persons had gone down, which would have been an obvious conclusion reached by any relative inequality measure.

In a second example, suppose due to any dire consequence, every poor person in a hypothetical society becomes deprived in an additional dimension in which they were not deprived earlier. It is hard to argue that inequality among deprivation scores among the poor should have decreased, as would have been claimed by any relative inequality measure. Now consider a third example which shows that a relative inequality measure may provide contradictory conclusions about the direction of change in inequality depending on whether poverty is measured by counting attainments or deprivations. Consider the two censored deprivation score vectors $c = (0.3, 0.4, 0.5, 0.6)$ and $c' = (0.7, 0.8, 0.9, 1)$. Any relative inequality measure would conclude that there is higher inequality in c than in c'. Whereas, if we use the corresponding attainment score vectors $\bar{c} = (0.7, 0.6, 0.5, 0.4)$ and $\bar{c}' = (0.3, 0.2, 0.1, 0)$, respectively, then inequality is higher in $\bar{c}'$ than that in $\bar{c}$ according to the same relative inequality measure. Thus, whether a society has less or more inequality among the poor depends on whether they are identified as poor by counting their deprivations or counting their attainments. This type of ambiguity is hard to justify. The only way to reflect the same level of inequality in this situation is to use an absolute inequality measure.

In order to incorporate our value judgment for absolute inequality, we, thus, require that inequality should remain unaltered when everybody's deprivation score increases by the same amount. The deprivation scores obtained from counting dimensions are already independent of any unit of measurement and so the unit consistency property is also not appealing in this situation. An absolute inequality measure must satisfy the *translation invariance* property.

Translation Invariance: For the deprivation score vector x such that $\max_i\{x_1, \ldots, x_t\} \leq 1 - \varepsilon$ and $\varepsilon \geq \delta > 0$,

$$I(x) = I(x + \delta 1^t).$$

Note that the property has been slightly modified in comparison with how it is usually presented. This is because the deprivation scores are bounded above and cannot be increased indefinitely. In order to increase everybody's deprivation score by an amount δ, the largest deprivation score should be such that it can be increased by δ and still does not surpass the upper bound.

There are a few additional well-known properties we deem essential to providing a well-behaved structure for the inequality measure:

Anonymity: If the deprivation score vector x' is a permutation of the deprivation score vector x, then

$$I(x') = I(x).$$

Replication Invariance: If the deprivation score vector x' is obtained from the deprivation score vector x by replicating x more than once, then

$$I(x') = I(x).$$

Normalization: For the deprivation score vector x such that $x_i = \delta \in [0,1]$ for all $i = 1, \ldots, t$,

$$I(x) = 0.$$

***Transfer Principle*:** If the deprivation score vector x' is obtained from the deprivation score vector x by a regressive transfer, then

$$I(x') > I(x).$$

The *anonymity* property requires that an inequality measure should not change by a permutation of deprivation scores across the society. The *replication invariance* property requires that the inequality measure should enable comparison across societies with different population sizes. Technically, if a society is obtained from another society by a merely duplicating or replicating the entire population with the same deprivation score vector, then the level of inequality should be the same. The *normalization* property is a calibration property, which requires that if everybody in the society has the same deprivation score, then the inequality measure should be equal to zero. Finally, an inequality measure must respect some version of the *transfer principle* property. This fundamental property requires that an inequality measure should increase due to a *regressive transfer*. What is a regressive transfer? Suppose, x' is obtained from x, such that $x_{i'}^{'} = x_{i'} - \delta \geq 0, x_{i''}^{'} = x_{i''} + \delta \leq 1, x_{i'} < x_{i''}$, $\delta > 0$ and $x_i^{'} = x_i$ for all $i \neq i^{'}, i^{''}$. Then, x' can be stated to be obtained from x by a *regressive transfer* across deprivation scores. One may question how a transfer is possible between non-transferable deprivation scores or attainment scores as it is possible between incomes.

Let us discuss the relevance of the transfer principle in the counting approach framework in terms of association between dimensions. Satisfying the transfer principle implies that inequality should increase due to multidimensional *association increasing rearrangements* between the poor in terms of the deprivation status values g_{ij}. To explain this concept, suppose the censored *deprivation status values* of all t persons in d dimensions are summarized by the $t \times d$-dimensional matrix $g(k)$. Thus, the ij^{th} element of $g(k)$ is $g_{ij}(k) = g_{ij} \times \rho_i(k)$. When we consider all persons within a society, i.e., $t = n$. In this case, the $g_{ij}(k) = 0$ even when person i is deprived in dimension j but not identified as poor by the poverty cutoff k. For the union approach, $g_{ij}(k) = g_{ij}$ for all i and j. When we focus only on those who have been identified as poor, i.e., $t = q$, then matrix g only contains deprivation status values of the poor people. We denote the i^{th} row of matrix $g(k)$ by $g_{i\cdot}$ and the j^{th} column by $g_{\cdot j}$.

A matrix $g^{'}(k)$ is obtained from matrix $g(k)$ by an association increasing rearrangement among the poor if the set of poor persons remain unchanged, yet $g^{'}(k) \neq g(k)$, $g^{'}(k)$ is not a permutation of $g(k)$, and there exist two persons i and i' such that $g^{'}_{i'\cdot} = (g_{i'\cdot} \vee g_{i''\cdot})$, $g^{'}_{i''\cdot} = (g_{i'\cdot} \wedge g_{i''\cdot})$, and $g^{'}_{i\cdot} = g_{i\cdot}$ for all $i \neq i^{'}, i^{''}$. Operator $\vee$ is the join of the two vectors $g_{i'\cdot}$ and $g_{i''\cdot}$, so that vector $g^{'}_{i'\cdot}$ has the maximum of each of the d elements; and operator $\wedge$ is the meet of vectors $g_{i'\cdot}$ and $g_{i''\cdot}$, so that vector $g^{'}_{i''\cdot}$ has the minimum of each of the d elements. If i' has a larger deprivation score than i'' to begin with, then this type of transfer increases the distance between the deprivation scores of persons $i^{'}$ and $i^{''}$ and so the inequality measure V should increase. Thus, the association increasing rearrangement is a type of regressive transfer. We may refer to this property as *increasing under increasing association*.[16]

Increasing under Increasing Association: If the deprivation score vectors x and x' correspond to deprivation status value matrices $g(k)$ and $g^{'}(k)$, respectively, and $g^{'}(k)$ is obtained from $g(k)$ by an association increasing rearrangement among the poor so that the set of poor remains unchanged, then

$$I(x') > I(x).$$ [17]

The following proposition characterizes the desired class of inequality measures.

Proposition: For any deprivation score vector x and any $\alpha > 0$, an inequality measure I satisfies anonymity, the transfer principle, replication invariance, additive decomposability, within-group mean independence and translation invariance if and only if:

$$I(x) = \frac{\alpha}{t}\sum_{i=1}^{n}[x_i - \mu(x)]^2. \tag{4.1}$$

Proof: See Appendix.[18]

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[16] This property has been primarily motivated by Boland and Proschan (1988). For a different statement of this property in counting approach for poverty measurement see Alkire and Foster (2013) and for a related statement in the context of welfare measurement, see Seth (2013). For weaker versions of this property, see Tsui (2002) and Alkire and Foster (2011).

[17] This property can be equivalently stated in terms of (0-1) attainments rather than deprivations.

[18] The proposition is analogous to Theorem 1 of Chakravarty (2001). However, we do not assume differentiability and population share weighted decomposability as assumed by Chakravarty (2001). Instead, we use the Within-group Mean Independence property to prove the proposition.

Thus, the class of inequality measures that satisfies the desired properties is some positive multiple (α) of what has been called "variance".[19]

The class of absolute inequality measures in (4.1) results in the same level of inequality whether people are identified as poor by counting the number of deprivations or alternatively by counting the number of attainments.[20]

By construction, the minimum possible value that $I(x)$ takes is zero, which is attained when the mean deprivation score is shared by all. The maximum possible value that variance takes is one fourth of the range of the deprivation score vector, which is attained when half of the population have the lowest deprivation scores and the other half have the highest deprivation scores. The value of α can be chosen in such a way that the value of the inequality measure is bounded between zero and one, as is true of any standard inequality measure. For example, suppose a counting measure uses five dimensions with equal weights and one is merely interested in the level of inequality among the poor. In this case, the deprivation scores among the poor range from 0.2 to 1, and so the maximum possible variance is 0.2. Thus, we should set $\alpha = 1/0.2 = 5$. Similarly, given that we are interested in exploring between-group disparity in poverty, which may range between zero and one, the maximum value that variance can take is 0.25 and so we choose $\alpha = 4$. Then, the inequality measure that we use for our analysis is:

$$V(x) = \frac{4}{t}\sum_{i=1}^{n}[x_i - \mu(x)]^2. \tag{4.2}$$

5. Inequality Decomposition in Poverty

We now explore the possible and meaningful decomposition formulation of the inequality measure.[21] To recall, the censored deprivation score vector c has $m \geq 2$ mutually exclusive and collectively exhaustive population subgroups, where the population share of subgroup ℓ to total

[19] Note that the unbiased sample estimate for variance is $\sum_{i=1}^{t}[x_i - \mu(x)]^2/(t-1)$, but this formulation does not satisfy population replication invariance.
[20] Alkire and Foster 2013 discuss the construction of an 'attainment' matrix in more detail.
[21] In this paper, we obtain the decomposition results focusing on the censored deprivation score vector to show their relevance to a dual cutoff approach to poverty measurement. One may in theory, however, apply these decomposition expressions across non-censored deprivation or attainment score vectors, where one may be interested in dividing the population using a vector of deprivation or attainment cutoffs but then study inequality decomposition across these two groups.

population is denoted by ν^ℓ and the share of poor in subgroup ℓ to total poor population by θ^ℓ. An immediate inequality decomposition that can be made is breaking down the overall inequality among the poor into a within-group and a between-group component. The following function decomposes the overall inequality among the poor, whose deprivation scores are summarized by vector a:

$$V(a) = \left[\sum_{\ell=1}^{m} \theta^\ell V(a^\ell)\right] + \left[V(\bar{\mu}_a; \bar{q})\right]. \tag{5.1}$$

Total Within-group Between-group

Thus the term $V(a)$ captures *inequality among the poor*. The first component in (5.1) captures the total within-group inequality among the poor; whereas, the second component captures the total between-group inequality or inequality across subgroup intensities. Following the definition in the previous section, the between-group term is $V(\bar{\mu}_a; \bar{q}) = V(\mu(a^1)1^{q^1}, \ldots, \mu(a^m)1^{q^m})$ and 1^{q^ℓ} is the q^ℓ-dimensional vector of ones for all $\ell = 1, \ldots, m$, which can be expressed as:

$$V(\bar{\mu}_a; \bar{q}) = 4\sum_{\ell=1}^{m} \theta^\ell \left[\mu(a^\ell) - \mu(a)\right]^2. \tag{5.2}$$

As we have argued in Section 3, merely looking at the between-group inequality does not provide the complete picture about disparity across subgroups. We have shown with an example that disparity in poverty may increase even when both within-group inequality and between-group inequality among the poor may have gone down. One way of incorporating the total within-group inequality among the poor and disparity across subgroup poverty is through applying the inequality measure to the censored deprivation score vector c. In fact, all counting poverty measures are based on the entire deprivation score vector c. For example, the adjusted headcount ratio poverty measure proposed by Alkire and Foster (2011) is obtained by taking an average across all elements of this vector. The inequality-adjusted poverty measures are also based on the deprivation score vector c presented in (3.1) and (3.2).

The inequality measure V applied on the entire censored deprivation score vector c can also be decomposed into a within-group and a between-group component as:

$$V(c) = \left[\sum_{\ell=1}^{m} \nu^{\ell} V(c^{\ell})\right] + \left[V(\bar{\mu}_c; \bar{n})\right]. \tag{5.3}$$

The function $V(c)$ captures *inequality in deprivation scores across the society*, but assuming that the non-poor are assigned a deprivation score equal to zero. The first term on the right-hand side of equation (5.3) is the population-weighted inequality across the deprivation scores within subgroups and the second term captures inequality in averages of all deprivation scores across m subgroups. Note that $V(\bar{\mu}_c; \bar{n}) = V(\mu(c^1)1^{n^1}, \ldots, \mu(c^m)1^{n^m})$ and 1^{n^ℓ} is the n^ℓ-dimensional vector of ones for all $\ell = 1, \ldots, m$, where $\mu(c^\ell)$ is the *adjusted headcount ratio* poverty measure of subgroup ℓ and so the second term captures disparity in poverty across subgroups. As in (5.2), the between-group tem can be expressed as:

$$V(\bar{\mu}_c; \bar{n}) = 4 \sum_{\ell=1}^{m} \nu^{\ell} \left[\mu(c^{\ell}) - \mu(c)\right]^2. \tag{5.4}$$

The ℓ^{th} within-group inequality component $V(c^\ell)$ captures inequality across all elements in c^ℓ and so it does not capture inequality only across the poor, which is important in order to understand whether a poverty alleviation policy has been equitable across the poor. To reflect inequality across the poor, the population within each subgroup can be divided into two further subgroups: the poor and the non-poor. The deprivation scores of all the poor in subgroup ℓ are summarized by a^ℓ. Note that the average deprivation score among the poor, $\mu(a^\ell)$, is referred to as the intensity of poverty by Alkire and Foster (2011). The average deprivation score among the non-poor, however, is zero by the poverty focus property and hence, the term $V(c^\ell)$ for each subgroup ℓ is further decomposed as:

$$V(c^{\ell}) = \left[\frac{q^{\ell}}{nn^{\ell}} V(a^{\ell})\right] + \left[V(\mu(a^{\ell}), 0)\right] = \left[H^{\ell} V(a^{\ell})\right] + \left[V(\mu(a^{\ell}), 0)\right]. \tag{5.5}$$

The first term on the right-hand side of (5.5) is the within-group inequality among the poor, which in this case is the headcount ratio or the share of the population poor in that subgroup (H^ℓ) times its within-group inequality among the poor ($V(a^\ell)$). Replacing the expression of $V(c^\ell)$ from (5.5) in (5.3), we obtain:

$$V(c) = H\sum_{\ell=1}^{m}\theta^{\ell}V(a^{\ell}) + V(\overline{\mu}_c;\overline{n}) + \sum_{\ell=1}^{m}\nu^{\ell}V(\mu(a^{\ell}),0). \quad (5.6)$$

Thus, the inequality measure $V(c)$, which measures inequality in deprivation scores across the society, can be broken down into three components. The first is the *total within-group inequality among the poor* ($\sum_{\ell=1}^{m}\theta^{\ell}V(a^{\ell})$) *times the share of the poor in the society* (H). The second component captures *inequality or disparity between the subgroups' adjusted headcount ratios* ($V(\overline{\mu}_c;\overline{n})$) as defined in (5.4). The third component represents the population-weighted average *inequality between the poor and the non-poor* in population subgroups, where $V(\mu(a^{\ell}),0)$ captures inequality between the average deprivation-score among the poor and the average deprivation-score among the non-poor, which is zero. Thus, $V(\mu(a^{\ell}),0) = 4[H^{\ell}[\mu(a^{\ell}) - \mu(c^{\ell})]^2 + (1-H^{\ell})[\mu(c^{\ell})]^2]$.

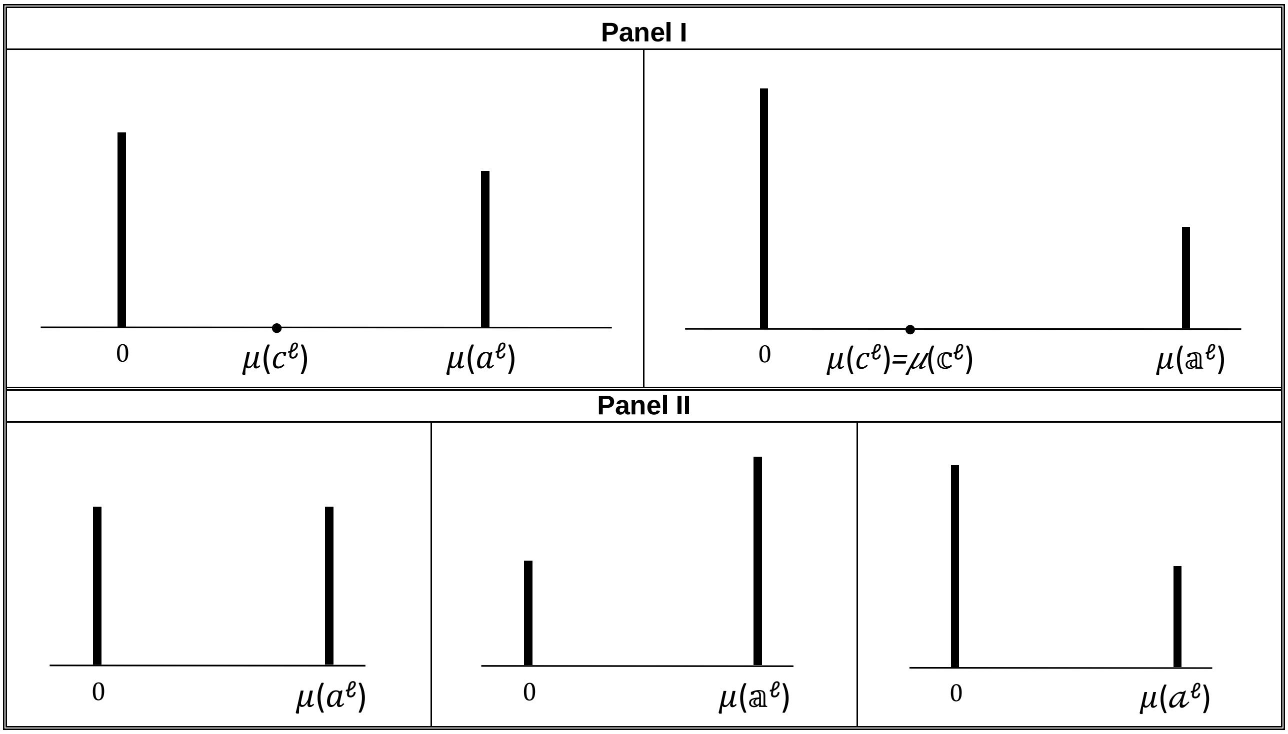
The first two components in (5.6) thus have clear interpretation and direct policy interest, but the interpretation of the third term, which captures inequality between poor and non-poor, is not straightforward. Expressing the term $V(\mu(a^{\ell}),0)$, which denotes the inequality between the average deprivation scores of the poor and the non-poor within subgroup ℓ, in two different ways can provide useful intuition. One way of expressing $V(\mu(a^{\ell}),0)$ is:

$$V(\mu(a^{\ell}),0) = 4\mu(c^{\ell})[\mu(a^{\ell})(1-H^{\ell})]. \quad (5.7)$$

Note that the population-weighted average of the average deprivation scores among the poor and the non-poor is $\mu(c^{\ell})$, i.e., $\mu(c^{\ell}) = H^{\ell}\times\mu(a^{\ell}) + (1-H^{\ell})\times 0$, where $\mu(c^{\ell})$ is the adjusted headcount ratio of subgroup ℓ. The same adjusted headcount ratio,, however, can be obtained by different combinations of H^{ℓ} or headcount ratio and $\mu(a^{\ell})$ or the intensity. If the same adjusted headcount ratio is obtained by a lower headcount ratio in combination with a higher intensity among the poor, then this term $V(\mu(a^{\ell}),0)$ is larger. In other words, in this case, $\mu(c^{\ell})$ is penalized by multiplying by both $\mu(a^{\ell})$ and $(1-H^{\ell})$. This is explained in Panel I of Figure 1. Suppose there are two distributions of deprivation scores c^{ℓ} and $\mathbb{c}^{\ell}$, such that $\mu(c^{\ell}) = \mu(\mathbb{c}^{\ell})$ but $\mu(\mathbb{a}^{\ell}) > \mu(a^{\ell})$ and so $\mathbb{H}^{\ell} < H^{\ell}$. Thus, $V(\mu(\mathbb{a}^{\ell}),0) > V(\mu(a^{\ell}),0)$.[22]

[22] We denote the headcount ratio of corresponding to distribution $\mathbb{c}$ by $\mathbb{H}$ and the vector of deprivation scores of the poor by $\mathbb{a}$.

Figure 1: Graphical Representation for Inequality between the Poor and the Non-poor



Another way $V(\mu(a^\ell),0)$ can be expressed as:

$$V(\mu(a^\ell),0) = 4\left(\mu(a^\ell)\right)^2 [H^\ell(1-H^\ell)]. \quad (5.8)$$

For a given value of the average deprivation scores $\mu(a^\ell)$ or intensity, the term $V(\mu(a^\ell),0)$ is maximized when $H = 0.5$. This means that inequality is maximized for a given $\mu(a^\ell)$ when there are two equal sized groups of poor and non-poor. This is explained in Panel II of Figure 1. Consider three distributions of deprivation scores c^ℓ, $\mathbb{c}^\ell$ and $\mathscr{c}^\ell$ such that $\mu(a^\ell) = \mu(\mathbb{a}^\ell) = \mu(\mathscr{a}^\ell)$, $H^\ell = 1/2$, and $\mathbb{H}^\ell = (1-\mathcal{H}^\ell)$, but $\mathbb{H}^\ell > H^\ell > \mathcal{H}^\ell$.[23] Thus, $V(\mu(a^\ell),0)$ is larger than both $V(\mu(\mathbb{a}^\ell),0)$ and $V(\mu(\mathscr{a}^\ell),0)$. However, how to compare the same between $\mathbb{c}^\ell$ and $\mathscr{c}^\ell$? The primary difference between $\mathbb{c}^\ell$ and $\mathscr{c}^\ell$ is that the population share has been swapped between the two groups so that the percentage of poor in one group is equal to the percentage of non-poor in other and vice versa with the same average deprivation scores among the poor. These two societies are considered to

[23] We denote the headcount ratios of corresponding to distributions $\mathbb{c}$ and $\mathscr{c}$ by $\mathbb{H}$ and $\mathcal{H}$, and the vectors of deprivation scores of the poor by $\mathbb{a}$ and $\mathscr{a}$, respectively.

have same inequality between the groups of poor and the non-poor. Thus, $V(\mu(\mathbb{a}^\ell),0) = V(\mu(a^\ell),0)$.

Although the second component in (5.6) has an intuitive interpretation, the first and the third terms are of the most policy interest: the overall within-group inequality among the poor and the disparity across subgroup poverty. Therefore, one may just be interested in focusing on these two terms controlling for the second term. We denote the summation of the two terms by:

$$\mathbb{V}(c) = H\sum_{\ell=1}^{m} \theta^\ell V(a^\ell) + V(\overline{\mu}_c;\overline{n}). \tag{5.9}$$

Note that $\mathbb{V}(c)$ is not invariant to different subgroup classifications of the same population. Technically, for a particular subgroup classification, $\mathbb{V}(c)$ can also be obtained by applying measure V to a transformed deprivation score vector $\hat{c}$ such that $\hat{c}_i^\ell = \mu(c^\ell)$ if $c_i^\ell = 0$ and $\hat{c}_i^\ell = c_i - [\mu(a^\ell) - \mu(c^\ell)]$, otherwise, for all $\ell = 1, \ldots, m$. In other words, the average of the deprivation score $\mu(c^\ell)$ is assigned to those who are not poor and the score $\mu(a^\ell) - \mu(c^\ell)$ is taken away from the deprivation score of those who are poor. Note that as a result, $\mu(c^\ell) = \mu(\hat{c}^\ell)$, $V(a^\ell) = V(\hat{a}^\ell)$, but $V(\mu(a^\ell),0) = 0\ \forall \ell$ and so, $\mathbb{V}(c) = V(\hat{c})$. Thus, if there are two hypothetical societies like c and $\hat{c}$, the total within-group inequality among the poor and the between-group disparity in poverty would be identical across these two societies. The decomposition expression in (5.9) allows us to understand how the contribution of each of these two terms has changed over time or vary across different countries.

We should also point out that in theory there is another pathway, although less intuitive, to decompose the overall societal inequality across the censored deprivation scores $V(c)$. Note that in (5.6), we first divided the entire population into population subgroups and then each subgroup across poor and non-poor. In the second decomposition, we can divide the population first across poor and non-poor, and then divide each of these two groups into m population subgroups. By decomposing the population across poor and non-poor, we obtain,

$$V(c) = HV(a) + V(\mu(a),0). \tag{5.10}$$

The second term $V(\mu(a),0)$ captures the overall inequality between the poor and the non-poor. The first term is the total within-group inequality, which is merely equal to the within-group inequality among the poor times the population share of the poor (the headcount ratio). The term $V(a)$ can be further decomposed into total within-group and between-group inequalities across subgroups as:

$$V(a) = \left[\sum_{\ell=1}^{m} \theta^{\ell} V(a^{\ell})\right] + \left[V(\overline{\mu}_a;\overline{q})\right]. \tag{5.11}$$

The first term $\sum_{\ell=1}^{m} \theta^{\ell} V(a^{\ell})$ denotes the total within-group inequality among the poor and the second term $V(\overline{\mu}_a;\overline{q})$ as defined in (5.2) captures the between-group inequality among the poor. Substituting (5.11) into (5.10) we obtain:

$$V(c) = H\left[\sum_{\ell=1}^{m} \theta^{\ell} V(a^{\ell})\right] + H\left[V(\overline{\mu}_a;\overline{q})\right] + V(\mu(a),0). \tag{5.12}$$

The right-hand side of (5.12) also has three terms. The first term captures the total within-group inequality times the headcount ratio. The second term is the between-group inequality among the poor, as defined in (5.2), times their population share or the headcount ratio. The final term assesses the inequality between the average deprivation share among the poor and that of the non-poor.

Comparing the decompositions of $V(c)$ in (5.6) and (5.12), we can see that the first term, $H[\sum_{\ell=1}^{m} \theta^{\ell} V(a^{\ell})]$, in each decomposition is the same, capturing the contribution of the total within-group inequality among the poor. The other two terms in each equation are not identical, however. In order to have a clearer understanding, we obtain vector $\bar{c}$ from c such that $c_i^{\ell} = \mu(a^{\ell})$ for all $i = 1, \dots, q^{\ell}$ and for all $\ell = 1, \dots, m$. In other words, $\bar{c}$ is a hypothetical distribution where each poor person within subgroup ℓ shares the same deprivation score $\mu(a^{\ell})$. Note that $\mu(\bar{c}) = \mu(c)$, $\mu(\bar{a}) = \mu(a)$ and $\mu(\bar{c}^{\ell}) = \mu(c^{\ell})$ and $\mu(\bar{a}^{\ell}) = \mu(a^{\ell})$ for all ℓ. As a result, the first term in each of (5.6) and (5.12) is equal to zero. We now obtain the following two expressions:

$$V(\bar{c}) = \sum_{\ell=1}^{m} v^{\ell} V\left(\mu(a^{\ell}),0\right) + V(\overline{\mu}_c;\overline{n}). \tag{5.13}$$

and

$$V(\bar{c}) = H\left[V(\overline{\mu}_a;\overline{q})\right] + V(\mu(a),0). \tag{5.14}$$

If the elements in $\bar{c}$ are first divided into m subgroups, then the between-subgroup inequality is captured by $V(\overline{\mu}_c;\overline{n})$ and the rest is captured by the within-group inequality. On the other hand, if the elements in $\bar{c}$ are first divided into two groups – poor and non-poor – then $V(\mu(a),0)$ is the between-group inequality and the rest captures the within-group inequality across the poor. The interpretation of $V(\mu(a),0)$ is same as in (5.7) and (5.8). From (5.13), we capture between-group disparity in poverty $V(\overline{\mu}_c;\overline{n})$ and from (5.14), we capture between-group inequality among the poor $V(\overline{\mu}_a;\overline{q})$.

Now, what is the relation between $V(\mu(a),0)$ and $\sum_{\ell=1}^{m}\nu^{\ell}V(\mu(a^{\ell}),0)$? Is one always larger than the other? Not necessarily. First, assume a situation where there is no inequality in average deprivation scores across subgroups, then $V(\overline{\mu}_a;\overline{q}) = 0$. From (5.13) and (5.14), we obtain $V(\mu(a),0) = \sum_{\ell=1}^{m}\nu^{\ell}V(\mu(a^{\ell}),0) + V(\overline{\mu}_c;\overline{n})$. In this case, $V(\mu(a),0) > \sum_{\ell=1}^{m}\nu^{\ell}V(\mu(a^{\ell}),0)$. On the other hand, when $\mu(c^{\ell}) = \mu(c''^{\ell'})$ for all $\ell \neq \ell'$, then $V(\overline{\mu}_c;\overline{n}) = 0$ and from (5.13) and (5.14), we obtain $H\left[V(\overline{\mu}_a;\overline{q})\right] + V(\mu(a),0) = \sum_{\ell=1}^{m}\nu^{\ell}V(\mu(a^{\ell}),0)$ and $V(\mu(a),0) < \sum_{\ell=1}^{m}\nu^{\ell}V(\mu(a^{\ell}),0)$. Finally, when there is no disparity in poverty and no inequality in average deprivation scores across subgroups, i.e., both $V(\overline{\mu}_a;\overline{q}) = 0$ and $V(\overline{\mu}_c;\overline{n}) = 0$, then $V(\mu(a),0) = \sum_{\ell=1}^{m}\nu^{\ell}V(\mu(a^{\ell}),0)$.

6. Applications using the Global Multidimensional Poverty Index

We now present two illustrations on how to analyze inequality across the poor and disparity in poverty across subgroups. In the first illustration, we present a cross-sectional analysis of inequality across the poor and sub-national disparity in multidimensional poverty using the DHS dataset for 23 countries. In the second illustration, we present an inter-temporal analysis of inequality in multidimensional poverty in the Indian context using the DHS datasets across two periods. For both examples, we use the global MPI or a variant of it as a measure of poverty.[24]

[24] The global MPI was developed by Alkire and Santos (2010) in collaboration with the United Nation Development Programme's Human Development Report Office (HDRO) and uses the adjusted headcount ratio.

An Illustration with Cross-Country Analysis

For the cross-sectional analysis, we use the DHS of 23 countries, which are a subset of 104 countries whose MPI was reported in the 2013 *Human Development Report*. The survey years range between 2007 and 2011 and use all ten indicators reported in Table 1 for measuring poverty.[25] In order to preserve strict comparability across countries, we have chosen not to use surveys other than DHS nor to include countries that have Demographic Health Surveys but with fewer than ten indicators. The weight vector w is presented in the second column and the deprivation cutoff vector z is summarized in the third column of the table. A person is identified as poor if the person's deprivation score is equal or higher than 1/3. The global MPI can be expressed as a product of the *incidence* or multidimensional headcount ratio and the *intensity* or average deprivation share among the poor.

Table 1: Indicators, Weights and Deprivation Cutoffs of the Global MPI

Indicator	Weight	Deprivation Cutoff
Schooling	1/6	No household member has completed five years of schooling
Attendance	1/6	Any school-aged child in the household is not attending school up to class 8
Nutrition	1/6	Any woman or child in the household with nutritional information is undernourished
Mortality	1/6	Any child has passed away in the household
Electricity	1/18	The household has no electricity
Sanitation	1/18	The household's sanitation facility is not improved or it is shared with other households
Water	1/18	The household does not have access to safe drinking water or safe water is more than a 30-minute walk (round trip)
Flooring material	1/18	The household has a dirt, sand or dung floor
Cooking fuel	1/18	The household cooks with dung, wood or charcoal
Assets	1/18	The household does not own more than one of: radio, telephone, TV, bike, motorbike or refrigerator; and does not own a car or truck

Source: Alkire, Roche, Santos, and Seth (2011).

Table 2 analyzes inequality among the poor and disparity across sub-national MPIs for 23 countries, which are ranked by their MPI values reported in the fourth column. The third and the fifth columns present the incidence and the intensity of poverty, respectively. The sixth column reports the overall inequality among the poor, which can be broken down into two components: the total within-group inequality and the between-group inequality among the poor. The total within-group inequality, presented in the seventh column, is the population-weighted average of sub-national within-group inequalities. Notice that the between-group inequality among the poor, reported in the

[25] The surveys need to be representative at the sub-national level and satisfy a few other criteria reported in Alkire, Roche and Seth (2011).

eighth column, is the inequality across sub-national intensities of poverty and not the disparity in sub-national MPIs, which are presented in the final column of the table.

Table 2: Inequality across the Poor and Between-group Disparity

1	2	3	4	5	6	7	8	9
Country	**Year**	**Incidence**	**MPI**	**Intensity**	**Inequality (Poor)**	**Total Within-Group**	**Between Intensity**	**Between MPI**
Jordan	2009	2.4%	0.008	34.4%	0.005	0.005	0.000	0.000
Dominican Republic	2007	4.6%	0.018	39.4%	0.025	0.022	0.002	0.001
Colombia	2010	5.4%	0.022	40.9%	0.041	0.037	0.003	0.001
Guyana	2009	7.7%	0.030	39.2%	0.023	0.021	0.003	0.004
Bolivia	2008	20.5%	0.089	43.7%	0.044	0.042	0.002	0.006
Ghana	2008	31.2%	0.144	46.2%	0.066	0.059	0.006	0.038
Sao Tome and Principe	2009	34.5%	0.154	44.7%	0.053	0.052	0.000	0.006
Lesotho	2009	35.3%	0.156	44.1%	0.042	0.040	0.001	0.014
Zimbabwe	2011	39.1%	0.172	44.0%	0.045	0.044	0.001	0.021
Cambodia	2010	45.9%	0.212	46.1%	0.068	0.064	0.004	0.023
Nepal	2011	44.2%	0.217	49.0%	0.083	0.082	0.001	0.010
Kenya	2009	47.8%	0.229	48.0%	0.076	0.070	0.005	0.025
Bangladesh	2007	57.8%	0.292	50.4%	0.071	0.070	0.001	0.003
Nigeria	2008	54.1%	0.310	57.3%	0.135	0.111	0.024	0.128
Zambia	2007	64.2%	0.328	51.2%	0.085	0.080	0.005	0.052
Malawi	2010	66.7%	0.334	50.1%	0.074	0.073	0.001	0.002
Rwanda	2010	69.0%	0.350	50.8%	0.078	0.077	0.001	0.017
Madagascar	2009	66.9%	0.357	53.3%	0.080	0.074	0.005	0.042
Timor Leste	2009	68.1%	0.360	52.9%	0.095	0.089	0.006	0.053
Senegal	2011	74.4%	0.439	58.9%	0.130	0.107	0.023	0.092
Sierra Leone	2008	77.0%	0.439	57.0%	0.101	0.095	0.007	0.056
Liberia	2007	83.9%	0.485	57.7%	0.106	0.095	0.011	0.049
Ethiopia	2011	87.3%	0.564	64.6%	0.129	0.126	0.002	0.039

Although the level of poverty in terms of the MPI and inequality across the poor are positively associated across 23 countries, there are certain interesting exceptions. Let us look at the comparison between Colombia and Lesotho. Colombia has a much lower level of MPI (0.022) and only 5.4 percent of the population is poor; whereas Lesotho's MPI (0.156) is more than seven times larger with 35.3 percent of the population being poor. However, when we compare these two countries' levels of inequality among the poor, these two countries are almost equally unequal. Similarly, inequality among the poor in Nepal, whose MPI value is 0.217 with 44.2 percent of the population being poor, is worse than that of Madagascar whose MPI value is 0.357 with 66.9 percent of the population being poor. Of all 23 countries, inequality among the poor is highest in Nigeria – larger than the countries with much higher poverty levels.

Next, we explore sub-national disparities within these 23 countries, which naturally may depend on the number of regions and their population sizes. The cross-country figures show empirically that merely decomposing overall inequality among the poor into a within-group component and a between-group component does not provide the entire picture on how diverse sub-national regions are in terms of poverty. Consider the comparison between Bolivia and Zimbabwe, which have similar numbers of sub-national regions. Both countries also have similar levels of overall inequality among the poor and similar sub-national disparity across intensities of poverty. In fact, Bolivia appears to be marginally more unequal in terms of sub-national intensities. This does not mean, however, that sub-national regions are more unequal in terms of the level of MPIs. We find that sub-national disparity in Zimbabwe is more three times larger than that in Bolivia. Similarly, Zambia and Madagascar appear to have similar levels of disparity across sub-national intensities. However, disparity across sub-national MPIs is larger in Zambia, despite the number of sub-national regions in Madagascar being more than double that of Zambia. In fact, we do not find any statistical relationship between the number of sub-national regions and disparity across sub-national regions.

An Illustration with Inter-temporal Analysis in India

Our second illustration presents an inter-temporal analysis of inequality among the poor and disparity in MPIs across castes in India. Various studies on multidimensional poverty in India have shown recently that poverty has gone down between the 1990s and early 2000s (Jayaraj and Subramanian 2009, Alkire and Seth 2013, Mishra and Ray 2013). Both Jayaraj and Subramanian (2009) and Misha and Ray (2013) use the measure proposed by Chakravarty and D'Ambrosio (2006) in (3.2). These studies find that the national reduction in multidimensional poverty has not been accompanied by uniform reductions across different population subgroups. When inequality-sensitive poverty measures are used to capture inequality in deprivation scores across the poor, then Jayaraj and Subramanian find that the ranking of population subgroups changed. The rank of a subgroup deteriorated if the reduction in poverty had not been equitable across the poor. Alkire and Seth found that the population subgroups (be they states, castes or religious groups) that were poorest in 1999 reduced poverty the least in the next seven years.

None of these studies, however, explicitly explored if overall inequality among the poor or the disparity in poverty across populations has gone down. In this second illustration, we use the slightly modified version of the global MPI.[26] We use two rounds of nationally representative Demographic Health Surveys (DHS) for years 1998/99 and 2005/06. The DHS for year 2005/06 has also been used by Alkire and Santos (2010) to compute the global MPI for India.[27] It is evident from Table 3 that poverty, in terms of MPI, as well as the intensity went down between 1999 and 2006. The national MPI went down from 0.300 in 1999 to 0.251 in 2006; whereas, the intensity went down from 52.9 percent to 51.7 percent.

Table 3: Inequality Decomposition across the Poor (Castes)

	1999				2006			
	Share of Poor	M_0	A	Inequality among the Poor	Share of Poor	M_0	A	Inequality among the Poor
India	100%	0.300	52.9%	0.100	100%	0.251	51.7%	0.097
Scheduled Castes	22.1%	0.378	55.0%	0.107	22.9%	0.307	52.6%	0.098
Scheduled Tribes	12.6%	0.458	57.0%	0.110	12.9%	0.417	56.3%	0.115
OBCs	33.3%	0.301	52.1%	0.095	42.1%	0.258	50.8%	0.090
General	32.0%	0.229	50.6%	0.089	22.0%	0.164	49.7%	0.092
Total Within-group				0.098				0.096
Within group Contribution				98.0%				98.3%
Between-group				0.0020				0.0017
Between-group Contribution				2.0%				1.7%

We further delve into whether the reduction in inequality among the poor has been obtained by a reduction in disparity in poverty across subgroups. In order to do so, we look at the inequality decomposition using equation (5.9) with two components: the total within-group inequality among the poor times their population share (the headcount ratio) and disparity in poverty (MPI) across castes. Table 4 shows that the disparity in MPIs across castes has not gone down, unlike a large reduction in the proportion of poor and a reduction in the inequality among the poor. Therefore, the contribution of the between-group disparity in poverty, based on the decomposition formulation in (5.9), has clearly increased.

Table 4: Inequality across the Poor and Disparity in Poverty across Castes

Year	H	Within-Group (Poor)	Within-Group $\times H$	Disparity in MPI
1999	56.8%	0.100	0.057	0.021
Contribution			*72.5%*	*27.5%*
2006	48.5%	0.097	0.047	0.021
Contribution			*68.7%*	*31.3%*

[26] The modifications were made in the definition of four indicators – attendance, mortality, nutrition, and flooring material – in order to preserve strictly comparability. However, it uses the same set of deprivation cut-offs and weights. See Alkire and Seth (2013).

[27] Given that samples in DHS2 covering 80.5 percent of the population were collected in 1999, and in DHS3 covering 92.6 percent of the population were collected in 2006, we consider 1999 and 2006 as the reference years for the surveys.

To illustrate the inequality decomposition, we divide the population first by four caste categories: Scheduled Castes (SC), Scheduled Tribes (ST), Other Backward Classes (OBC), and General. The M_0 and A, although not uniformly, went down for each of the four subgroups. The picture is not as encouraging when we look at inequality across the poor. At the national level, the inequality across the poor went down from 0.100 to 0.097, but inequality across the poor did not go down within all subgroups. It went down within scheduled castes and OBCs, but not within scheduled tribes and the general category. The bottom half of Table 3 reports the decomposition of overall inequality across the poor into total within-group and between-group components following equation (5.1). There has been a reduction in both within-group inequality and between-group inequality, but the contribution of the between-group component has decreased slightly. Thus, Table 1 shows that although both the overall within-group inequality and the between-group inequality have gone down, inequality across the poor within the poorest subgroup has gone up.

Table 5: Inequality across the Poor and Disparity in Poverty for Different population Subgroups

Subgroups	1999 Total Within-group	1999 Between-group (Poor)	2006 Total Within-group	2006 Between-group (Poor)
Caste	0.098	0.0020	0.096	0.0017
contribution	98.0%	2.0%	98.3%	1.7%
Religion	0.099	0.0005	0.096	0.0009
contribution	99.5%	0.5%	99.0%	1.0%
States	0.094	0.0053	0.091	0.0065
contribution	94.7%	5.3%	93.3%	6.7%
Head's Education	0.090	0.0091	0.089	0.0084
contribution	90.8%	9.2%	91.4%	8.6%
Household Size	0.098	0.0013	0.095	0.0020
contribution	98.7%	1.3%	98.0%	2.0%
	Within-group $\times H$	**Between M_0**	**Within-group $\times H$**	**Between M_0**
Caste	0.055	0.021	0.046	0.021
contribution	72.1%	27.9%	68.3%	31.7%
Religion	0.056	0.004	0.047	0.004
contribution	93.0%	7.0%	91.5%	8.5%
States	0.054	0.031	0.044	0.041
contribution	63.0%	37.0%	51.9%	48.1%
Head's Education	0.051	0.070	0.043	0.064
contribution	42.3%	57.7%	40.1%	59.9%
Household Size	0.056	0.005	0.046	0.008
contribution	92.3%	7.7%	84.9%	15.1%

We extend these two inequality decomposition frameworks to other population subgroups grouped by religion, state, household head's education, and household size. Although, the total within-group inequality, albeit to a greater or smaller extent, has gone down for all subgroups, between-group inequality has not. This is evident from the upper half of Table 5. Besides castes, the between-group inequality among the poor went down across education categories of household heads.

In the lower half of the table we report the second kind of inequality decomposition, analyzing disparity in MPIs across different subgroup classifications. The disparity in MPIs across religious groups appears to be much lower compared to that of across castes, but the disparity across religious groups did not go down between 1999 and 2006. Strikingly, disparity in MPIs across states clearly increased between 1999 and 2006, which supports the finding of other studies.

7. Concluding Remarks

There have been recent developments in both theory and practice in the measurement of multidimensional poverty based on counting approaches. The categorical or binary nature of dimensions and the fact that the counting measures of poverty are based on direct deprivations make the use the counting approaches more amicable. Even in counting approaches, however, it is important that all three 'I's of poverty – incidence, intensity, and inequality among the poor – are incorporated. If the object of a policy maker is to reduce only the incidence of poverty, then only the marginally poor people would be driven out of poverty ignoring the poorest of the poor completely. If the objective is to reduce both the incidence and intensity of poverty, then while the policy maker has no reason to focus on the marginally poor instead of the poorest of the poor, the policy maker has no strong incentive to assist the poorest of the poor either. It is only when the consideration of inequality is brought to the table that a policy maker has greater incentives to assist the poorest of the poor.

The most common approach to incorporating inequality into poverty measurement has been to adjust a poverty measure in such a way that the measure is sensitive to distributional changes among the poor. This approach, however, has certain limitations. First, the inequality-adjusted poverty measures may lack intuitive interpretations. Second is that it does not allow the possibility of breaking down a counting measure by dimensions in order to understand the contribution of each dimension to overall poverty, as shown by Alkire and Foster (2011). Third, the integrated measures often involve selecting a particular value of an inequality-aversion parameter, which often becomes a subject of debate. Fourth, a poverty measure that combines incidence, intensity and inequality into one may not make transparent the relative weight that the measure places on each of these aspects. Finally, this approach does not give a clear picture about the disparity in poverty between population subgroups.

In this paper, thus, we suggest the use of a separate inequality measure to capture inequality among the poor and disparity across population subgroups. Then the question is, which inequality measure is to be used? Our choice of inequality measure is determined by certain desirable properties, in addition to the standard properties of inequality measures, that we require it to satisfy. We first require that the inequality measure is additively decomposable so that it can be expressed as a sum of total within-group inequality and between-group inequality. Moreover, the total within-group inequality should not change as long as the inequality within each population subgroup does not change. Second, we require that inequality across deprivation scores to remain unchanged when the deprivation score increases by the same amount rather than by the same factor. In other words, we require that inequality should be perceived through absolute distances between deprivation scores rather than relative levels. The only inequality measure that satisfies our requirements is a positive multiple of variance.

We provide two illustrations to show the application of the inequality measure to analyze inequality among the poor and disparity across population subgroups. In our first illustration, we use the DHS of 23 developing countries around the world. Although we find that the level of inequality among the poor and the level of poverty in terms of the MPI are positively associated across these countries, there were several exceptions. For example, Colombia's MPI was less than one-seventh of Lesotho's but with the same level of inequality among the poor. Similarly, the MPI of Madagascar was more than 50 percent larger than that of Nepal, but inequality among the poor in Nepal was much higher.

Moreover, we show that looking at the sub-national inequality in intensity of poverty is not enough. It conceals the high level of disparity that exists across sub-national MPIs. For example, Bolivia and Zimbabwe have similar levels of between-group inequality across the poor, but sub-national disparity in Zimbabwe is more than three times worse than that in Bolivia. Our illustration on India also shows a similar result but through an inter-temporal analysis. When decomposed across castes, we find that although the MPI and its components – the incidence and the intensity of poverty – went down for each subgroup, the inequality among the poor became higher for the poorest subgroup – the Schedule Tribes.

So what is the value added of using the proposed inequality measure? First, the inequality measure adds valuable information to the adjusted headcount ratio proposed by Alkire and Foster (2011), which has been adopted by international organizations and country governments without sacrificing the dimensional breakdown property that allows understanding the contribution of each dimension to overall poverty. Second, the inequality measure does not involve any inequality-aversion parameter, whose selection may cause wide disagreement among policy makers. Thus, the additive decomposability property allows overall poverty to be decomposed into within-group and between-group components. Although the contribution of within-group and between-group components to overall poverty is subject to debate (Kanbur 2006), it is not difficult to argue that understanding their change over time and across countries may provide valuable information. Finally, the inequality measure reflects the same level of inequality whether the poor are identified in an achievement/attainment space or in a deprivation space.

At the same time, this research agenda raises a number of interesting questions regarding the dynamics of inequality among the poor. For example, in situations in which the intensity of poverty is exceedingly high – approaching 100 percent – then progress in reducing the intensity of poverty is likely to involve a temporary increase in inequality among the poor as the intensity of deprivations for some are reduced. Using the proposed inequality measure 'variance' alongside the adjusted headcount ratio will enable researchers to identify various patterns of progression of inequality among the poor in different countries and to link these to other patterns such as conflict, migration, and local or regional activities. It will also be interesting to compare multidimensional 'variance' with income inequality among the income poor in order to assess whether diverse kinds of inequality among the poor converge or diverge.

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8. Appendix

The proof has two parts: sufficiency and necessity. For the first, it is straightforward to show that the inequality measure $I(x) = \frac{\alpha}{t}\sum_{i=1}^{n}[x_i - \mu(x)]^2$ satisfies anonymity, the transfer principle, replication invariance, subgroup decomposability, within-group mean independence, and translation invariance.

Let us show that this is the only inequality measure that satisfies the five properties. If an inequality measure satisfies *population share weighted decomposability*, then it satisfies the *decomposability* property introduced by Shorrocks (1984), which requires that the overall inequality measure should be expressed as a function of subgroup means, subgroup population sizes, and subgroup inequality levels.

Now, the theorem in Bosmans and Cowell (2010) shows that the only class of inequality measures that satisfies anonymity, the transfer principle, replication invariance, decomposability and translation invariance is:

$$f\big(I(c)\big) = \begin{cases} \frac{1}{t}\sum_{i=1}^{t}\{\exp(\gamma[x_i - \mu(x)]) - 1\} & \text{if } \gamma \neq 0 \\ \frac{1}{t}\sum_{i=1}^{t}[x_i - \mu(x)]^2 & \text{if } \gamma = 0 \end{cases};$$

where γ a real number and $f : \mathbb{R} \to \mathbb{R}$ is a continuous and strictly increasing function, with $f(0) = 0$.

The additive decomposability property along with $f(0) = 0$ and the functional restriction on f requires $f(x) = \alpha x$ for any $\alpha > 0$. Thus,

$$I(c) = \begin{cases} \frac{\alpha}{t}\sum_{i=1}^{t}\{\exp(\gamma[x_i - \mu(x)]) - 1\} & \text{if } \gamma \neq 0 \\ \frac{\alpha}{t}\sum_{i=1}^{t}[x_i - \mu(x)]^2 & \text{if } \gamma = 0 \end{cases};$$

Next we need to check which of these measures satisfies the property of within-group mean independence. From our previous notation, recall that there are $m \geq 2$ mutually exclusive and collectively exhaustive subgroups. The deprivation score vector and the population size of any subgroup ℓ are denoted by x^ℓ and n^ℓ.

Consider $\gamma \neq 0$. The corresponding measures can be decomposed into within-group inequalities and between group inequality components as:

$$I(x) = \sum_{\ell=1}^{m} \frac{t^\ell \exp\left[\gamma\mu\left(x^\ell\right)\right]}{t \exp[\gamma\mu(x)]} I(x^\ell) + I(\bar{\mu}_x; \bar{t}).$$

The indices with $\gamma \neq 0$ do not satisfy the property of within-group mean independence, which can be shown as follows. Suppose for any two deprivation score vectors x and $\mathscr{x}$, $n^\ell = \mathscr{n}^\ell$ and $I\left(x^\ell\right) = I(\mathscr{x}^\ell)$ for all $\ell = 1, \ldots m$, but $\mu\left(x^{\ell'}\right) \neq \mu\left(\mathscr{x}^{\ell'}\right)$ and $\mu\left(x^\ell\right) = \mu\left(\mathscr{x}^\ell\right)$ for all $\ell \neq \ell'$. Clearly, as for $\gamma \neq 0$, $I_W(x) \neq I_W(\mathscr{x})$.

Thus, the only class of inequality indices that satisfies all the requires property is $I(x) = \frac{\alpha}{t}\sum_{i=1}^{t}[x_i - \mu(x)]^2$.